MSE228 Engineering Quantum Mechanics
Quiz 4 Duration: 30 minutes Open Book Quiz

1. Use the momentum and energy operators with the conservation of energy to produce the Schrödinger wave equation.

$E = i\hbar \frac{\partial}{\partial t}$

$p = \frac{\hbar}{i} \frac{\partial}{\partial x}$

Conservation of energy: $E_{total} = K + U$

$E = K + U \rightsquigarrow i\hbar \frac{\partial}{\partial t}\psi = \left(\frac{\hat{p}^2}{2m} + U\right)\psi$

$E\psi = (K+U)\psi$

$\hat{E}\psi = \left(\frac{\hat{p}^2}{2m} + U\right)\psi$

$$\boxed{i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + U\psi}$$

Time-dependent Schrödinger equation

2. A typical diameter of a nucleus is about 10^{-14} m. Use the infinite square-well potential to calculate the transition energy from the first excited state to the ground state for a proton confined to the nucleus.

- Diameter: $10^{-14}\,m = L$
- Infinite square-well potential
- Transition: $n=2 \rightarrow n=1$
- $m_p = 1.673 \times 10^{-27}\,kg$

Particle in a box: $E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$ $n = 1, 2, 3, \ldots$

$\Delta E = E_2 - E_1 = 4E_1 - E_1 = 3E_1 = \frac{3\pi^2 (6.62\times10^{-34}\,J.s/2\pi)^2}{2(1.67\times10^{-27}\,kg)(10^{-14}\,m)^2}$

$= \underline{\underline{9.84\times10^{-13}}}\,J = \underline{\underline{6.15\,MeV}}$