MSE228 Engineering Quantum Mechanics
Quiz 5 Duration: 30 minutes Open Book Quiz

1. Enumerate all states of the hydrogen atom corresponding to the principal quantum number n=2, giving the spectroscopic designation for each. Calculate the energies of these states. How many possible states are there for the n=3 level of hydrogen? For the n=4 level?

$n=2 \rightsquigarrow l=0,1,\ldots n-1 \Rightarrow \underline{l=0,1}$ & $m_l=0,\pm1,\pm2,\ldots\pm l$

$\rightsquigarrow$ if $l=0 \rightsquigarrow m_l=0$
if $l=1 \rightsquigarrow m_l=-1,0,+1$

$n=2 \Rightarrow 2s$ & $2p$
2s: $n=2$ $l=0$ $m_l=0$
2p: $n=2$ $l=1$ $m_l=-1$
$n=2$ $l=1$ $m_l=0$
$n=2$ $l=1$ $m_l=1$

$\underline{4 \text{ states}}$
$E_n=\dfrac{E_1}{n^2}(Z^2)$
$=\dfrac{-13.6\,eV}{4}=\underline{-3.4\,eV}$

$n=3 \rightsquigarrow l=0,1,2$, $m_l=0,\pm1,\ldots,\pm l$
$n=3$ $l=0$ $m_l=0$
$l=1$ $m_l=-1$ $m_l=0$ $m_l=1$
$l=2$ $m_l=-2$ $m_l=-1$ $m_l=0$ $m_l=1$ $m_l=2$

$n=3$: $\underline{9 \text{ states}}$ & $n=4$: $\underline{16 \text{ states}}$

2. Show that the hydrogen wave function ψ_{211} is normalized.
Hints: $\int_0^\infty r^n e^{-cr}dr=\frac{n!}{c^{n+1}}$ & $\int_0^\pi sin^3\theta d\theta=\frac{4}{3}$

$\int |\psi|^2 dv = 1$: normalization

$\int \psi^*_{211}\psi_{211}\,dv=\int \psi^*_{211}\psi_{211}\underbrace{r^2\sin\theta\,dr\,d\theta\,d\phi}_{dV}$

$\rightsquigarrow \psi_{211}=\dfrac{1}{8\sqrt{\pi}\,a_0^{3/2}}\dfrac{r}{a_0}e^{-r/2a_0}\sin\theta e^{+i\phi} \rightsquigarrow \int \dfrac{r^2}{64\pi a_0^5}e^{-r/a_0}\sin^2\theta\, r^2\sin\theta\,dr\,d\theta\,d\phi$

$\rightsquigarrow \int_0^\infty \dfrac{r^2}{64\pi a_0^5}e^{-r/a_0}r^2dr\int_0^\pi \sin^3\theta\,d\theta\int_0^{2\pi}d\phi=1$

$\dfrac{1}{64\pi a_0^5}\underbrace{\int_0^\infty r^4e^{-r/a_0}dr}_{\frac{4!}{(\frac{1}{a_0})^5}}\ \underbrace{\quad}_{4/3}\ \underbrace{\quad}_{2\pi}$

$\dfrac{1}{64\pi a_0^5}\dfrac{24}{1/a_0^5}\dfrac{4}{3}2\pi=1$ ✓

$\underline{\text{confirmed}}$

$\int_0^\pi \sin^3\theta\,d\theta=\int(1-\cos^2\theta)\sin\theta\,d\theta$

$u=\cos\theta$
$du=-\sin\theta\,d\theta$

$\int(1-u^2)du\left(\dfrac{\sin\theta}{-\sin\theta}\right)=-\int(1-u^2)du$
$=-\left(u-\dfrac{u^3}{3}\right)=-u+\dfrac{u^3}{3}$
$=-\cos\theta+\dfrac{\cos^3\theta}{3}\Big|_0^\pi=4/3$