Lecture 3

Performance Analysis

Performance Metrics, Postulates

Ceng505 *Parallel Computing* at October 11, 2010

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Contents

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- The scalability of parallel systems.

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  - to the time taken by a parallel system consisting of n processors to solve the same problem.

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- This equation indicates that the **relative values of t_s and t_c affect the achieved speed-up factor**.

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- The resulting measure is called the efficiency, E.
- The efficiency is a measure of the **speed-up achieved per processor**.
- According to the simple equal duration model, the efficiency E is equal to 1, if the communication overhead is ignored.
- However if the communication overhead is taken into consideration, the efficiency can be expressed as

$$E = \frac{1}{1 + n * t_c/t_s} \tag{3}$$

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- However, real algorithms contain some (serial) parts that cannot be divided among processors.
- These (serial) parts must be executed on a single processor.

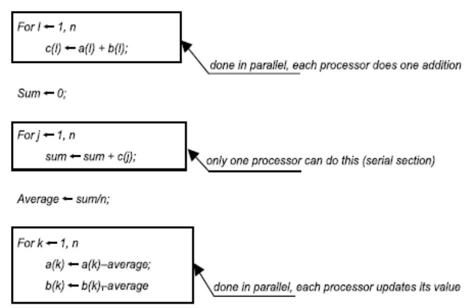

Figure: Example program segments.

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  - The last program segment can be done in parallel. Each processor can update its elements of *a* and *b*.

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- If the task (program) is completely serial, that is, $f = 1$, then no speed-up can be achieved regardless of the number of processors used.

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- The maximum speed-up factor under such conditions is given by

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- As the number of processors increases, it may become difficult to use those processors efficiently.

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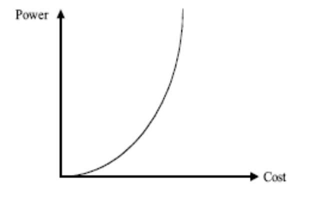

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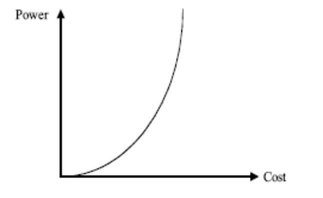

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- It was as early as the late 1940s that H. Grosch studied the relationship between the power of a computer system, P, and its cost, C.
- He postulated that $P = K * C^s$, where s and K are positive constants. Grosch postulated further that the value of s would be close to 2.

Postulates - Grosch's Law II

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- Alternatively, to do a computation twice as cheaply, one has to do it four times slower.
- With the advances in computing, it is easy to see that Grosch's law is overturned, and it is possible to build faster and less expensive computers over time.

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- This is clearly in contradiction to Amdahl's law.

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- Assume further that the solution of such a problem is performed on a binary tree architecture consisting of n nodes (processors).
- Initially, the root node stores the vector $X(m)$ and the matrix $A(m \times m)$ is distributed row-wise among the n processors such that the maximum number of rows in any processor is $m/n + 1$.

Postulates - Amdahl's Law III

A simple algorithm to perform such computation consists of the following three steps:

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- Therefore, the fraction of computation that is indivisible

$$f(m) = \frac{(O(m) + O(m * logn))}{O(m^2)} = O(\frac{(1 + logn)}{m})$$

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- The Gustafson-Barsis law makes use of this argument.

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- They found that to a first approximation the parallel part of the program, not the serial part, scales up with the problem size.

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- This shows clearly that it is possible, even easier, to achieve efficient parallel performance than is implied by Amdahl's speed-up formula.
- Speed-up should be measured by scaling the problem to the number of processors, not by fixing the problem size.