

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 2.988 \times 10^8 \text{ m/s}$$

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$E_0 = m_e c^2 = 0.511 \text{ MeV}$$

$$m_p = 1.67 \times 10^{-27} \text{ kg}$$

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$h = 6.626 \times 10^{-34} \text{ J.s}$$

$$k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ N.m}^2/\text{C}^2$$

$$e = 1.602 \times 10^{-19} \text{ C}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

$$R = 1.097 \times 10^7 \text{ m}^{-1}$$

$$G(\nu)d\nu = \frac{8\pi\nu^2 d\nu}{c^3} \quad (2)$$

$$u(\nu)d\nu = \bar{\epsilon}G(\nu)d\nu = \frac{8\pi kT}{c^3} \nu^2 d\nu \quad (3)$$

$$u(\nu)d\nu = \frac{8\pi h}{c^3} \frac{\nu^3 d\nu}{e^{h\nu/kT} - 1} \quad (4)$$

$$\epsilon_n = nh\nu \quad n = 0, 1, 2, \dots \quad (5)$$

$$\epsilon = \frac{h\nu}{e^{h\nu/kT} - 1} \quad (6)$$

$$\phi = h\nu_0 \quad (7)$$

$$h\nu = KE_{\max} + \phi \quad (8)$$

$$E = \left( \frac{6.626 \times 10^{-34} \text{ J.s}}{1.602 \times 10^{-19} \text{ J/eV}} \right) \nu = (4.136 \times 10^{-15}) \nu \text{ eV.s} \quad (9)$$

$$V_e = h\nu_{\max} = \frac{hc}{\lambda_{\min}} \quad (10)$$

$$\lambda_{\min} = \frac{hc}{V_e} = \frac{1.24 \times 10^{-6}}{V} \text{ V.m} \quad (11)$$

$$2d\sin\theta = n\lambda \quad n = 1, 2, 3, \dots \quad (12)$$

$$h\nu - h\nu' = KE \quad (13)$$

$$p = \frac{E}{c} = \frac{h\nu}{c} \quad (14)$$

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c} \cos\phi + p \cos\theta \quad (15)$$

$$0 = \frac{h\nu'}{c} \sin\phi - p \sin\theta \quad (15)$$

$$\lambda' - \lambda = \lambda_C(1 - \cos\phi) \quad (16)$$

$$\lambda_C = \frac{h}{mc} \quad (17)$$

$$p = \frac{h\nu}{c} = \frac{h}{\lambda}, \quad \lambda = \frac{h}{p} \quad (18)$$

$$\lambda = \frac{h}{\gamma mv} \quad (19)$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \quad (20)$$

$$E = \sqrt{E_0^2 + p^2 c^2} = KE + E_0 \quad (21)$$

$$v_p = \nu\lambda = \left( \frac{\gamma mc^2}{h} \right) \left( \frac{h}{\gamma mv} \right) = \frac{c^2}{v} \quad (22)$$

$$y = A \cos(2\pi vt) = A \cos \left( 2\pi\nu \left( t - \frac{x}{v_p} \right) \right) \quad (23)$$

$$\omega = 2\pi\nu \quad (24)$$

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{v_p} \quad (25)$$

$$y = A \cos(\omega t - kx) \quad (26)$$

$$v_g = v \quad \& \quad v_p = \frac{\omega}{k} = \frac{c^2}{v} \quad (27)$$

$$\lambda = \frac{2L}{n} \quad n = 1, 2, 3, \dots \quad (28)$$

$$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{(mv)^2}{2m} = \frac{h^2}{2m\lambda^2} \quad (29)$$

$$E_n = \frac{n^2 h^2}{8mL^2} \quad n = 1, 2, 3, \dots \quad (30)$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h} \quad \& \quad p = \frac{hk}{2\pi} \quad (31)$$

$$\Delta x \Delta p \geq \frac{\hbar}{2} \quad \& \quad \Delta E \Delta t \geq \frac{\hbar}{2} \quad (32)$$

$$\sigma = \sqrt{\left( \frac{1}{N} \sum_{i=1}^N (x_i - x_0)^2 \right)} \quad (33)$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-x_0)^2/2\sigma^2} \quad (34)$$

$$P_{x_1 x_2} = \int_{x_1}^{x_2} f(x) dx \quad (35)$$

$$P_{x_0 \pm \sigma} = \int_{x_0 - \sigma}^{x_0 + \sigma} f(x) dx = 0.683 \quad (36)$$

$$F_e = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \quad \& \quad F_c = \frac{mv^2}{r} \quad (37)$$

$$n\lambda = 2\pi r_n \quad n = 1, 2, 3, \dots \quad (38)$$

$$r_n = \frac{n^2 h^2 \epsilon_0}{\pi m e^2} = n^2 a_0, \quad v_n = \sqrt{\frac{e^2}{4\pi\epsilon_0 m r_n}}, \quad n = 1, 2, 3, \dots \quad (39)$$

$$a_0 = r_1 = 5.292 \times 10^{-11} \text{ m} \quad (40)$$

$$E_n = -\frac{e^2}{8\pi\epsilon_0 r_n} = -\frac{me^4}{8\epsilon_0^2 h^2} \left( \frac{1}{n^2} \right) \quad n = 1, 2, 3, \dots$$

$$E_n = \frac{E_1}{n^2}, \quad E_n = \frac{Z^2 E_1}{n^2} \text{ (hydrogen like atom)} \quad (41)$$

$$E_1 = -2.18 \times 10^{-18} \text{ J} = -13.6 \text{ eV}$$

$$\nu = \frac{E_i - E_f}{h} = -\frac{E_1}{h} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right), \quad \lambda = \frac{c}{\nu} \quad (42)$$

$$\text{Lyman } n_f = 1: \quad \frac{1}{\lambda} = -\frac{E_1}{ch} \left( \frac{1}{1^2} - \frac{1}{n^2} \right) \quad n = 2, 3, 4, \dots$$

$$\text{Balmer } n_f = 2: \quad \frac{1}{\lambda} = -\frac{E_1}{ch} \left( \frac{1}{2^2} - \frac{1}{n^2} \right) \quad n = 3, 4, 5, \dots$$

$$\text{Paschen } n_f = 3: \quad \frac{1}{\lambda} = -\frac{E_1}{ch} \left( \frac{1}{3^2} - \frac{1}{n^2} \right) \quad n = 4, 5, 6, \dots$$

$$\text{Brackett } n_f = 4: \quad \frac{1}{\lambda} = -\frac{E_1}{ch} \left( \frac{1}{4^2} - \frac{1}{n^2} \right) \quad n = 5, 6, 7, \dots$$

$$\text{Pfund } n_f = 5: \quad \frac{1}{\lambda} = -\frac{E_1}{ch} \left( \frac{1}{5^2} - \frac{1}{n^2} \right) \quad n = 6, 7, 8, \dots \quad (43)$$

$$\Psi = A + iB \quad \& \quad \Psi^* = A - iB \quad (44)$$

$$|\Psi|^2 = \Psi^* \Psi = A^2 - i^2 B^2 = A^2 + B^2 \quad (45)$$

$$\int_{-\infty}^{\infty} |\Psi|^2 dV = 0 \quad \& \quad \int_{-\infty}^{\infty} |\Psi|^2 dV = 1 \quad (46)$$

$$P_{x_1 x_2} = \int_{-x_1}^{x_2} |\Psi|^2 dV = 1 \quad (47)$$

$$E = h\nu = 2\pi h\lambda \quad \& \quad \lambda = \frac{h}{p} = \frac{2\pi h}{p} \quad (48)$$

$$\Psi = A e^{-(i/\hbar)(Et - px)} = A e^{-(iE/\hbar)t} e^{+(ip/\hbar)x} = \psi e^{-(iE/\hbar)t} \quad (49)$$

$$\frac{\partial^2 \Psi}{\partial x^2} = -\frac{p^2}{\hbar^2} \Psi \Rightarrow p^2 \Psi = -\hbar^2 \frac{\partial^2 \Psi}{\partial x^2} \Rightarrow \hat{p} = \frac{\hbar}{i} \frac{\partial}{\partial x} \quad (50)$$

$$\frac{\partial \Psi}{\partial t} = -\frac{iE}{\hbar} \Psi \Rightarrow E \Psi = -\frac{\hbar}{i} \frac{\partial \Psi}{\partial t} \Rightarrow \hat{E} = i\hbar \frac{\partial}{\partial t} \quad (51)$$

$$E = \frac{p^2}{2m} + U(x, t) \Rightarrow \hat{E} = \hat{K}E + \hat{U} \quad (52)$$

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \left( \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} \right) + U \Psi \quad (53)$$

$$\bar{x} = \frac{N_1 x_1 + N_2 x_2 + N_3 x_3 + \dots}{N_1 + N_2 + N_3 + \dots} = \frac{\sum N_i x_i}{\sum N_i} \quad (54)$$

$$\langle x \rangle = \int_{-\infty}^{\infty} x |\Psi|^2 dx \quad (55)$$

$$\langle p \rangle = \int_{-\infty}^{\infty} \Psi^* \left( \frac{\hbar}{i} \frac{\partial}{\partial x} \right) \Psi dx \quad (56)$$

$$\langle E \rangle = \int_{-\infty}^{\infty} \Psi^* \left( i\hbar \frac{\partial}{\partial t} \right) \Psi dx \quad (57)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{2m}{\hbar^2} (E - U) \psi = 0 \quad (58)$$

$$E_n = -\frac{me^4}{32\pi^2 \epsilon_0^2 \hbar^2} \left( \frac{1}{n^2} \right) \quad n = 1, 2, 3, \dots \quad (59)$$

$$L = \sqrt{l(l+1)} \hbar \quad l = 0, 1, 2, \dots, (n-1) \quad (60)$$

$$\hat{H} \psi_n = E_n \psi_n, \quad \hat{H} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + U \quad (61)$$

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \quad n = 1, 2, 3, \dots \quad (62)$$

$$\psi_n = A \sin \frac{\sqrt{2mE_n}}{\hbar} x = A \sin \frac{n\pi x}{L}, \quad A = \sqrt{\frac{2}{L}} \quad (63)$$

$$\frac{d^2 \psi}{dx^2} - a^2 \psi = 0 \quad \begin{matrix} x < 0 \\ x > L \end{matrix}; \quad a = \frac{\sqrt{2m(U-E)}}{\hbar} \quad (64)$$

$$\psi_I = C e^{ax} + D e^{-ax} \rightarrow \psi_I = C e^{ax}$$

$$\psi_{II} = A \sin \frac{\sqrt{2mE}}{\hbar} x + B \cos \frac{\sqrt{2mE}}{\hbar} x \quad (65)$$

$$\psi_{III} = F e^{ax} + G e^{-ax} \rightarrow \psi_{III} = G e^{-ax}$$

$$\frac{d^2 \psi}{dx^2} + \frac{2m}{\hbar^2} \left( E - \frac{1}{2} kx^2 \right) \psi = 0 \quad (66)$$

$$E_n = \left( n + \frac{1}{2} \right) h\nu, \quad n = 0, 1, 2, 3, \dots \quad (67)$$

$$U = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r}, \quad r = \sqrt{x^2 + y^2 + z^2} \quad (68)$$

$$\psi(r, \theta, \phi) = R(r) \Theta(\theta) \Phi(\phi), \quad \psi = R_{nl} \Theta_{lm} \Phi_{m_l} \quad (69)$$

$$\frac{\sin^2 \theta}{R} \frac{d}{dr} \left( r^2 \frac{dR}{dr} \right) + \frac{\sin \theta}{\Theta} \frac{d}{d\theta} \left( \sin \theta \frac{d\Theta}{d\theta} \right) + \frac{1}{\Phi} \frac{d^2}{d\phi^2} + \frac{2mr^2 \sin^2 \theta}{\hbar^2} \left( \frac{e^2}{4\pi\epsilon_0 r} + E \right) = 0 \quad (70)$$

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dR}{dr} \right) + \left[ \frac{2m}{\hbar^2} \left( \frac{e^2}{4\pi\epsilon_0 r} + E \right) - \frac{l(l+1)}{r^2} \right] R = 0 \quad (71)$$

$$E_n = -\frac{me^4}{32\pi^2 \epsilon_0^2 \hbar^2} \left( \frac{1}{n^2} \right) = \frac{E_1}{n^2} \quad n = 1, 2, 3, \dots \quad (72)$$

$$\text{Principal quantum number } n = 1, 2, 3, \dots \quad (73)$$

$$\text{Orbital quantum number } l = 0, 1, 2, \dots, (n-1)$$

$$\text{Magnetic quantum number } m_l = 0, \pm 1, \pm 2, \dots, \pm l$$

$$L = rmv_{\text{orbital}} \Rightarrow L = \sqrt{l(l+1)} \hbar, \quad l = 0, 1, 2, \dots, (n-1) \quad (74)$$

$$\begin{array}{l} \text{Angular-} \\ \text{momentum states} \end{array} \quad \begin{array}{l} l = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad \dots \\ s \quad p \quad d \quad f \quad g \quad h \quad i \quad \dots \end{array} \quad (75)$$

$$L_z = m_l \hbar \quad m_l = 0, \pm 1, \pm 2, \dots, \pm l \quad (76)$$

$$dV = (dr)(r d\theta)(r \sin \theta d\phi) = r^2 \sin \theta dr d\theta d\phi \quad (77)$$

$$P(r) dr = \quad (78)$$

$$r^2 |R|^2 dr \int_0^\pi |\Theta|^2 \sin \theta d\theta \int_0^{2\pi} |\Phi|^2 d\phi = r^2 |R|^2 dr \int_{-\infty}^{\infty} u \psi_{n,l,m_l} \psi_{n',l',m_l'}^* dV \neq 0 \quad (79)$$

$$\begin{array}{l} \text{Selection rules } \Delta l = \pm 1 \\ \Delta m_l = 0, \pm 1 \end{array} \quad (80)$$

$$\mu = -ef\pi r^2, \quad \mathbf{L} = m\mathbf{v}r = 2\pi mfr^2, \quad \mu = -\left(\frac{e}{2m}\right)\mathbf{L} \quad (81)$$

$$\mathbf{U}_m = -\mu\mathbf{B}\cos\theta \rightarrow U_m = m_l\left(\frac{e\hbar}{2m}\right)\mathbf{B} = m_l\mu_B\mathbf{B} \quad (82)$$

$$\mu_B = \frac{e\hbar}{2m} = 9.274 \times 10^{-24} \text{ J/T} = 5.788 \times 10^{-5} \text{ eV/T} \quad (83)$$

$$\nu_1 = \nu_0 - \mu_B \frac{\mathbf{B}}{\hbar} = \nu_0 - \frac{e}{4\pi m} \mathbf{B}$$

Normal Zeeman effect  $\nu_2 = \nu_0$

$$\nu_3 = \nu_0 + \mu_B \frac{\mathbf{B}}{\hbar} = \nu_0 + \frac{e}{4\pi m} \mathbf{B}$$

$$\int_0^\infty x^n e^{-cx} dx = \frac{n!}{c^{n+1}} \quad \& \quad \int_0^\pi \sin^3\theta d\theta = \frac{4}{3} \quad (85)$$

$$\int x^n e^{-cx} dx = -\frac{e^{-cx}}{c} \left( x^n + \frac{nx^{n-1}}{c} + \frac{n(n-1)x^{n-2}}{c^2} + \dots + \frac{n!}{c^n} \right) \quad (86)$$

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta) \quad (87)$$

$$S = \sqrt{s(s+1)}\hbar = \frac{\sqrt{3}}{2}\hbar, \quad S_z = m_s\hbar = \pm\frac{1}{2}\hbar \quad (88)$$

$$\vec{\mu}_s = -\frac{e}{m}\mathbf{S}_i, \quad \mu_{sz} = \pm\frac{e\hbar}{2m} = \pm\mu_B \quad (89)$$

$$n = 1 \ 2 \ 3 \ 4 \ 5 \ \dots \quad \text{K L M N O } \dots \quad (90)$$

$$1s^2 \ 2s^2 \ 2p^6 \ 3s^2 \ 3p^6 \ 4s^2 \ 3d^{10} \ 4p^6 \ 5s^2 \ 4d^{10} \ 5p^6 \ 6s^2 \ 4f^{14} \ 5d^{10} \ 6p^6 \ 7s^2 \ 6d^{10} \ 5f^{14} \quad (91)$$

$$\nu = \frac{m(Z-1)^2 e^4}{8\epsilon_0^2 h^3} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \quad (92)$$

$$(2 \rightarrow 1) \nu = cR(Z-1)^2 \left( \frac{1}{1^2} - \frac{1}{2^2} \right) = \frac{3cR(Z-1)^2}{4}$$

**Table 6.1** Normalized Wave Functions of the Hydrogen Atom for  $n = 1, 2,$  and  $3^*$

$n$	$l$	$m_l$	$\Phi(\phi)$	$\Theta(\theta)$	$R(r)$	$\psi(r, \theta, \phi)$
1	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{2}{a_0^{3/2}} e^{-r/a_0}$	$\frac{1}{\sqrt{\pi} a_0^{3/2}} e^{-r/a_0}$
2	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2\sqrt{2}} \frac{r}{a_0^{3/2}} \left( 2 - \frac{r}{a_0} \right) e^{-r/2a_0}$	$\frac{1}{4\sqrt{2\pi} a_0^{3/2}} \left( 2 - \frac{r}{a_0} \right) e^{-r/2a_0}$
2	1	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2} \cos\theta$	$\frac{1}{2\sqrt{6}} \frac{r}{a_0^{3/2}} e^{-r/2a_0}$	$\frac{1}{4\sqrt{2\pi} a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0} \cos\theta$
2	1	$\pm 1$	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$	$\frac{\sqrt{3}}{2} \sin\theta$	$\frac{1}{2\sqrt{6}} \frac{r}{a_0^{3/2}} e^{-r/2a_0}$	$\frac{1}{8\sqrt{\pi} a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0} \sin\theta e^{\pm i\phi}$
3	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{2}{81\sqrt{3}} \frac{r^2}{a_0^{3/2}} \left( 27 - 18\frac{r}{a_0} + 2\frac{r^2}{a_0^2} \right) e^{-r/3a_0}$	$\frac{1}{81\sqrt{3\pi} a_0^{3/2}} \left( 27 - 18\frac{r}{a_0} + 2\frac{r^2}{a_0^2} \right) e^{-r/3a_0}$
3	1	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2} \cos\theta$	$\frac{4}{81\sqrt{6}} \frac{r^2}{a_0^{3/2}} \left( 6 - \frac{r}{a_0} \right) e^{-r/3a_0}$	$\frac{\sqrt{2}}{81\sqrt{\pi} a_0^{3/2}} \left( 6 - \frac{r}{a_0} \right) \frac{r}{a_0} e^{-r/3a_0} \cos\theta$
3	1	$\pm 1$	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$	$\frac{\sqrt{3}}{2} \sin\theta$	$\frac{4}{81\sqrt{6}} \frac{r^2}{a_0^{3/2}} \left( 6 - \frac{r}{a_0} \right) e^{-r/3a_0}$	$\frac{1}{81\sqrt{\pi} a_0^{3/2}} \left( 6 - \frac{r}{a_0} \right) \frac{r}{a_0} e^{-r/3a_0} \sin\theta e^{\pm i\phi}$
3	2	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{10}}{4} (3\cos^2\theta - 1)$	$\frac{4}{81\sqrt{30}} \frac{r^2}{a_0^{3/2}} e^{-r/3a_0}$	$\frac{1}{81\sqrt{6\pi} a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0} (3\cos^2\theta - 1)$
3	2	$\pm 1$	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$	$\frac{\sqrt{15}}{2} \sin\theta \cos\theta$	$\frac{4}{81\sqrt{30}} \frac{r^2}{a_0^{3/2}} e^{-r/3a_0}$	$\frac{1}{81\sqrt{\pi} a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0} \sin\theta \cos\theta e^{\pm i\phi}$
3	2	$\pm 2$	$\frac{1}{\sqrt{2\pi}} e^{\pm 2i\phi}$	$\frac{\sqrt{15}}{4} \sin^2\theta$	$\frac{4}{81\sqrt{30}} \frac{r^2}{a_0^{3/2}} e^{-r/3a_0}$	$\frac{1}{162\sqrt{\pi} a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0} \sin^2\theta e^{\pm 2i\phi}$

\*The quantity  $a_0 = 4\pi\epsilon_0\hbar^2/m_e e^2 = 5.292 \times 10^{-11} \text{ m}$  is equal to the radius of the innermost Bohr orbit.