MSE228 Engineering Quantum Mechanics
Quiz 3 Duration: 30 minutes Open Book Quiz

1. The Balmer series for the hydrogen atom corresponds to electronic transitions that terminate in the state of quantum number n=2
   (a) Find the longest-wavelength photon emitted and determine its energy.
   (b) Find the shortest-wavelength photon emitted and determine its energy.

Balmer series hydrogen atom

i) longest-wavelength photon: $n=3$ (smallest frequency)

$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$ $\quad \frac{1}{\lambda} = (1.097\times10^{7}\,m^{-1})\left(\frac{1}{4} - \frac{1}{9}\right) \rightsquigarrow \lambda_{max} = 656.3\,nm$

its energy: $E = h\nu = \frac{hc}{\lambda_{max}} = 3.03\times10^{-19}\,J = 1.89\,eV$

ii) shorthest-wavelength photon, $n=\infty$ (series limit, largest frequency)

$\frac{1}{\lambda} = (1.097\times10^{7}\,m^{-1})\left(\frac{1}{4} - \frac{1}{\infty}\right) \rightsquigarrow \lambda_{min} = 364.6\,nm$

its energy: $E = \frac{hc}{\lambda_{min}} = \frac{(6.62\times10^{-34}\,J\cdot s)(3\times10^{8}\,m/s)}{364.6\times10^{-9}\,m} = 5.45\times10^{-19}\,J = 3.40\,eV$

2. Are we justified in using a nonrelativistic treatment for the speed of an electron in the hydrogen atom? (Hint: $r=a_0= 0.5$ Å)

Hydrogen atom; $v = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{mr}}$; $r_n = n^2 a_0 = \left(\frac{\varepsilon_0 h^2}{\pi m e^2}\right) n^2 = (0.5\,Å)\,n^2$

$\rightsquigarrow v = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{m(0.5\,Å\,n^2)}}$ ($n = 1$) $\rightsquigarrow v_{n=1} = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{(0.5\,Å)m}} = \sqrt{\frac{1}{4\pi 8.85\times10^{-12}\frac{C^2}{Nm^2}}\frac{(1.6\times10^{-19}\,C)^2}{9.8\times10^{-31}\,kg\; 0.5\times10^{-10}\,m}}$

$= 2.25\times10^{6}\,m/s$

$\Rightarrow \frac{2.25\times10^{6}\,m/s}{3\times10^{8}\,m/s} = 0.0075 \rightsquigarrow 2.25\times10^{6}\,m/s = 0.0075\,c < 0.01\,c$

nonrelativistic treatment