İzmir Kâtip Çelebi University
Materials Science and Engineering
Mse228 Engineering Quantum Mechanics
Midterm Examination
April 11, 2018 09:30 – 11:30
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

DURATION: 120 minutes

◇ **Answer all the questions.**
◇ **Write the solutions explicitly and clearly. Use the physical terminology.**
◇ **Calculator is allowed.**
◇ **You are not allowed to use any other electronic equipment in the exam.**

Question	Grade	Out of
1A		10
1B		10
2		30
3A		10
3B		10
4		20
5A		10
5B		10
TOTAL		110

1. A) The work function of a tungsten surface is 5.4 eV. When the surface is illuminated by light of wavelength 175 nm, the maximum photo-electron energy is 1.7 eV. Find Planck's constant from these data.

$$\left.\begin{aligned} \phi &= 5.4\,eV \\ \lambda &= 175\,nm \\ KE_{max} &= 1.7\,eV \end{aligned}\right\} \begin{aligned} KE_{max} &= h\nu - \phi = h\frac{c}{\lambda} - 5.4\,eV = 1.7\,eV \\ \rightsquigarrow h &= \frac{\lambda}{c}(7.1\,eV) = \frac{175\times10^{-9}\,m}{3\times10^{8}\,m}\,7.1\,eV \\ &= \frac{175\times10^{-9}\,m}{3\times10^{8}\,m}(7.1\,eV)(1.6\times10^{-19}\,J/eV) \\ &\boxed{\begin{aligned} &= 6.626\times10^{-34}\,Js \\ &= 4.14\times10^{-15}\,eV \end{aligned}} \end{aligned}$$

B) An electron has a de Broglie wavelength equal to the diameter of the hydrogen atom. What is the kinetic energy of the electron? How does this energy compare with the ground-state energy of the hydrogen atom?

Diameter of the hydrogen atom: $2a_0 = 1.06\,\mathring{A} = 10.6\times10^{-9}\,m$

$$\lambda = 2a_0 = 10.6\times10^{-9}\,m = \frac{h}{p}$$

$$p = \frac{h}{\lambda} \Rightarrow KE = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{(6.626\times10^{-34}\,J.s)^2}{2(9.1\times10^{-31}\,kg)(10.6\times10^{-9}\,m)^2}$$

$$= 2.15\times10^{-21}\,J = 0.0134\,eV$$

Ground state of hydrogen atom: $E_1 = 13.6\,eV$

$\Rightarrow$ Comparing $\dfrac{0.0134\,eV}{13.6\,eV} \sim \dfrac{1}{1000}$

2. An x-ray of wavelength 0.050 nm scatters from a gold target.

a) Can the x-ray be Compton-scattered from an electron bound by as much as 62 keV ?

b) What is the largest wavelength of scattered photon that can be observed?

c) What is the kinetic energy of the most energetic recoil electron?

i) x-ray: $\lambda = 0.050\,nm \rightsquigarrow E = h\nu = (6.626\times10^{-34}\,J\cdot s)\dfrac{3\times10^{8}\,m/s}{0.050\times10^{-9}\,m}$

(initial photon energy) $= \dfrac{hc}{\lambda} = 3.972\times10^{-15}\,J$

$= 24.825\times10^{3}\,eV = 24.8\,keV$

This is the energy of the x-ray and less than the bond energy 62 keV. NO. it can not

ii) The longest wavelength occur when $\Delta\lambda$ is a maximum or when $\theta = 180°$

$\lambda' - \lambda = \dfrac{h}{mc}(1 - \cos 180°) = \dfrac{2h}{mc}$

$\lambda' = 0.050\times10^{-9}\,m + \dfrac{2(6.626\times10^{-34}\,J\cdot s)}{(9.1\times10^{-31}\,kg)(3\times10^{8}\,m/s)}$

$= 0.050\times10^{-9}\,m + 0.00485\times10^{-9}\,m = 0.055\,nm$

$\rightsquigarrow E' = h\nu' = \dfrac{hc}{\lambda'} = \dfrac{(6.626\times10^{-34}\,J\cdot s)(3\times10^{8}\,m/s)}{(0.055\times10^{-9}\,m)}$

$= 3.63\times10^{-15}\,J = 22.65\,keV$ (final photon energy)

iii) $E - E' = KE_{electron}$

$= (24.8 - 22.65)\,keV = 2.15\,keV$

3. A) A particle of charge q and mass m is accelerated from rest through a small potential difference V.

   a) Find its de Broglie wavelength, assuming that the particle is nonrelativistic.

   b) Calculate λ if the particle is an electron and V=50 V.

charge q
mass m
from rest
V

a) $KE = PE \rightsquigarrow \frac{m^2v^2}{2m} = qV$

$\frac{1}{2}mv^2 = qV \qquad \frac{p^2}{2m} = qV$

$\lambda = ?\ \frac{h}{p} \qquad \rightarrow \lambda = \frac{h}{\sqrt{2mqV}}$ //

b) particle: e^- & $V = 50V$

$$\lambda = \frac{6.63\times10^{-34}\,J.s}{\sqrt{2(9.1\times10^{-31}\,kg)(1.6\times10^{-19}\,C)(50V)}}$$

$= 1.7\times10^{-10}\,m = \underline{\underline{17\,nm}}$

B) A proton has a kinetic energy of 1.0 MeV. If its momentum is measured with an uncertainty of 5.0%, what is the minimum uncertainty in its position?

$$KE_{proton} = \frac{m_p v^2}{2} = \frac{p^2}{2m_p} \rightsquigarrow p^2 = 2(1.67\times10^{-27}\,kg)(1\times10^{6}\,eV)(1.6\times10^{-19}\,J/eV)$$

$$\rightsquigarrow \underline{\underline{p = 2.312\times10^{-20}\,kgm/s}}$$

$$\Rightarrow \Delta p = 0.05p = \underline{\underline{1.160\times10^{-21}\,kgm/s}}$$

$$\Rightarrow \Delta x \Delta p \geq \frac{\hbar}{2} \rightsquigarrow \Delta x = \frac{(6.626\times10^{-34}\,J.s)/2\pi}{2 * 1.160\times10^{-21}\,kgm/s} = \boxed{4.56\times10^{-14}\,m}$$

4. An atom in an excited state normally remains in that state for a very short time ($\sim 10^{-8}\ s$) before emitting a photon and returning to a lower energy state. The "lifetime" of the excited state can be regarded as an uncertainty in the time Δt associated with a measurement of the energy of the state. This, in turn, implies an "energy width", namely, the corresponding energy uncertainty ΔE. Calculate

   a) the characteristic "energy width" of such a state,

   b) the uncertainty ratio of the frequency $\Delta\nu/\nu$ if the wavelength of the emitted photon is 300 nm.

i) $\Delta t = 10^{-8} s$ & $\Delta E \Delta t \geq \hbar/2$

$$\rightsquigarrow \Delta E = \frac{(6.626\times10^{-34} J.s)/2\pi}{2(10^{-8} s)} = \underline{\underline{5.3\times10^{-27} J}} = \underline{\underline{3.3\times10^{-8} eV}}$$

ii) $\lambda = \frac{c}{\nu} = 300\, nm \rightsquigarrow \nu = \frac{3\times10^{8} m}{300\times10^{-9} m/s} = 10^{15}\, 1/s = \underline{10^{15} Hz}$

$$\Delta E = h\Delta\nu \rightsquigarrow \Delta\nu = \frac{5.3\times10^{-27} J}{6.626\times10^{-34} J.s} = 8\times10^{6} Hz$$

$$\rightsquigarrow \frac{\Delta\nu}{\nu} = \frac{8\times10^{6} Hz}{1\times10^{15} Hz} = \underline{\underline{8\times10^{-9}}}$$

5. A) Show that the speed of an electron in the n^{th} Bohr orbit of hydrogen is $\alpha c/n$, where α is the fine structure constant, equal to $e^2/4\pi\epsilon_0\hbar c$. What would be the speed in a hydrogen-like atom with a nuclear charge of Ze?

a nuclear charge of Ze?

$E = pc = h\nu$ $\{c = \lambda\nu\}$ $p\lambda\nu = h\nu \to p = \frac{h}{\lambda} = mv$ | for v, $F_c = F_{coulomb}$

$m\frac{v^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2} \to v = \sqrt{\frac{e^2}{4\pi\epsilon_0 m r}}$ } Condition for orbital stability

$n\lambda = 2\pi r_n$

$\lambda = \frac{h}{mv} = \frac{h}{m}\sqrt{\frac{4\pi\epsilon_0 m r}{e^2}} \to n\frac{h}{e}\sqrt{\frac{4\pi\epsilon_0 r_n}{m}} = 2\pi r_n$ } $n^2\frac{h^2}{e^2}\frac{4\pi\epsilon_0 r_n}{m} = 4\pi^2 r_n^2$

$\to r_n = \frac{h^2\epsilon_0}{e^2\pi m} n^2 = a_0 n^2$

$v = \frac{h}{m\lambda} = \frac{h}{m}\frac{m}{h}\sqrt{\frac{e^2}{4\pi\epsilon_0 m r_n}}$

$v_n = \sqrt{\frac{e^2}{4\pi\epsilon_0 m}\frac{e^2\pi m}{h^2\epsilon_0 n^2}} = \frac{e^2}{2\epsilon_0 h n}$ & we are given $v_n = \frac{\alpha c}{n} = \frac{e^2}{4\pi\epsilon_0\hbar c}\frac{c}{n}$

✓

They are equal $v_n = \frac{e^2}{2\epsilon_0 h n}$

when the nuclear charge is Ze, we must replace e^2 with Ze^2 $\Rightarrow$ $\boxed{v_n = \frac{Ze^2}{2\epsilon_0 h n}}$

B) The electron in a hydrogen atom at rest makes a transition from the n=2 energy state to the n=1 ground state. Find the wavelength, frequency, and energy (eV) of the emitted photon.

$n=2 \rightsquigarrow n=1$

- $\frac{1}{\lambda} = \frac{-E_1}{ch}\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{-(-13.6 eV)(1.6\times10^{-19} J/eV)}{(3\times10^8)(6.626\times10^{-34} J.s)}\left(\frac{3}{4}\right)$

$\rightsquigarrow \frac{1}{\lambda} = 8.2\times10^{6} m^{-1} \rightsquigarrow \lambda = \underline{\underline{1.22\times10^{-7} m}} = \underline{\underline{122 nm}}$

- $\nu = \frac{c}{\lambda} = \frac{3\times10^8 m/s}{1.22\times10^{-7} m} = \underline{\underline{2.46\times10^{15} Hz}}$
- $E = h\nu = (6.626\times10^{-34} J.s)(2.46\times10^{15} 1/s) = \underline{\underline{1.6\times10^{-18} J}}$

$\underline{\underline{= 10.2 eV}}$