İzmir Kâtip Çelebi University
Materials Science and Engineering
Mse228 Engineering Quantum Mechanics
Final Examination
June 07, 2017 15:30 – 17:30
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

DURATION: 120 minutes

◇ **Answer all the questions.**
◇ **Write the solutions explicitly and clearly. Use the physical terminology.**
◇ **You are allowed to use Formulae Sheet.**
◇ **Calculator is allowed.**
◇ **You are not allowed to use any other electronic equipment in the exam.**

Question	Grade	Out of
1A		15
1B		15
2		20
3		20
4		20
5		20
TOTAL		110

Table 6.1 Normalized Wave Functions of the Hydrogen Atom for $n = 1, 2,$ and 3*

n	l	m_l	$\Phi(\phi)$	$\Theta(\theta)$	$R(r)$	$\psi(r, \theta, \phi)$
1	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{2}{a_0^{3/2}} e^{-r/a_0}$	$\frac{1}{\sqrt{\pi}\, a_0^{3/2}} e^{-r/a_0}$
2	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2\sqrt{2}\, a_0^{3/2}}\left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$	$\frac{1}{4\sqrt{2\pi}\, a_0^{3/2}}\left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$
2	1	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2}\cos\theta$	$\frac{1}{2\sqrt{6}\, a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0}$	$\frac{1}{4\sqrt{2\pi}\, a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0}\cos\theta$
2	1	±1	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$	$\frac{\sqrt{3}}{2}\sin\theta$	$\frac{1}{2\sqrt{6}\, a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0}$	$\frac{1}{8\sqrt{\pi}\, a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0}\sin\theta\, e^{\pm i\phi}$
3	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{2}{81\sqrt{3}\, a_0^{3/2}}\left(27 - 18\frac{r}{a_0} + 2\frac{r^2}{a_0^2}\right) e^{-r/3a_0}$	$\frac{1}{81\sqrt{3\pi}\, a_0^{3/2}}\left(27 - 18\frac{r}{a_0} + 2\frac{r^2}{a_0^2}\right) e^{-r/3a_0}$
3	1	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2}\cos\theta$	$\frac{4}{81\sqrt{6}\, a_0^{3/2}}\left(6 - \frac{r}{a_0}\right)\frac{r}{a_0} e^{-r/3a_0}$	$\frac{\sqrt{2}}{81\sqrt{\pi}\, a_0^{3/2}}\left(6 - \frac{r}{a_0}\right)\frac{r}{a_0} e^{-r/3a_0}\cos\theta$
3	1	±1	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$	$\frac{\sqrt{3}}{2}\sin\theta$	$\frac{4}{81\sqrt{6}\, a_0^{3/2}}\left(6 - \frac{r}{a_0}\right)\frac{r}{a_0} e^{-r/3a_0}$	$\frac{1}{81\sqrt{\pi}\, a_0^{3/2}}\left(6 - \frac{r}{a_0}\right)\frac{r}{a_0} e^{-r/3a_0}\sin\theta\, e^{\pm i\phi}$
3	2	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{10}}{4}(3\cos^2\theta - 1)$	$\frac{4}{81\sqrt{30}\, a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}$	$\frac{1}{81\sqrt{6\pi}\, a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}(3\cos^2\theta - 1)$
3	2	±1	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$	$\frac{\sqrt{15}}{2}\sin\theta\cos\theta$	$\frac{4}{81\sqrt{30}\, a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}$	$\frac{1}{81\sqrt{\pi}\, a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}\sin\theta\cos\theta\, e^{\pm i\phi}$
3	2	±2	$\frac{1}{\sqrt{2\pi}} e^{\pm 2i\phi}$	$\frac{\sqrt{15}}{4}\sin^2\theta$	$\frac{4}{81\sqrt{30}\, a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}$	$\frac{1}{162\sqrt{\pi}\, a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}\sin^2\theta\, e^{\pm 2i\phi}$

*The quantity $a_0 = 4\pi\epsilon_0\hbar^2/me^2 = 5.292 \times 10^{-11}$ m is equal to the radius of the innermost Bohr orbit.

1. A) Show that the speed of an electron in the n^{th} Bohr orbit of hydrogen is $\alpha c/n$, where α is the fine structure constant, equal to $e^2/4\pi\epsilon_0\hbar c$. What would be the speed in a hydrogen-like atom with a nuclear charge of Ze?

B) Draw an energy-level diagram showing the lowest four levels of singly ionized helium. Show all possible transitions from the levels and label each transition with its wavelength.

2. Find the probability that a particle in a box L wide can be found between $x = 0$ and $x = L/n$ when it is in the n^{th} state.

3. Calculate the probability that the electron in the ground state of hydrogen will be found outside the first Bohr radius. Hints: Use the table at cover page and integral; $\int x^2 e^{ax} dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3} \right) e^{ax}$

4. i List the six possible sets of quantum numbers (n, l, m_l, m_s) of a 2p electron.

ii Suppose we have an atom such as carbon, which has two 2p electrons. Ignoring the Pauli principle, how many different possible combinations of quantum numbers of the two electrons are there?

iii How many of the possible combinations of part (ii) are eliminated by applying the Pauli principle?

iv Suppose carbon is in an excited state with configuration $2p^1 3p^1$. Does the Pauli principle restrict the choice of quantum numbers for the electrons? How many different sets of quantum numbers are possible for the two electrons?

5. i The ionization energy of Li is 5.4 eV and the electron affinity of chlorine is 3.61 eV. The Madelung constant for the LiCl structure is 1.748 and the distance between ions of opposite sign is 0.257 nm. On the basis of these data only, compute the cohesive energy of LiCl.

ii The observed cohesive energy of LiCl is 6.8 eV per ion pair. On the assumption that the difference between this figure and that obtained in (i) is due to the exclusion-principle repulsion, find the exponent n in the formula Br^{-n} for the potential energy arising from this source.