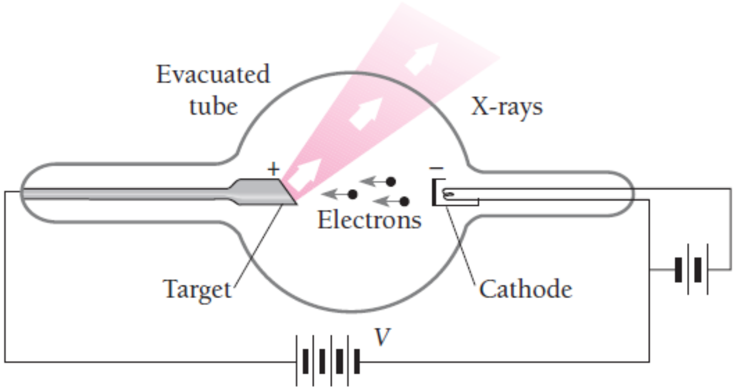
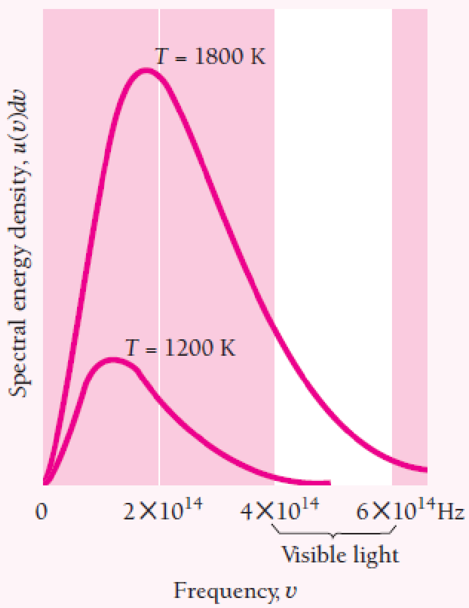
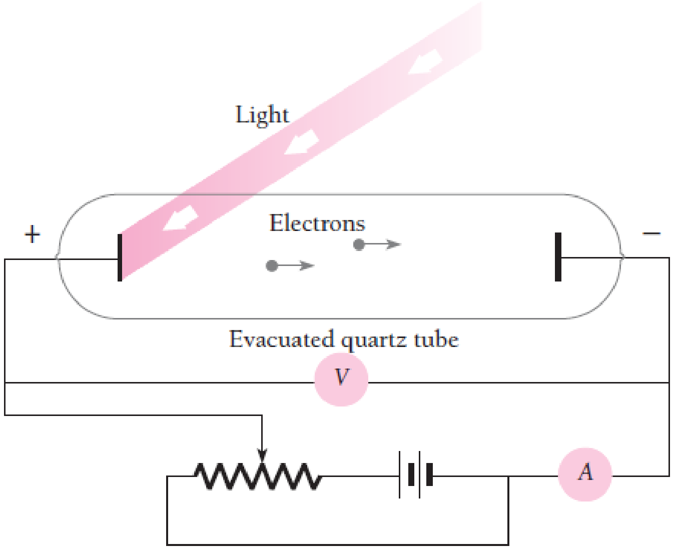


# Chapter 2

# Particle Properties of Waves





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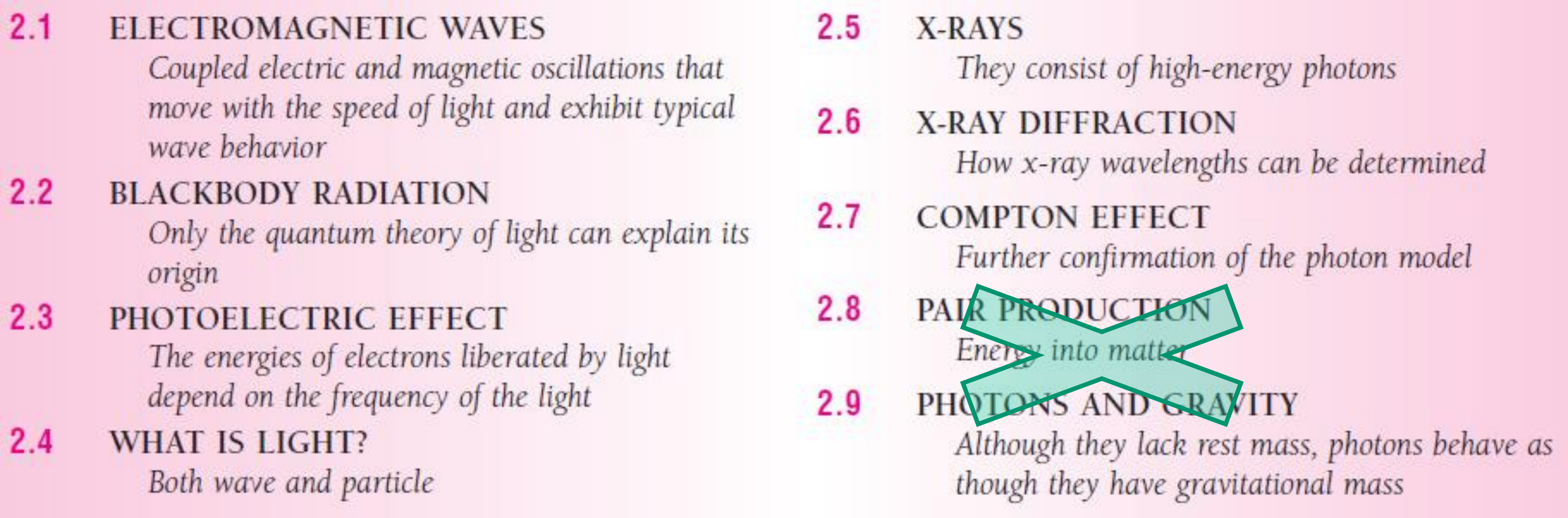
# 2 Particle Properties of Waves



**2.1** ELECTROMAGNETIC WAVES
*Coupled electric and magnetic oscillations that move with the speed of light and exhibit typical wave behavior*

**2.2** BLACKBODY RADIATION
*Only the quantum theory of light can explain its origin*

**2.3** PHOTOELECTRIC EFFECT
*The energies of electrons liberated by light depend on the frequency of the light*

**2.4** WHAT IS LIGHT?
*Both wave and particle*

**2.5** X-RAYS
*They consist of high-energy photons*

**2.6** X-RAY DIFFRACTION
*How x-ray wavelengths can be determined*

**2.7** COMPTON EFFECT
*Further confirmation of the photon model*

**2.8** ~~PAIR PRODUCTION~~
~~*Energy into matter*~~

**2.9** ~~PHOTONS AND GRAVITY~~
*Although they lack rest mass, photons behave as though they have gravitational mass*

- Particle ==> Mechanics
- Waves ==> Optics
    - *Carry energy and momentum from one place to another.*
- Microscopic world of atoms and molecules, electrons and nuclei
    - in this world, there are neither particles nor waves in our sense.
- We regard electrons as particles because they possess *charge* and *mass* and behave according to the laws of particle mechanics.
- However, it is just as correct to interpret a moving electron as a wave as it is to interpret it as a particle.
- We regard electromagnetic waves as *waves* because under suitable circumstances they exhibit *diffraction*, *interference*, *and polarization*.
- Similarly, we shall see that under other circumstances *electromagnetic waves* behave as though they consist of *streams of particles*.
- **The wave-particle duality is central to an understanding of modern physics.**

# 2.1 Electromagnetic Waves

- *Coupled* electric and magnetic oscillations that move with the speed of light and exhibit typical wave behavior.
- *Accelerated electric charges* generate linked electric and magnetic disturbances that can *travel* indefinitely through space.
- **If the charges oscillate periodically**, the disturbances are waves whose electric and magnetic components are perpendicular to each other and to the direction of propagation, as in Fig. 2.1

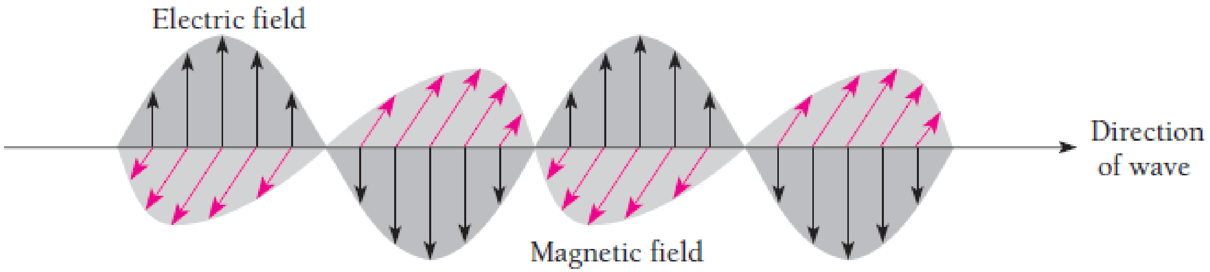


Figure 2.1 The electric and magnetic fields in an electromagnetic wave vary together. The fields are perpendicular to each other and to the direction of propagation of the wave.

James Clerk Maxwell (1831–1879)

- Maxwell was able to show that the speed c of electromagnetic (EM) waves in free space is given by

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 2.988 \times 10^8 \ m/s$$

where $\varepsilon_0$ the electric permittivity of free space and $\mu_0$ is its magnetic permeability. This is the same as the speed of light waves.

# 2.1 Electromagnetic Waves

- Many features of EM waves interaction with matter depend upon their *frequencies*.
- Light waves, which are EM waves the eye responds to, span only a brief frequency interval,
  - from about 4.3x$10^{14}$ Hz for red light
  - to about 7.5x$10^{14}$ Hz for violet light.

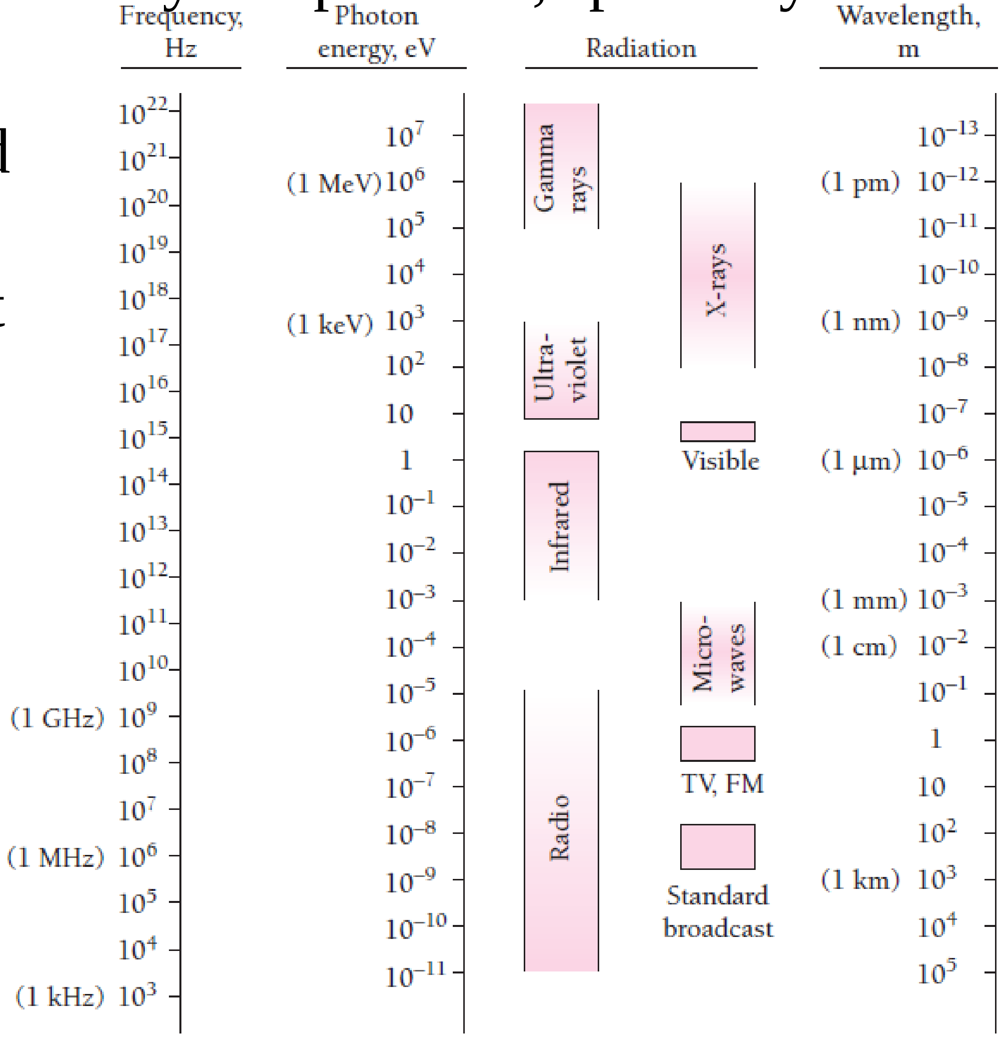


Figure 2.2 The spectrum of electromagnetic radiation.

# 2.1 Electromagnetic Waves: Principle of superposition

- When two or more waves of the same nature travel past a point at the same time, the instantaneous amplitude there is the *sum* of the instantaneous amplitudes of the individual waves.
  - Amplitude of a *water wave* is the height of the water surface relative to its normal level.
  - Amplitude of a *sound wave* is the change in pressure relative to the normal pressure.
  - Amplitude of a ***light wave*** (E=cB) can be taken as either E or B. Usually E is used, since its interactions with matter give rise to nearly all common optical effects.

# 2.1 Electromagnetic Waves: Interference

- When two or more trains of light waves meet in a region, they *interfere* to produce a new wave there whose instantaneous amplitude is the sum of those of the original waves.
  - **Constructive** interference refers to the reinforcement of waves with the same phase to produce a greater amplitude. Fig 2.3a
  - **Destructive** interference refers to the partial or complete cancellation of waves whose phases differ. Fig 2.3b
  - If the original waves have different frequencies, the result will be a *mixture* of constructive and destructive interference.

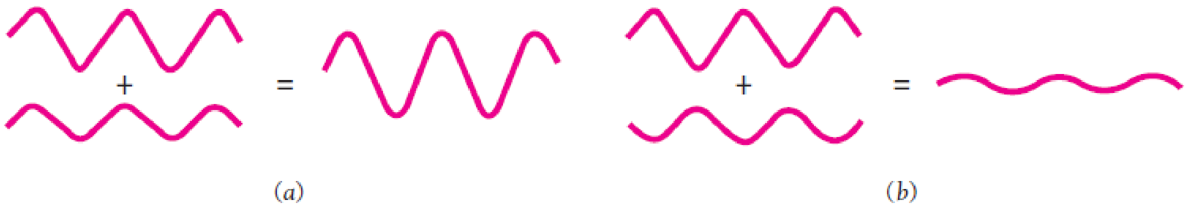


Figure 2.3 (a) In constructive interference, superposed waves in phase reinforce each other. (b) In destructive interference, waves out of phase partially or completely cancel each other.

# 2.1 Electromagnetic Waves:Young’s Double Slit Experiment

- Interference of light waves Fig. 2.4.
- From each *slit* secondary waves spread out as though originating at the slit. **Diffraction.**
- At those places on the screen where the path lengths from the two slits differ by an odd number of half wavelengths, λ (1/2, 3/2, 5/2, …), *destructive interference* occurs and a dark line is the result.
- At those places where the path lengths are equal or differ by a whole number of wavelengths, λ (1, 2, 3, …), *constructive interference* occurs and a bright line is the result.
- At intermediate places the interference is only partial, so the light intensity on the screen varies gradually between the bright and dark lines.

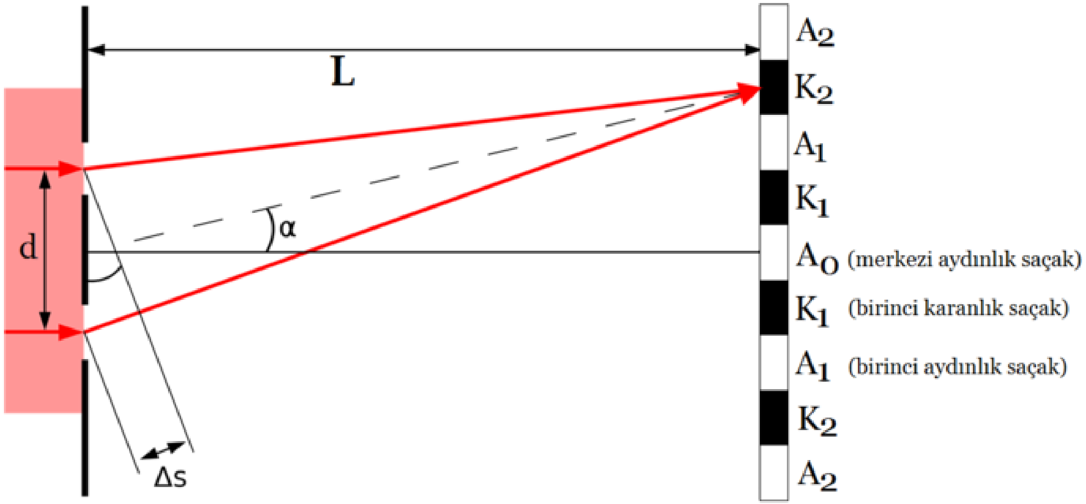
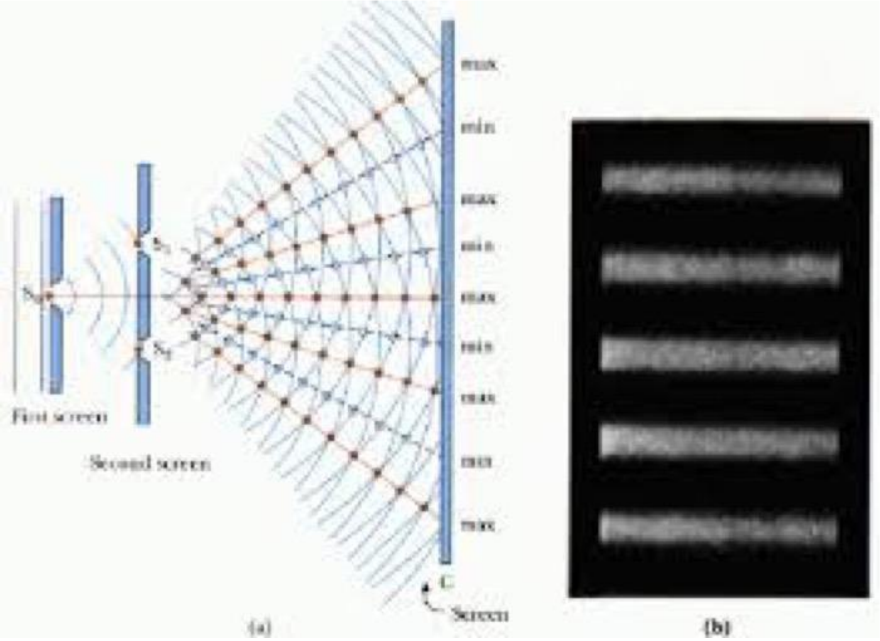
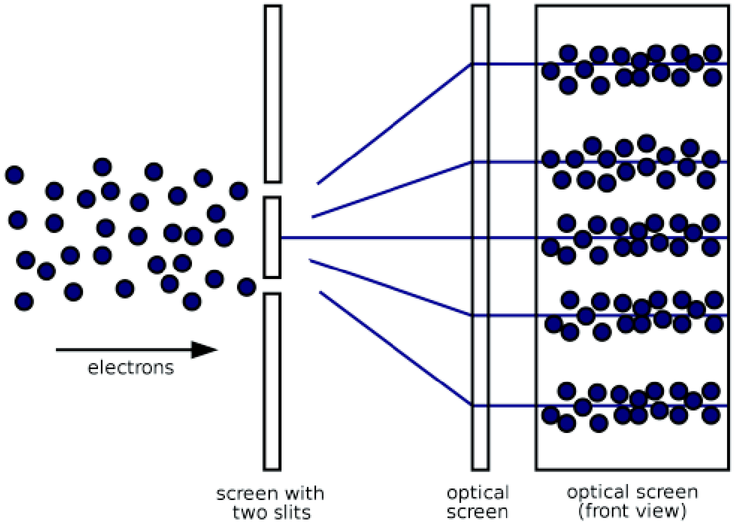


Figure 2.4 Origin of the interference pattern in Young’s experiment.

Thomas Young
1773-1829

# **2.1** Electromagnetic Waves





- **Interference and diffraction are found only in waves**.
- If light consisted of a stream of **classical** particles, the entire screen would be dark.
- Thus, Young’s experiment is proof that light consists of waves.

## **2.2** Blackbody Radiation

- Until the end of the nineteenth century the nature of light seemed settled forever.
- Attempts to understand the **origin of the radiation emitted** by bodies of matter.
  - All objects radiate energy continuously whatever their temperatures. Which frequencies predominate depends on the temperature.
  - At room temperature, most of the radiation is in the infrared part of the spectrum and hence is invisible.

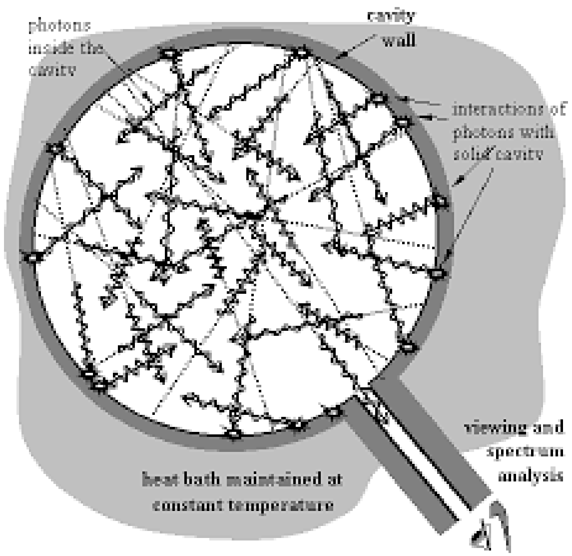


Figure 2.5 A hole in the wall of a hollow object is an excellent approximation of a blackbody.

- *Any radiation striking the hole* enters the cavity, where it is trapped by reflection back and forth until it is absorbed.
- The cavity walls are constantly emitting and absorbing radiation, and it is in the properties of this radiation (blackbody radiation).
- Sample blackbody radiation simply by inspecting what *emerges from the hole* in the cavity.

# 2.2 Blackbody Radiation

- A blackbody radiates more when it is hot than when it is cold, and the spectrum of a hot blackbody has its peak at a higher frequency than the peak in the spectrum of a cooler one.
- The spectrum of blackbody radiation is shown in Fig. 2.6 for two temperatures.
- Why does the blackbody spectrum have the shape shown in Fig. 2.6?
- Only the quantum theory of light can explain its origin.

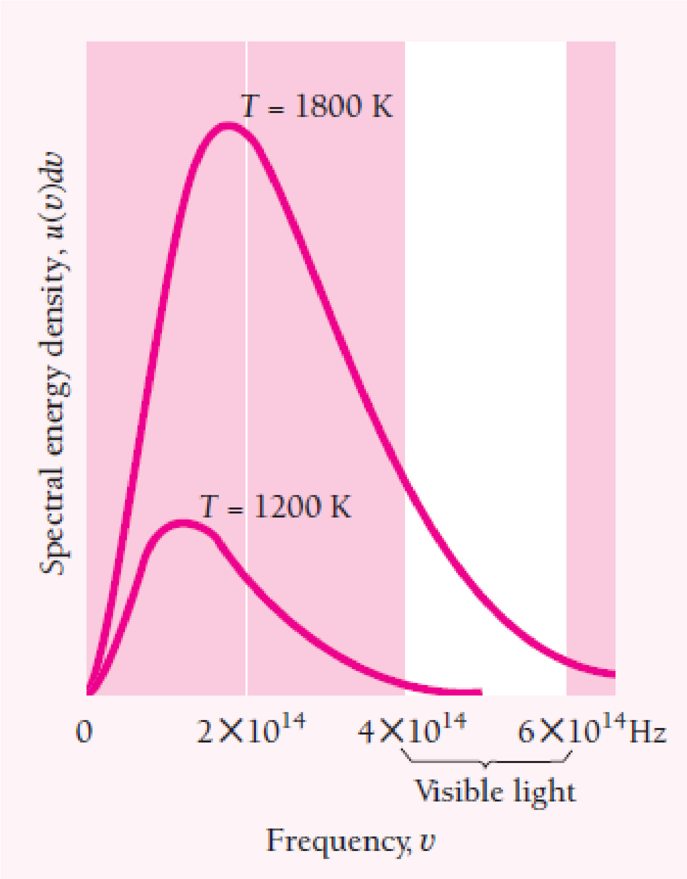


Figure 2.6 Blackbody spectra. The spectral distribution of energy in the radiation depends only on the temperature of the body. The higher the temperature, the greater the amount of radiation an”d the higher the frequency at which the maximum emission occurs.

- Lord Rayleigh and James Jeans started by considering the radiation inside a cavity of absolute temperature to be a series of standing EM waves (Fig. 2.7).
- The condition for standing waves in such a cavity is that the path length from wall to wall, whatever the direction, must be a whole number of *half-wavelengths*, so that a node occurs at each reflecting surface.
- The number of independent standing waves $G(\nu)d\nu$ in the frequency interval per unit volume in the cavity turned out to be

$$G(\nu)d\nu = \frac{8\pi\nu^2 d\nu}{c^3}$$ (Density of standing waves in cavity)

- Because each standing wave in a cavity originates in an oscillating electric charge in the cavity wall, two degrees of freedom are associated with the wave and it should have an average energy of $\overline{\epsilon} = kT$ (Classical average energy per standing wave)

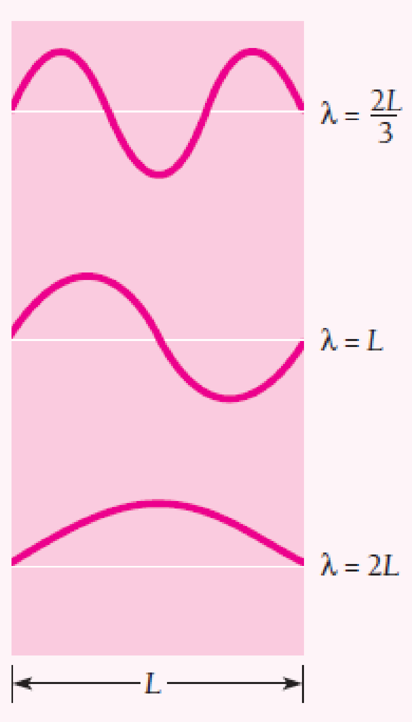


Figure 2.7 Standing waves that have nodes at the walls, which restricts their possible wavelengths. Shown are three possible wavelengths when the distance between opposite walls is L

# 2.2 Blackbody Radiation: The Ultraviolet Catastrophe

- The total energy per unit volume in the cavity in the frequency interval is therefore

$$u(\nu)d\nu = \bar{\epsilon}G(\nu)d\nu = \frac{8\pi kT}{c^3}\nu^2 d\nu$$ (Rayleigh-Jeans formula)

- *The Rayleigh-Jeans formulae, contains everything that classical physics can say about the spectrum of blackbody radiation.*

- In reality, of course, the energy density (and radiation rate) falls to 0 as frequency goes to infinity (see Fig. 2.8).
- This discrepancy became known as the **ultraviolet catastrophe** of classical physics.
- Where did Rayleigh and Jeans go wrong?

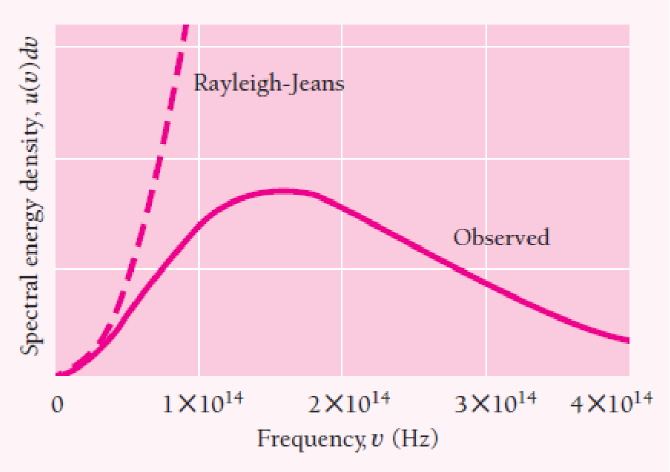
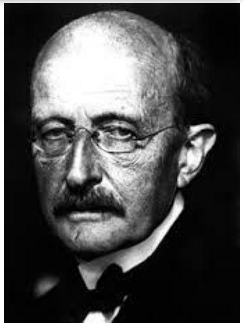


Figure 2.8 Comparison of the Rayleigh-Jeans formula for the spectrum of the radiation from a blackbody at 1500 K with the observed spectrum.

# 2.2 Blackbody Radiation: The Ultraviolet Catastrophe



Max Planck
(1858–1947)
Nobel Prize in
Physics in 1918

- This **failure of classical physics** led Planck to the discovery that *radiation is emitted in quanta* whose energy is hν.

$$u(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{e^{h\nu/kT}-1}$$ (Planck radiation formula)

$$h = 6.626 \times 10^{-34} J.s$$ (Planck's constant)

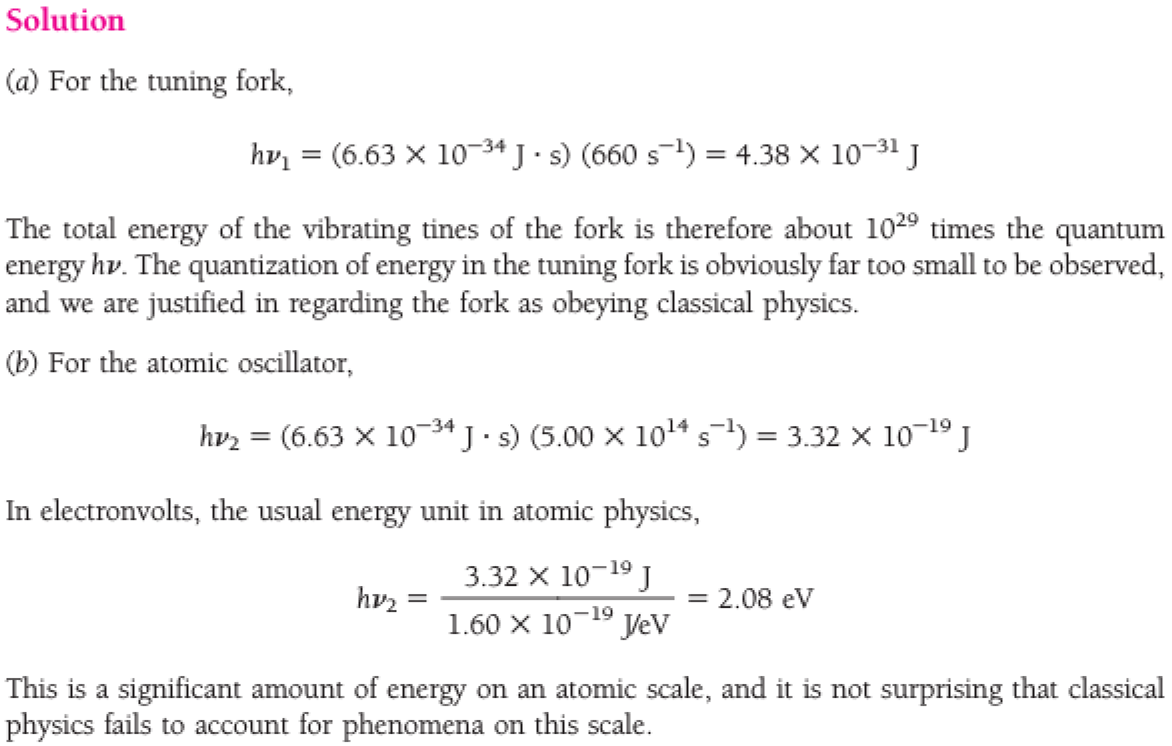
- Considered to mark the start of modern physics.
- At high frequencies and at low frequencies Planck's formula becomes Rayleigh-Jeans formula.
- *The oscillators in the cavity walls could not have a continuous distribution of possible energies ε but must have only the specific energies* $\epsilon_n = nh\nu \quad n = 0, 1, 2, \ldots$ (Oscillator energies)
- An oscillator emits radiation of frequency ν when it drops from one energy state to the next lower one, and it jumps to the next higher state when it absorbs radiation of frequency ν.
- Each discrete bundle of energy hν is called a quantum (plural quanta) from the Latin for "how much."

# 2.2 Blackbody Radiation: The Ultraviolet Catastrophe

- With oscillator energies limited to nhν, the average energy per oscillator in the cavity walls $$\epsilon = \frac{h\nu}{e^{h\nu/kT} - 1}$$ (Actual average energy per standing wave)

Example 2.1

Assume that a certain 660-Hz tuning fork can be considered as a harmonic oscillator whose vibrational energy is 0.04 J. Compare the energy quanta of this tuning fork with those of an atomic oscillator that emits and absorbs orange light whose frequency is $5.00\text{x}10^{14}$ Hz.



**Solution**

(*a*) For the tuning fork,

$$h\nu_1 = (6.63 \times 10^{-34}\ \text{J} \cdot \text{s})\ (660\ \text{s}^{-1}) = 4.38 \times 10^{-31}\ \text{J}$$

The total energy of the vibrating tines of the fork is therefore about $10^{29}$ times the quantum energy $h\nu$. The quantization of energy in the tuning fork is obviously far too small to be observed, and we are justified in regarding the fork as obeying classical physics.

(*b*) For the atomic oscillator,

$$h\nu_2 = (6.63 \times 10^{-34}\ \text{J} \cdot \text{s})\ (5.00 \times 10^{14}\ \text{s}^{-1}) = 3.32 \times 10^{-19}\ \text{J}$$

In electronvolts, the usual energy unit in atomic physics,

$$h\nu_2 = \frac{3.32 \times 10^{-19}\ \text{J}}{1.60 \times 10^{-19}\ \text{J/eV}} = 2.08\ \text{eV}$$

This is a significant amount of energy on an atomic scale, and it is not surprising that classical physics fails to account for phenomena on this scale.

- The energies of electrons liberated by light depend on the frequency of the light.
  - Electrons emitted when the frequency of the light was *sufficiently high*.
- This phenomenon is known as the **photoelectric effect** and the emitted electrons are called **photoelectrons**.

- Some of the photoelectrons that emerge from this surface have *enough energy to reach* the cathode despite its negative polarity (the measured current).
- When the voltage is increased to a certain value $V_0$, no more photoelectrons arrive (the current dropping to zero).
- Maximum photoelectron kinetic energy.

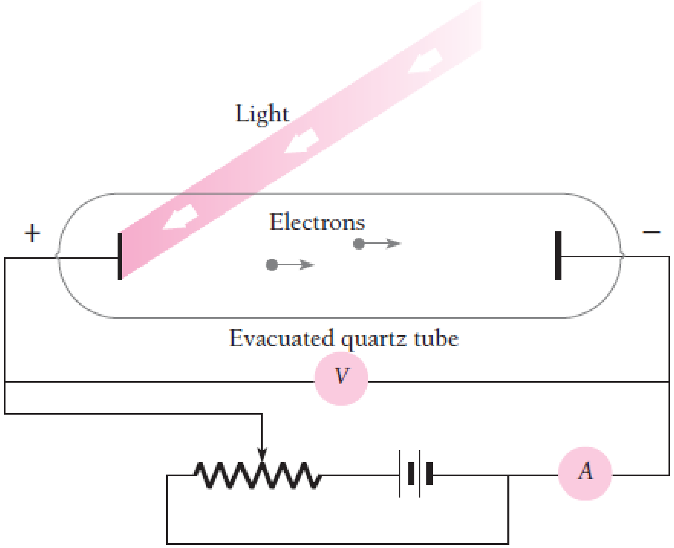


Figure 2.9 Experimental observation of the photoelectric effect..

- Three experimental findings;

1. There is no time interval between the arrival of light at a metal surface and the emission of photoelectrons.
    - ✖ However, because the energy in an EM wave is supposed to be spread across the wavefronts, *a period of time should elapse* before an individual electron accumulates enough energy (several eV) to leave the metal.
2. A bright light yields more photoelectrons than a dim one of the same frequency, but the electron energies remain the same (Fig. 2.10).
    - ✖ The EM theory of light, on the contrary, predicts that the *more intense the light, the greater the energies of the electrons*.

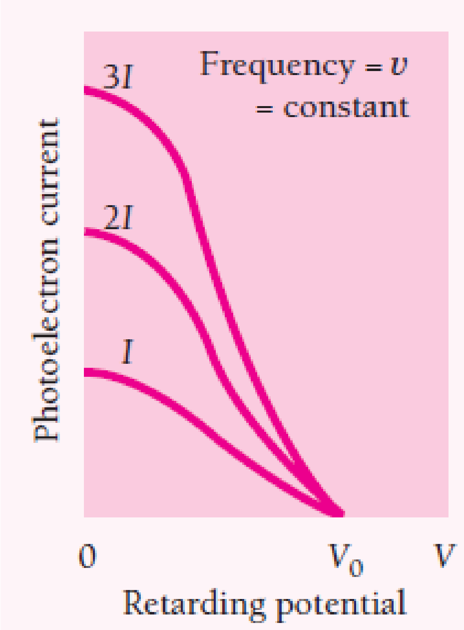


Figure 2.10 Photoelectron current is proportional to light intensity I for all retarding voltages. The stopping potential $V_0$, which corresponds to the maximum photoelectron energy, is the same for all intensities of light of the same frequency.

# 2.3 Photoelectric Effect

*3.The higher the frequency of the light, the more energy the photoelectrons have* (Fig. 2.11).

- At frequencies below a certain critical frequency $\nu_0$, which is characteristic of each particular metal, no electrons are emitted.
- Above $\nu_0$ the photoelectrons range in energy from 0 to a maximum value that increases linearly with increasing frequency (Fig.2.12).

✖ This observation, also, cannot be explained by the em theory of light.

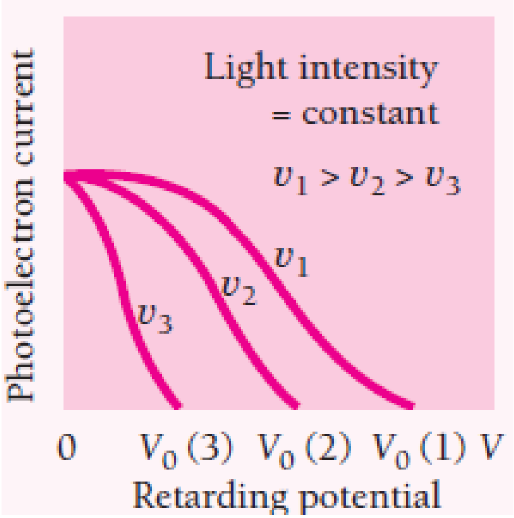


Figure 2.11 The stopping potential $V_0$, and hence the maximum photoelectron energy, depends on the frequency of the light. When the retarding potential is V=0, the photoelectron current is the same for light of a given intensity regardless of its frequency.

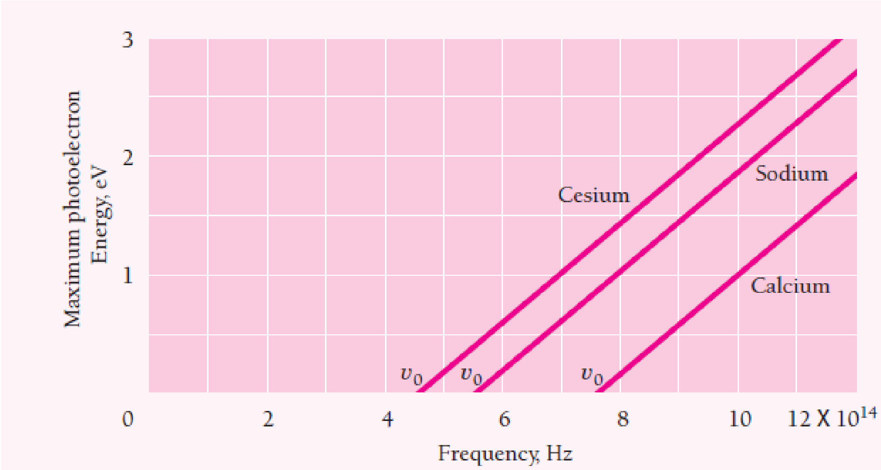


Figure 2.12 Maximum photoelectron kinetic energy $KE_{max}$ versus frequency of incident light for three metal surfaces.

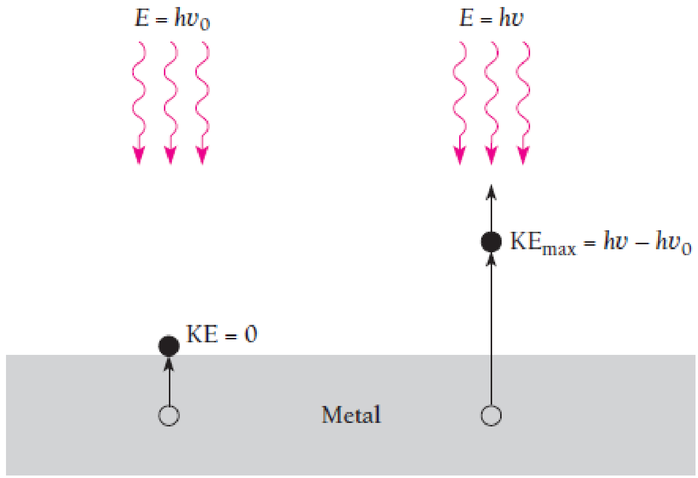
Albert Einstein
1879 – 1955
Nobel Prize in
Physics in 1921

- In 1905, Einstein realized that the photoelectric effect could be understood if the energy in light is *not spread out over wavefronts but is concentrated in small packets*, or photons.
- Each photon of light of frequency ν has the energy hν, the same as Planck's quantum energy
- **Energy was not only given to EM waves in separate quanta but was also carried by the waves in separate quanta.**
- The three experimental observations listed *above* follow directly from Einstein's hypothesis.
    1. Because EM wave energy is concentrated in photons and not spread out, there should be no delay in the emission of photoelectrons.
    2. All photons of frequency have the same energy, so changing the intensity of a monochromatic light beam will change the number of photoelectrons but not their energies.
    3. The higher the frequency ν, the greater the photon energy hν and so the more energy the photoelectrons have.

# 2.3 Photoelectric Effect; Quantum Theory of Light

- There must be a minimum energy φ for an electron to escape from a particular metal surface (Fig 2.13).
- This energy is called the *work function* of the metal, and is related to $\nu_0$ by the formula $\phi = h\nu_0$ (Work function)
- The greater the work function of a metal, the more energy is needed for an electron to leave its surface, and the higher the critical frequency for photoelectric emission to o $h\nu = KE_{max} + \phi$ (Photoelectric effect)

Figure 2.13 If the energy $h\nu_0$ (the work function of the surface) is needed to remove an electron from a metal surface, the maximum electron kinetic energy will be $h\nu - h\nu_0$ when light of frequency is directed at the surface.



where hν is the photon energy, $KE_{max}$ is the maximum photoelectron energy (which is proportional to the stopping potential), and φ is the minimum energy needed for an electron to leave the metal.

- In terms of electron volts, the formula E=hν for photon energy becomes

$$E = \left(\frac{6.626 \times 10^{-34} \ J.s}{1.602 \times 10^{-19} \ J/eV}\right)\nu = (4.136 \times 10^{-15})\nu \ \ eV.s$$

(Photon energy)

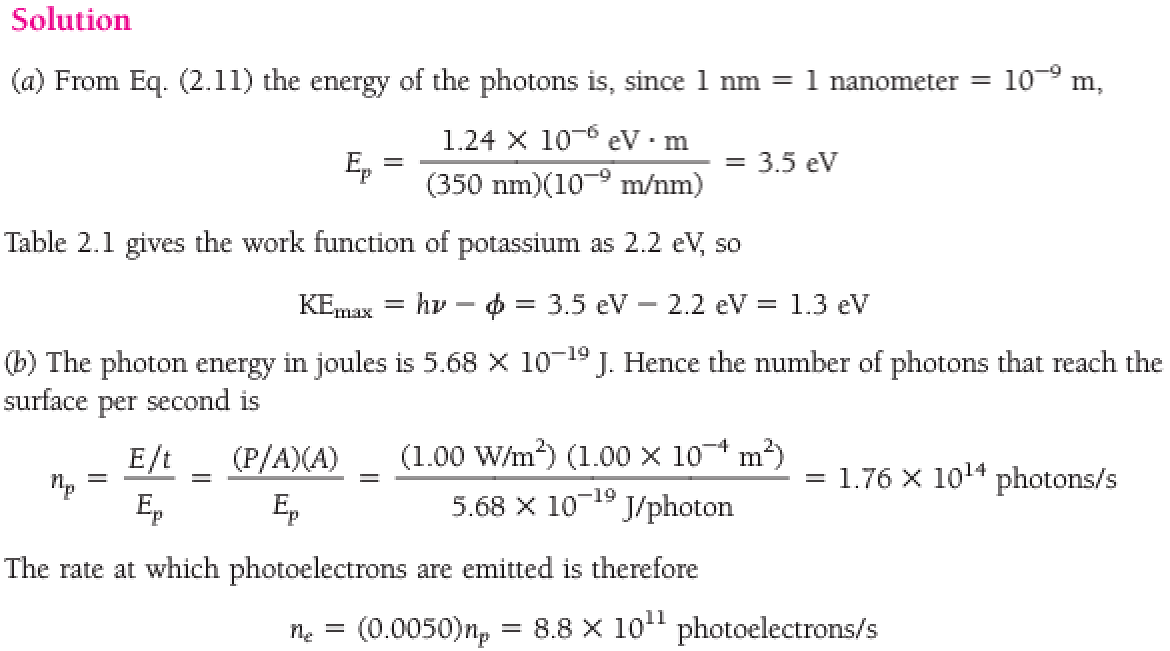
# 2.3 Photoelectric Effect; Quantum Theory of Light

## Example 2.2

Ultraviolet light of wavelength 350 nm and intensity 1.00 W/m$^2$ is directed at a potassium surface.

(a) Find the maximum KE of the photoelectrons.

(b) If 0.50 percent of the incident photons produce photoelectrons, how many are emitted per second if the potassium surface has an area of 1.00 cm$^2$?



**Solution**

(*a*) From Eq. (2.11) the energy of the photons is, since 1 nm = 1 nanometer = $10^{-9}$ m,

$$E_p = \frac{1.24 \times 10^{-6}\ \text{eV}\cdot\text{m}}{(350\ \text{nm})(10^{-9}\ \text{m/nm})} = 3.5\ \text{eV}$$

Table 2.1 gives the work function of potassium as 2.2 eV, so

$$\text{KE}_{\text{max}} = h\nu - \phi = 3.5\ \text{eV} - 2.2\ \text{eV} = 1.3\ \text{eV}$$

(*b*) The photon energy in joules is $5.68 \times 10^{-19}$ J. Hence the number of photons that reach the surface per second is

$$n_p = \frac{E/t}{E_p} = \frac{(P/A)(A)}{E_p} = \frac{(1.00\ \text{W/m}^2)\,(1.00 \times 10^{-4}\ \text{m}^2)}{5.68 \times 10^{-19}\ \text{J/photon}} = 1.76 \times 10^{14}\ \text{photons/s}$$

The rate at which photoelectrons are emitted is therefore

$$n_e = (0.0050)n_p = 8.8 \times 10^{11}\ \text{photoelectrons/s}$$

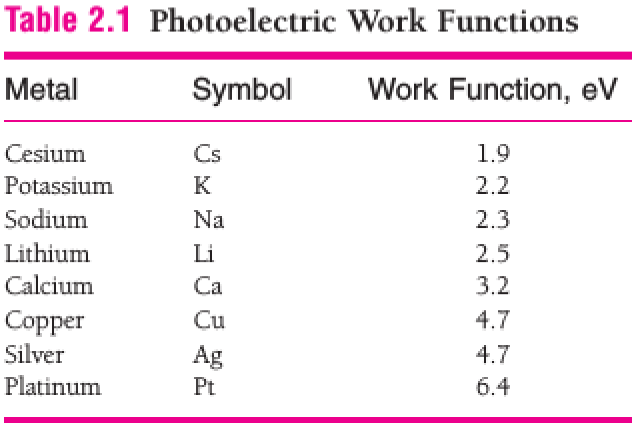


**Table 2.1** Photoelectric Work Functions

| Metal | Symbol | Work Function, eV |
|---|---|---|
| Cesium | Cs | 1.9 |
| Potassium | K | 2.2 |
| Sodium | Na | 2.3 |
| Lithium | Li | 2.5 |
| Calcium | Ca | 3.2 |
| Copper | Cu | 4.7 |
| Silver | Ag | 4.7 |
| Platinum | Pt | 6.4 |

## 2.4 What is Light?

- **Both wave and particle**.
- The wave theory of light explains diffraction and interference, for which the quantum theory cannot account.
  - According to the wave theory, light waves leave a source with their energy spread out continuously through the wave pattern.
- The quantum theory explains the photoelectric effect, for which the wave theory cannot account.
  - According to the quantum theory, light consists of individual photons, each small enough to be absorbed by a single electron.
- *Think of light as having a dual character.*
  - *The wave theory and the quantum theory complement each other.*
  - *Either theory by itself is only part of the story and can explain only certain effects*.

# 2.4 What is Light?

- In double-slit interference pattern on a screen.
  - When it passes through the slits, light is behaving as a wave does.
  - When it strikes the screen, light is behaving as a particle does.
  - Apparently light travels as a wave but absorbs and gives off energy as a series of particles.
- *In the wave model*, the light intensity at a place on the screen **depends on $E^2$**, the average over a complete cycle of the square of the instantaneous magnitude E of the EM wave’s electric field.
- *In the particle model*, this intensity **depends on Nhν**, where N is the number of photons per second per unit area that reach the same place on the screen.

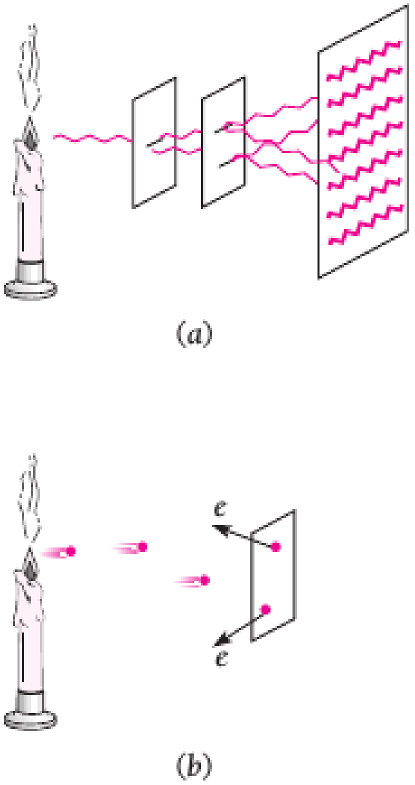


Figure 2.14 (a) The wave theory of light explains diffraction and interference, which the quantum theory cannot account for.
(b) The quantum theory explains the photoelectric effect, which the wave theory cannot account for.

- High-energy photons.
- The photoelectric effect provides convincing evidence that photons of light can transfer energy to electrons.
- Is the inverse process also possible? That is, can part or all of the kinetic energy of a moving electron be converted into a photon?
- In 1895 Wilhelm Roentgen found that a highly penetrating radiation of *unknown nature* is produced when fast electrons impinge on matter.
- The faster the original electrons, the more penetrating the resulting x-rays.
- The greater the number of electrons, the greater the intensity of the x-ray beam.
- Electromagnetic theory predicts that an *accelerated electric charge* will radiate EM waves.
  - Acceleration: Rapidly moving electron suddenly brought to rest.

Wilhelm Konrad Roentgen
(1845–1923)
First Nobel Prize in Physics in 1902

# 2.4 X-Rays: Bremsstrahlung (“braking radiation”)

- In 1912 a method was devised for measuring the wavelengths of x-rays.
- The spacing between adjacent lines on a *diffraction* grating must be of the *same order of magnitude as the wavelength* of the light.
- Max von Laue realized that the wavelengths suggested for x-rays were comparable to the ***spacing between adjacent atoms in crystals***.
- He therefore proposed that crystals be used to diffract x-rays, with their regular lattices acting as a kind of three-dimensional grating (Fig. 2.15).

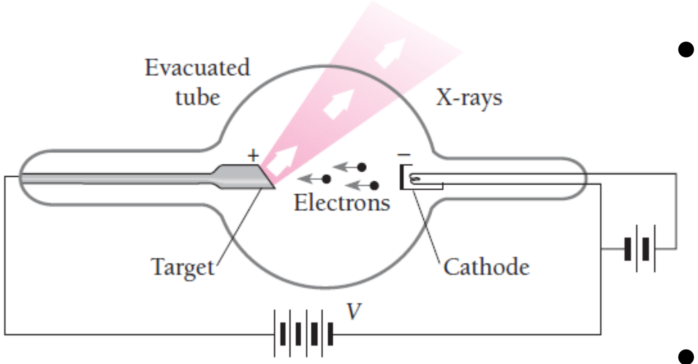


Figure 2.15 An x-ray tube. The higher the accelerating voltage V, the faster the electrons and the shorter the wavelengths of the x-rays.

- Classical electromagnetic theory predicts bremsstrahlung when electrons are accelerated, which accounts in general for the x-rays produced by an x-ray tube.
- Figures 2.16 and 2.17 show the x-ray spectra that result when tungsten and molybdenum targets are *bombarded by electrons at several different accelerating potentials*.

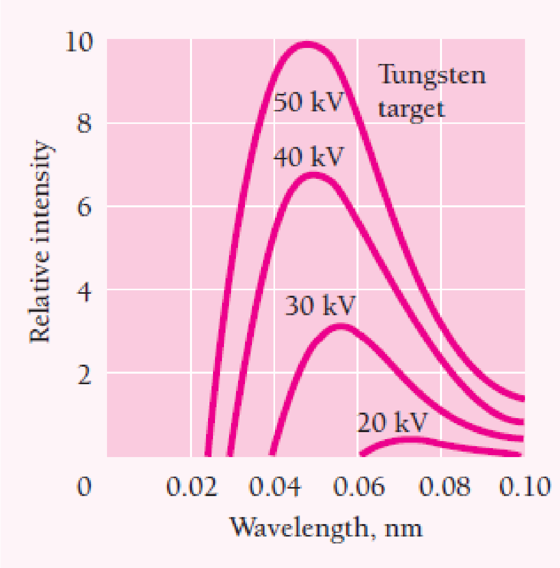


Figure 2.16 X-ray spectra of tungsten at various accelerating potentials.

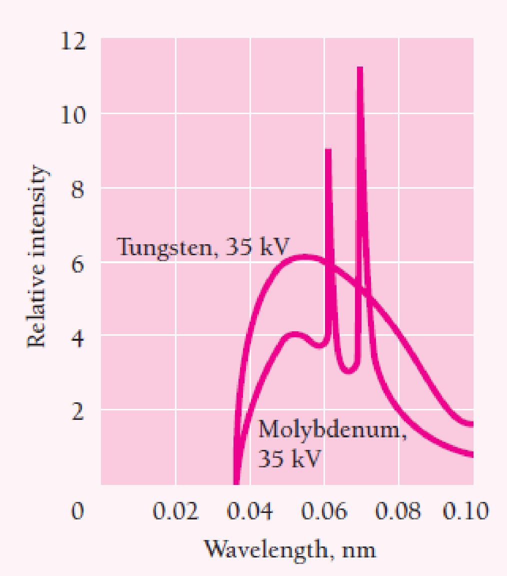


Figure 2.17 X-ray spectra of tungsten and molybdenum at 35 kV accelerating potential.

- The curves exhibit two features electromagnetic theory cannot explain:

1. In the case of molybdenum, intensity peaks occur that indicate the enhanced production of x-rays at certain wavelengths. These peaks occur at specific wavelengths for each target material and originate in rearrangements of the electron structures of the target atoms after having been disturbed by the bombarding electrons. The presence of x-rays of specific wavelengths, a decidedly non-classical effect, in addition to a continuous x-ray spectrum.
2. The x-rays produced at a given accelerating potential V vary in wavelength, but none has a wavelength shorter than a certain value $\lambda_{min}$. Increasing V decreases $\lambda_{min}$. Duane and Hunt found experimentally that

$$\lambda_{min} = \frac{1.24 \times 10^{-6}}{V} V.m$$

(X-ray production)

2.4 X-Rays: Inverse photoelectric effect

- The second observation fits in with the quantum theory of radiation.
  - Most of the electrons that strike the target undergo numerous glancing collisions, with their energy going simply into heat.
- A few electrons lose most or all of their energy in single collisions with target atoms. **This is the energy that becomes x-rays**.
- Since work functions are only a few electron-volts (eVs) whereas the accelerating potentials in x-ray tubes are typically tens or hundreds of thousands of volts, *we can ignore the work function*.
- Iinterpret the *short wavelength limit* of X-ray production Equation as corresponding to the case where the entire kinetic energy KE=Ve of a bombarding electron is given up to a single photon of energy $h\nu_{max}$.

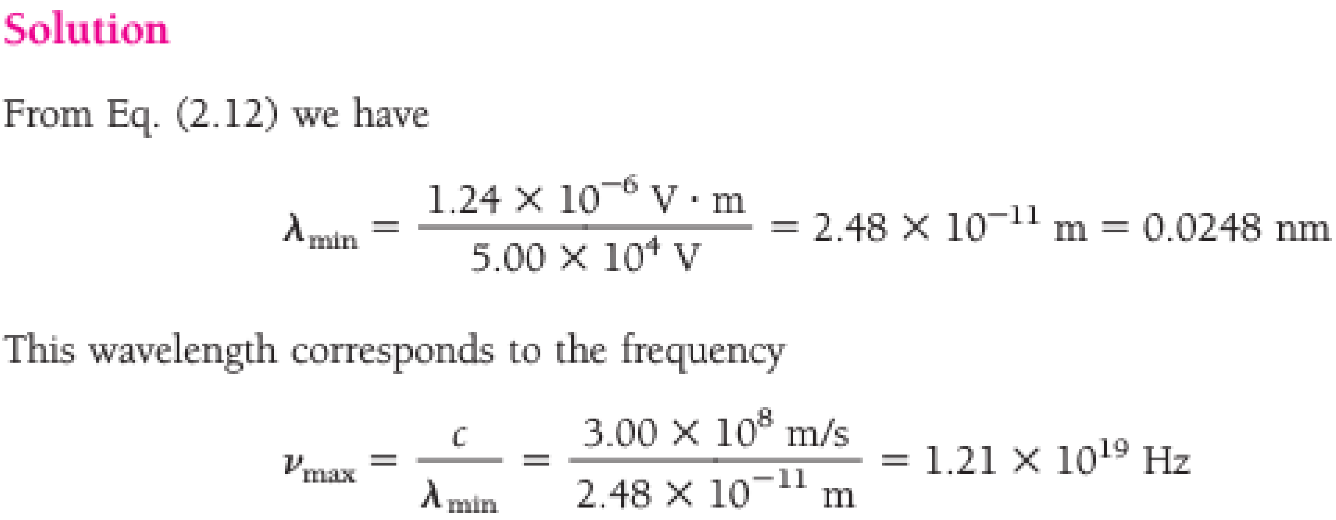
$$Ve = h\nu_{max} = \frac{hc}{\lambda_{min}}$$

(Duane-Hunt formula of X-ray production Equation)

$$\lambda_{min} = \frac{hc}{Ve} = \frac{1.24 \times 10^{-6}}{V}\ V.m$$

## Example 2.3

Find the shortest wavelength present in the radiation from an x-ray machine whose accelerating potential is 50.000 V.



**Solution**

From Eq. (2.12) we have

$$\lambda_{\min} = \frac{1.24 \times 10^{-6}\ \mathrm{V \cdot m}}{5.00 \times 10^{4}\ \mathrm{V}} = 2.48 \times 10^{-11}\ \mathrm{m} = 0.0248\ \mathrm{nm}$$

This wavelength corresponds to the frequency

$$\nu_{\max} = \frac{c}{\lambda_{\min}} = \frac{3.00 \times 10^{8}\ \mathrm{m/s}}{2.48 \times 10^{-11}\ \mathrm{m}} = 1.21 \times 10^{19}\ \mathrm{Hz}$$

# 2.6 X-Ray Diffraction: How x-ray wavelengths can be determined?

- A crystal consists of a regular array of atoms, each of which can scatter EM waves.
- The mechanism of scattering is straightforward.
  - An atom in a constant electric field becomes *polarized* since its negatively charged electrons and positively charged nucleus experience forces in opposite directions.
  - So, the result is a *distorted charge distribution* equivalent to an electric dipole.
  - In the presence of the alternating electric field of an EM wave of frequency ν, the polarization changes back and forth with the same frequency ν.
  - An **oscillating** electric dipole is thus created at the expense of some of the energy of the incoming wave.
- The **oscillating** dipole in turn radiates EM waves of frequency ν, and these secondary waves go out in all directions except along the dipole axis. (see Fig. 2.18).

# 2.6 X-Ray Diffraction: Bragg planes

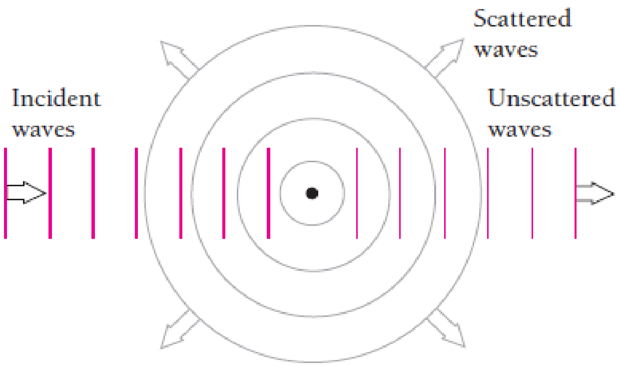
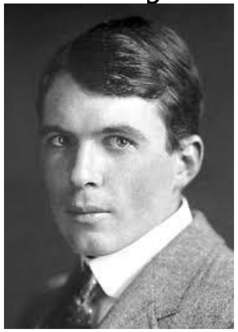


Figure 2.18 The scattering of electromagnetic radiation by a group of atoms. Incident plane waves are reemitted as spherical waves.

- A monochromatic beam of x-rays that falls upon a crystal will be *scattered in all directions* inside it.
- However, owing to the regular arrangement of the atoms, in certain directions the scattered waves will constructively interfere with one another while in others they will destructively .



William Lawrence Bragg (1890–1971) Nobel Prize in Physics in 1915

- The atoms in a crystal may be thought of as defining families of parallel planes with each family having a characteristic separation between its component planes.
- This analysis was suggested in 1913 by W. L Bragg, in honor of whom the above planes are called Bragg planes.

# 2.6 X-Ray Diffraction: Conditions

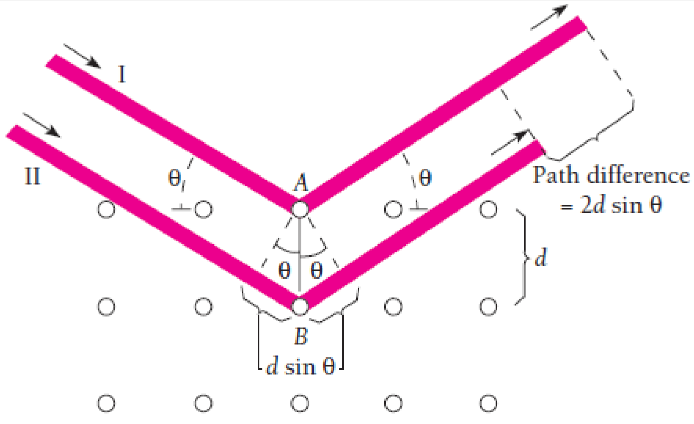


Figure 2.20 X-ray scattering from a cubic crystal.

The **conditions** that must be fulfilled for radiation scattered by crystal atoms to undergo constructive interference may be obtained from a diagram like that in Fig.2.20. A beam containing x-rays of wavelength λ is incident upon a crystal at an angle θ with a family of Bragg planes whose spacing is d.

- The beam goes past atom A in the first plane and atom B in the next, and each of them scatters part of the beam in random directions.
- Constructive interference takes place only between those scattered rays that are parallel and whose *paths differ by exactly* λ, 2λ, 3λ, and so on.
- That is, the path difference must be nλ, where n is an integer.

1.The first **condition** on I and II is that their common scattering angle be equal to the angle of incidence θ of the original beam.

2.The second **condition** is that $2dsin\theta = n\lambda \qquad n = 1, 2, 3, \ldots$

since ray II must travel the distance 2d sinθ farther than ray I. The integer *n* is the *order* of the scattered beam.

# 2.6 X-Ray Diffraction: Experimental Demonstration

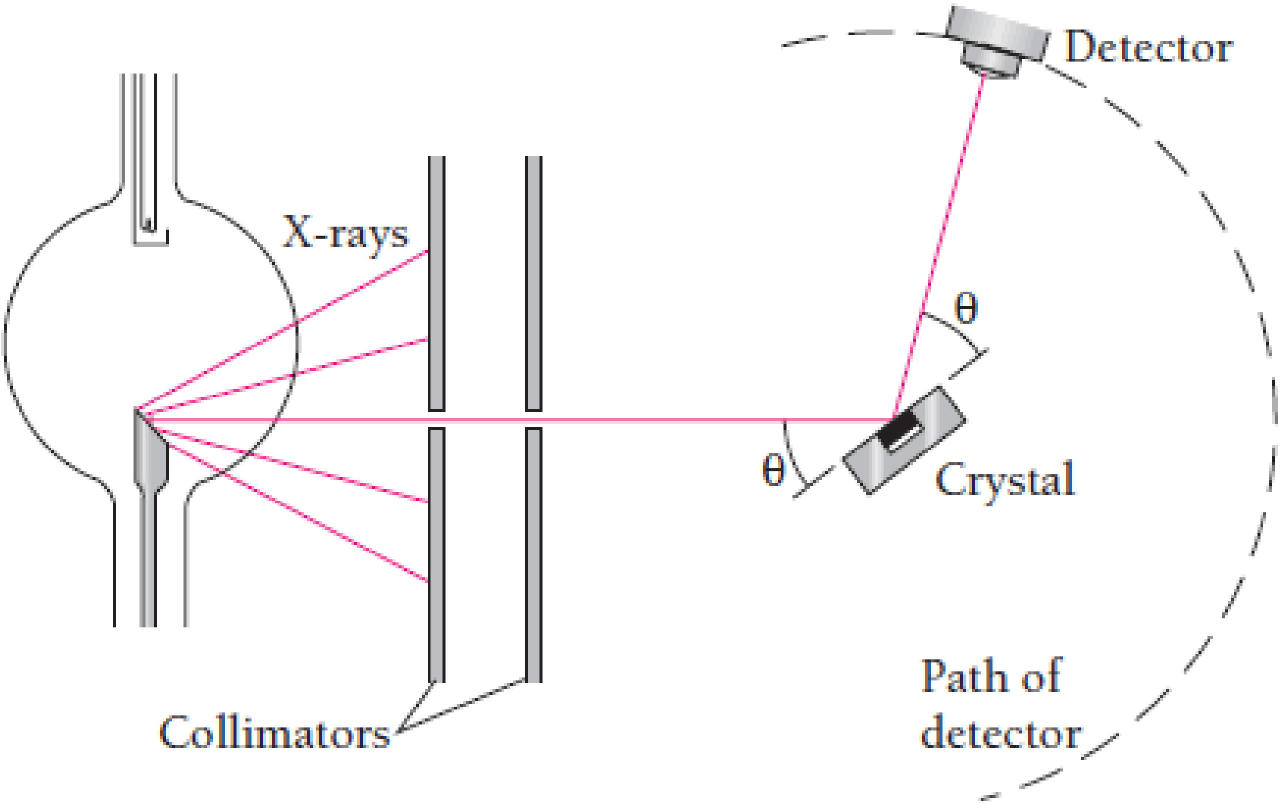


Figure 2.21 X-ray ray spectrometer.

If the spacing d between adjacent Bragg planes in the crystal is known, the x-ray wavelength λ may be calculated.

# 2.7 Compton Effect

- *Further confirmation of the photon model.*
- According to the quantum theory of light, photons behave like particles except for their lack of rest mass.
- Figure 2.22 shows a collision: an x-ray photon strikes an electron (assumed to be initially at rest in the laboratory coordinate system).

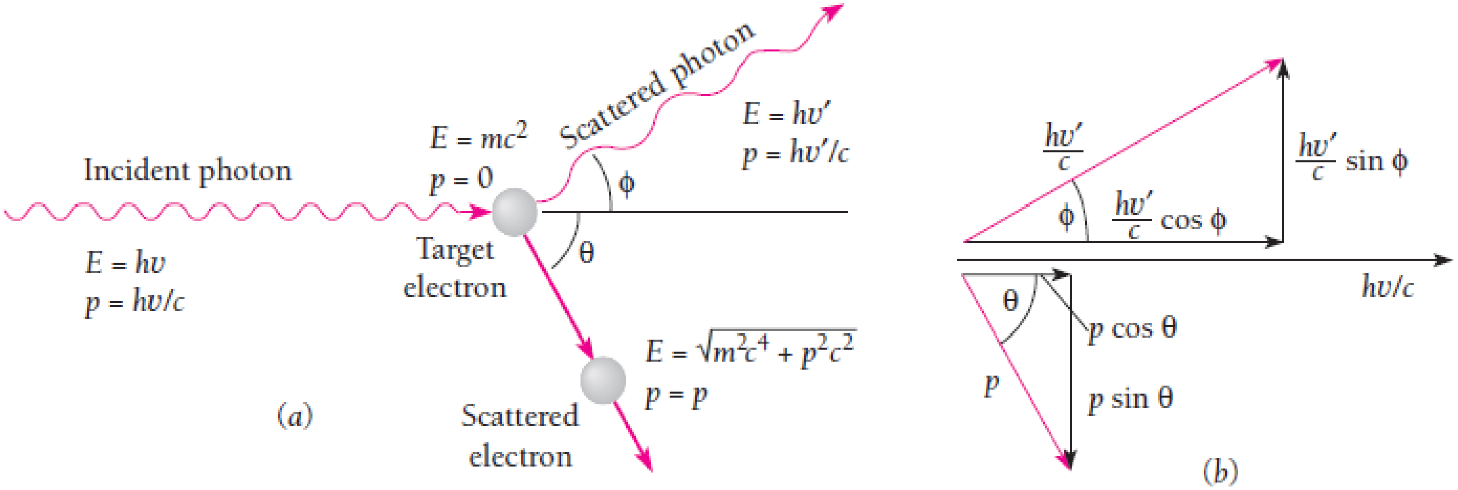


Figure 2.22 (a) The scattering of a photon by an electron is called the Compton effect. **Energy and momentum are conserved** in such an event, and as a result the scattered photon has less energy (longer wavelength) than the incident photon. (b) Vector diagram of the momenta and their components of the incident and scattered photons and the scattered electron.

- The momentum of a massless particle is related to its energy by the formula $E = pc$

# 2.7 Compton Effect: Conservations

- We can think of the photon as losing an amount of energy in the collision that is the same as the kinetic energy KE gained by the electron.

$$Loss\ in\ photon\ energy = gain\ in\ electron\ energy$$

$$h\nu - h\nu' = KE \qquad (1)$$

- Since the energy of a photon is hν, its momentum is

$$p = \frac{E}{c} = \frac{h\nu}{c} \qquad \text{(Photon momentum)}$$

- In the collision *momentum must be conserved* in each of two mutually perpendicular directions.

$$initial\ momentum = final\ momentum$$

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c} cos\phi + pcos\theta$$

(In the original photon direction) (2)

$$initial\ momentum = final\ momentum$$

$$0 = \frac{h\nu'}{c} sin\phi - psin\theta$$

(In perpendicular to original photon direction) (3)

- The angle ϕ is that between the directions of the initial and scattered photons, and θ is that between the directions of the initial photon and the recoil electron.

From Eqns. (1-3) we can find a formula that relates ….

# 2.7 Compton Effect: **Compton wavelength**

- From Eqns. (1-3) we can find a formula that relates the **wavelength difference** between initial and scattered photons with the angle ϕ between their directions, both of which are readily measurable quantities (unlike the energy and momentum of the recoil electron).

$$\lambda' - \lambda = \lambda_C(1 - cos\phi)$$ (Compton effect)

- Compton effect equation was derived by Arthur H. Compton in the early 1920s, and the phenomenon it describes is known as the Compton effect.
- *It constitutes very strong evidence in support of the quantum theory of radiation.*
- Change in wavelength is independent of the wavelength λ of the incident photon.
- The quantity

$$\lambda_C = \frac{h}{mc}$$ (Compton wavelength)

Arthur Holly Compton (1892–1962) Nobel Prize in Physics in 1927

is called the **Compton wavelength** of the scattering particle.

For an electron $\lambda_C = 2.426 \times 10^{-12}$ m, which is 2.426 pm. (1 pm=1 picometer=$10^{-12}$ m)

# 2.7 Compton Effect: Experimental Demonstration

- From Compton effect equation, we note that the greatest wavelength change possible corresponds to ϕ=180°. $\lambda' - \lambda = \lambda_C(1 - cos\phi)$
- Changes of this magnitude or less are readily *observable only in x-rays*.
- The Compton effect is the main reasoning that which *x-rays lose energy when they pass through matter*.
- The experimental demonstration of the Compton effect is straightforward.

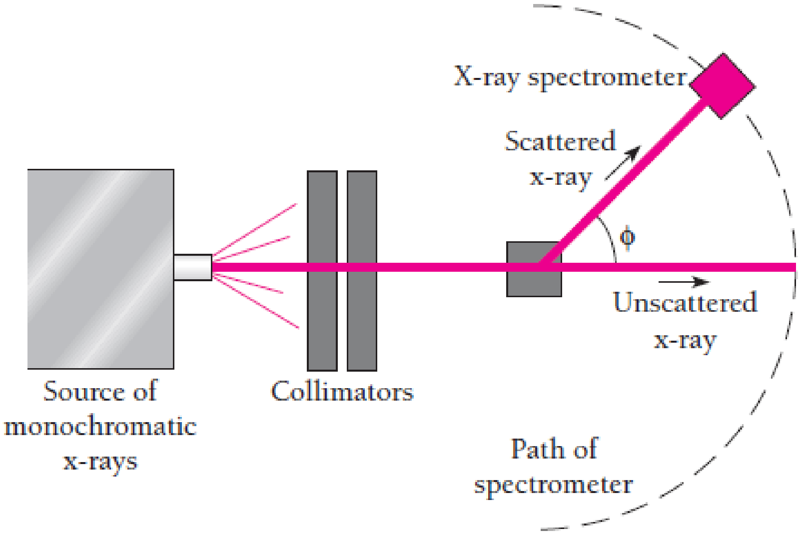


Figure 2.23 Experimental demonstration of the Compton effect.

- As in Fig. 2.23, a beam of x-rays of a single, known wavelength is directed at a target, and the wavelengths of the scattered x-rays are determined at various angles.
- The results are shown in Fig. 2.24.

# 2.7 Compton Effect

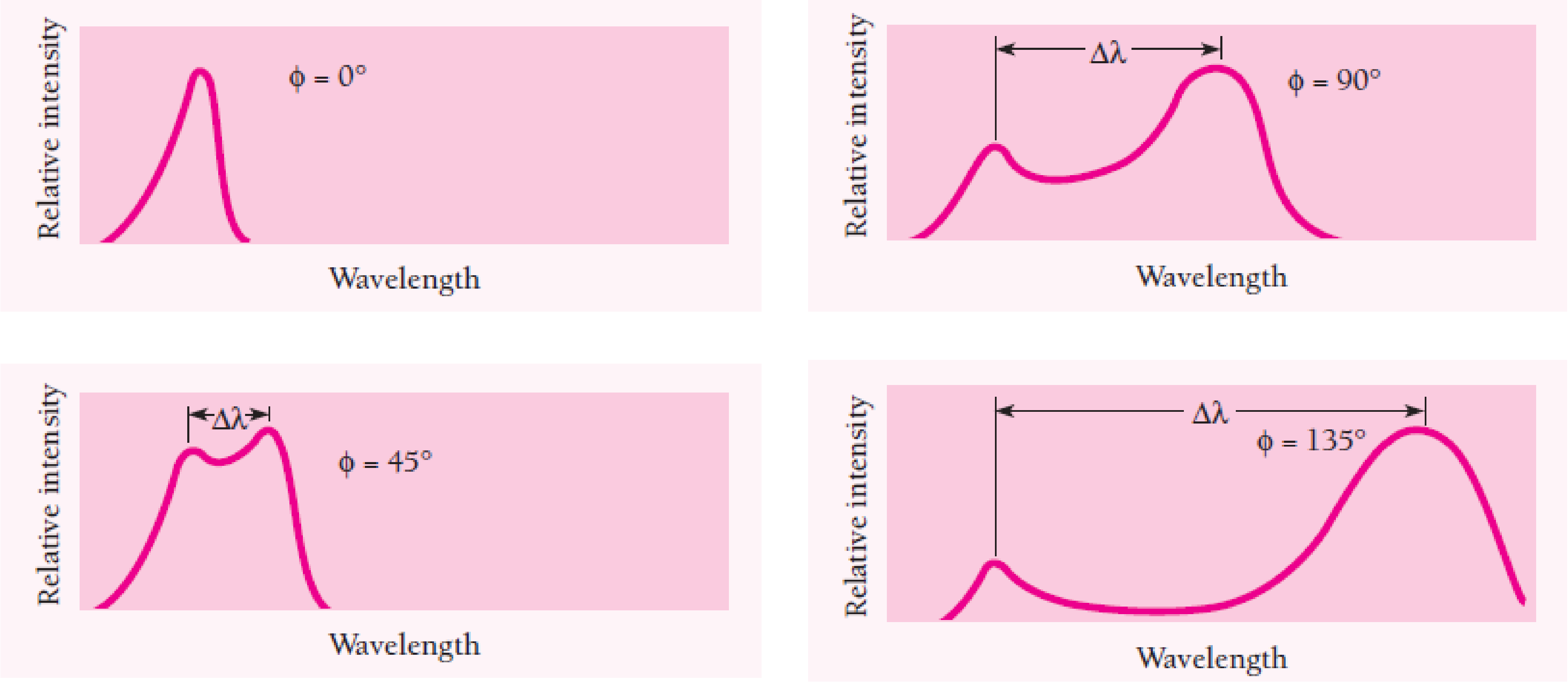
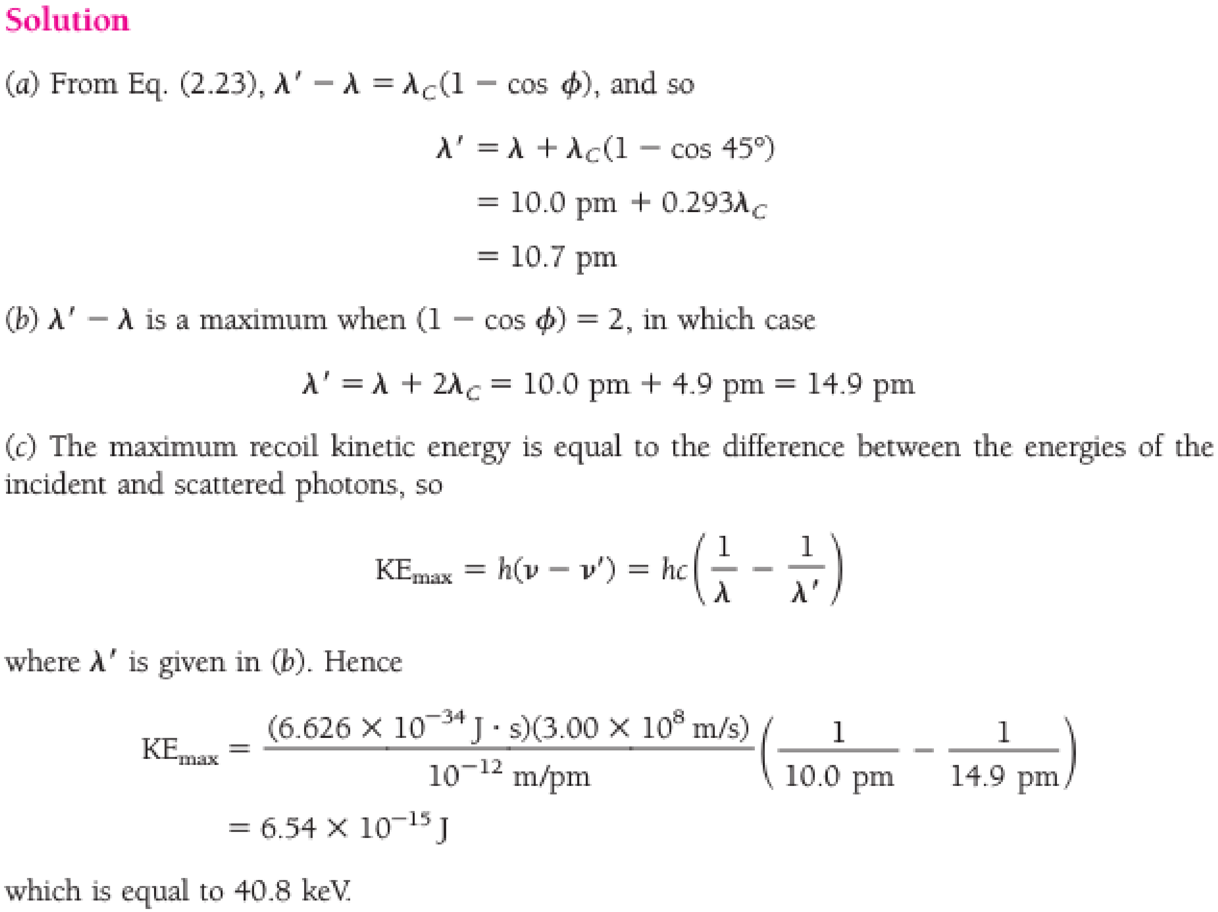


Figure 2.24 Experimental confirmation of Compton scattering. The greater the scattering angle, the greater the wavelength change, in accord with Compton effect equation.

# Example 2.4

X-rays of wavelength 10.0 pm are scattered from a target. (a) Find the wavelength of the x-rays scattered through 45°. (b) Find the maximum wavelength present in the scattered x-rays. (c) Find the maximum kinetic energy of the recoil electrons.



**Solution**

(*a*) From Eq. (2.23), $\lambda' - \lambda = \lambda_C(1 - \cos\phi)$, and so

$$\begin{aligned}\lambda' &= \lambda + \lambda_C(1 - \cos 45°)\\ &= 10.0\ \text{pm} + 0.293\lambda_C\\ &= 10.7\ \text{pm}\end{aligned}$$

(*b*) $\lambda' - \lambda$ is a maximum when $(1 - \cos\phi) = 2$, in which case

$$\lambda' = \lambda + 2\lambda_C = 10.0\ \text{pm} + 4.9\ \text{pm} = 14.9\ \text{pm}$$

(*c*) The maximum recoil kinetic energy is equal to the difference between the energies of the incident and scattered photons, so

$$\text{KE}_{\text{max}} = h(\nu - \nu') = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

where $\lambda'$ is given in (*b*). Hence

$$\begin{aligned}\text{KE}_{\text{max}} &= \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{10^{-12}\ \text{m/pm}}\left(\frac{1}{10.0\ \text{pm}} - \frac{1}{14.9\ \text{pm}}\right)\\ &= 6.54\times10^{-15}\ \text{J}\end{aligned}$$

which is equal to 40.8 keV.

1. (a) What are the energy and momentum of a photon of red light of wavelength 650 nm? (b) What is the wavelength of a photon of energy 2.40 eV?

A photon of red light

$\lambda = 650\ nm$

Energy and momentum

$h\nu \qquad pc$

a) $E = h\nu = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ J.s)(3\times10^{8}\ m/s)}{650\times10^{-9}\ m} = \underline{\underline{3.06\times10^{-19}\ J}}$

$$\frac{3.06\times10^{-19}\ J}{1.60\times10^{-19}\ J/eV} = \underline{\underline{1.91\ eV}}$$

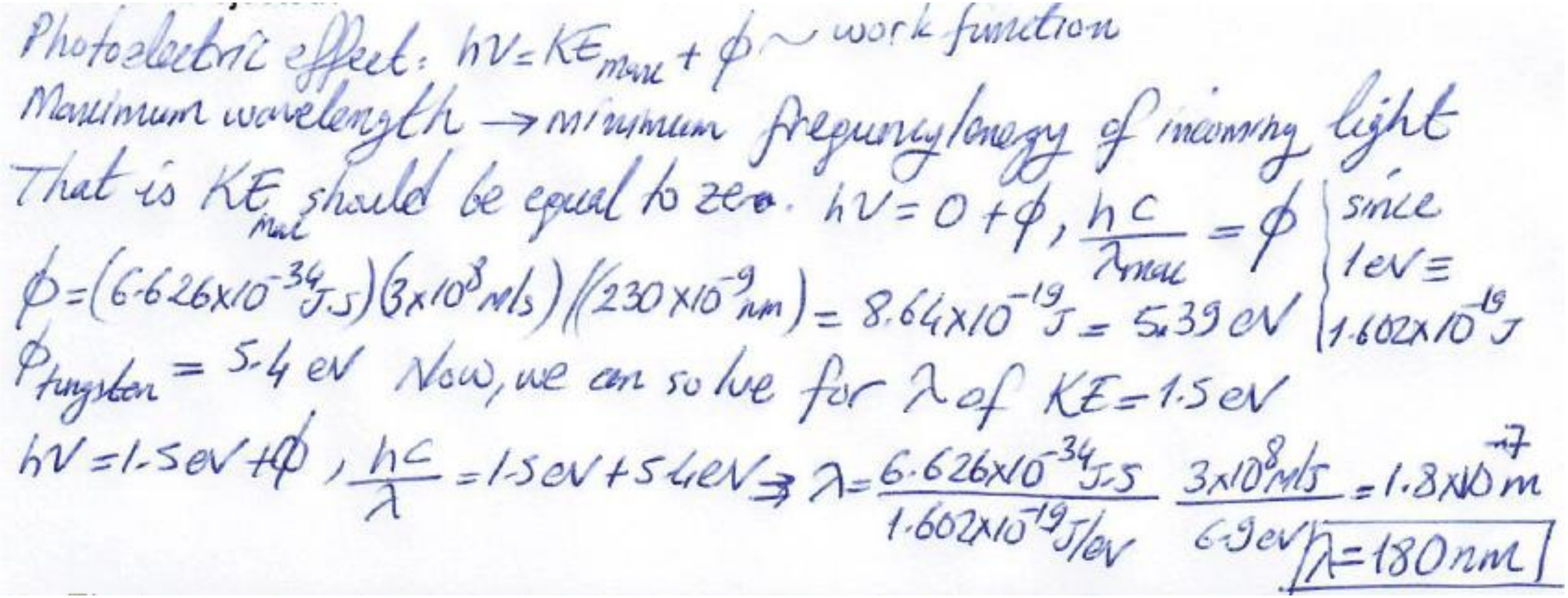
$E = pc \Rightarrow 3.06\times10^{-19}\ J = p(3\times10^{8}\ m/s) \rightarrow p = \underline{\underline{1.02\times10^{-27}\ \frac{J.s}{m}}}$

b) $E = 2.40\ eV \quad \lambda = ?$

$E = h\nu = \frac{hc}{\lambda} \rightarrow \lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\ J.s)(3\times10^{8}\ m/s)}{(2.40\ eV)(1.6\times10^{-19}\ J/eV)}$

$= 5.18\times10^{-7}\ m = \underline{\underline{518\ nm}}$

2. The maximum wavelength for photoelectric emission in tungsten is 230 nm. What wavelength of light must be used in order for electrons with a maximum energy of 1.5 eV to be ejected?



Photoelectric effect: $h\nu = KE_{max} + \phi$ ~ work function

Maximum wavelength → minimum frequency/energy of incoming light

That is $KE_{max}$ should be equal to zero. $h\nu = 0 + \phi$, $\frac{hc}{\lambda_{max}} = \phi$

$\phi = (6.626\times10^{-34} J.s)(3\times10^{8} m/s)/(230\times10^{-9} nm) = 8.64\times10^{-19} J = 5.39 eV$

since $1 eV \equiv 1.602\times10^{-19} J$

$\phi_{tungsten} = 5.4 eV$ Now, we can solve for $\lambda$ of $KE = 1.5 eV$

$h\nu = 1.5 eV + \phi$, $\frac{hc}{\lambda} = 1.5 eV + 5.4 eV \Rightarrow \lambda = \frac{6.626\times10^{-34} J.s}{1.602\times10^{-19} J/eV} \frac{3\times10^{8} m/s}{6.9 eV} = 1.8\times10^{-7} m$

$\boxed{\lambda = 180 nm}$

3. The work function for tungsten metal is 4.52 eV. (a) What is the cutoff wavelength $\lambda_c$ for tungsten? (b) What is the maximum kinetic energy of the electrons when radiation of wavelength 198 nm is used? (c) What is the stopping potential in this case?

Tungsten
$\phi = 4.52\,eV$

a) $\lambda_{cut\text{-}off} = ?$

$$\phi = h\nu = \frac{hc}{\lambda_c} = 4.52\,eV$$

$$\rightarrow \lambda_c = \frac{hc}{4.52\,eV} = \frac{(6.63\times10^{-34}\,J.s)(3\times10^{8}\,m/s)}{4.52\,eV\ \ 1.6\times10^{-19}\,J/eV} = 275\times10^{-9}\,m$$

$$= \underline{\underline{275\,nm}}$$

b) $\lambda = 198\,nm$
$KE_{max} = ?$

$$h\nu = KE_{max} + \phi$$

$$KE_{max} = \frac{hc}{\lambda} - \phi = \frac{(6.63\times10^{-34}\,J.s)(3\times10^{8}\,m/s)}{198\times10^{-9}\,m} - 4.52\,eV\ 1.6\times10^{-19}\,\frac{J}{eV}$$

$$= 2.81\times10^{-19}\,J = \underline{\underline{1.76\,eV}}$$

c) stopping voltage corresponds to $KE_{max}$ : $V_s = \dfrac{KE_{max}}{e} = \dfrac{1.74\,eV}{e}$

$$= \underline{\underline{1.74\,V}}$$

# 2 Solved Problems

4. A laser beam with an intensity of 120W/m$^2$ (roughly that of a small helium-neon laser) is incident on a surface of sodium. It takes a minimum energy of 2.3 eV to release an electron from sodium (the work function φ of sodium). Assuming the electron to be confined to an area of radius equal to that of a sodium atom (0.10 nm), how long will it take for the surface to absorb enough energy to release an electron?

Laser beam

Intensity $= 120\ W/m^2$

$\phi = 2.3\ eV$

target area } $\pi R^2$

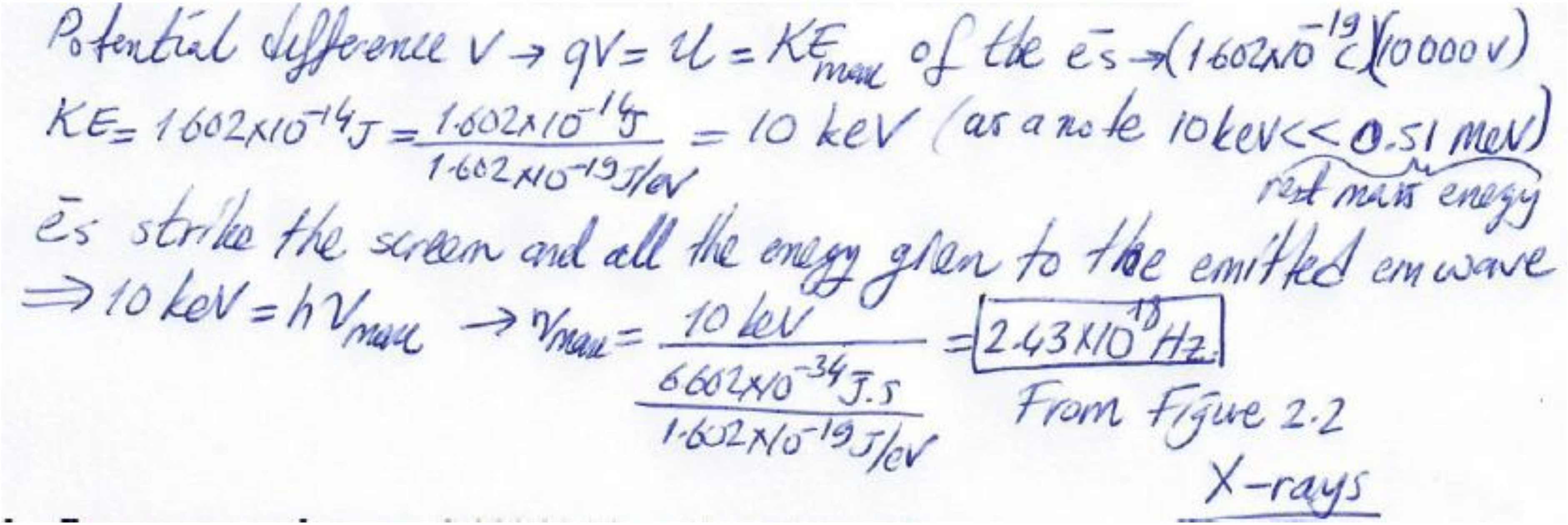
$R = 0.10 \times 10^{-9}\ m$

$\Delta t = ?$ $\quad P = \frac{\Delta E}{\Delta t} \rightarrow IA = \frac{\Delta E}{\Delta t} \rightarrow IA = \frac{\phi}{\Delta t} \rightarrow \Delta t = \frac{\phi}{IA}$

$I = \frac{P}{A}$

$$\rightarrow \Delta t = \frac{(2.3\,eV)(1.6 \times 10^{-19}\ J/eV)}{\left(120\ \frac{J}{s\,m^2}\right)\pi(0.1 \times 10^{-9}\ m)^2} = \underline{\underline{0.10\ s}}$$

5. Electrons are accelerated in television tubes through potential differences of about 10 kV. Find the highest frequency of the electromagnetic waves emitted when these electrons strike the screen of the tube. What kind of waves are these?



Potential difference V $\rightarrow qV = \mathcal{U} = KE_{max}$ of the $\bar{e}s \rightarrow (1.602\times10^{-19}\,C)(10000\,V)$

$KE = 1.602\times10^{-14}\,J = \frac{1.602\times10^{-14}\,J}{1.602\times10^{-19}\,J/eV} = 10\,keV$ (as a note $10\,keV \ll \underbrace{0.51\,MeV}_{\text{rest mass energy}}$)

$\bar{e}s$ strike the screen and all the energy given to the emitted em wave

$\Rightarrow 10\,keV = h\nu_{max} \rightarrow \nu_{max} = \frac{10\,keV}{\frac{6.602\times10^{-34}\,J.s}{1.602\times10^{-19}\,J/eV}} = \boxed{2.43\times10^{18}\,Hz}$

From Figure 2.2

X-rays

6. A single crystal of table salt (NaCl) is irradiated with a beam of X rays of wavelength 0.250 nm, and the first Bragg reflection is observed at an angle of 26.3°. What is the atomic spacing of NaCl?

Single crystal NaCl
$\lambda = 0.25\,nm$ (X-ray)
FIRST Bragg reflection $\rightarrow n = 1$
$\theta = 26.3°$

$d = ?$ $\quad 2d\sin\theta = n\lambda$

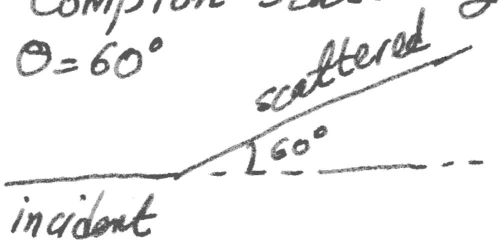
$$d = \frac{(1)(0.25\times10^{-9}\,m)}{2\sin 26.3°} = \underline{\underline{0.282\,nm}}$$

7. X rays of wavelength 0.2400 nm are Compton-scattered, and the scattered beam is observed at an angle of 60.0◦ relative to the incident beam. Find: (a) the wavelength of the scattered X rays, (b) the energy of the scattered X-ray photons, and (c) the kinetic energy of the scattered electrons

$\lambda = 0.2400\ nm$ (X-ray)

Compton Scattering

$\Theta = 60^\circ$



$\lambda' - \lambda = \lambda_c(1-\cos\phi)$

a) $\lambda' = ?$ $\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\phi) \rightarrow \lambda' = \lambda + \frac{h}{m_e c}(1-\cos\phi)$

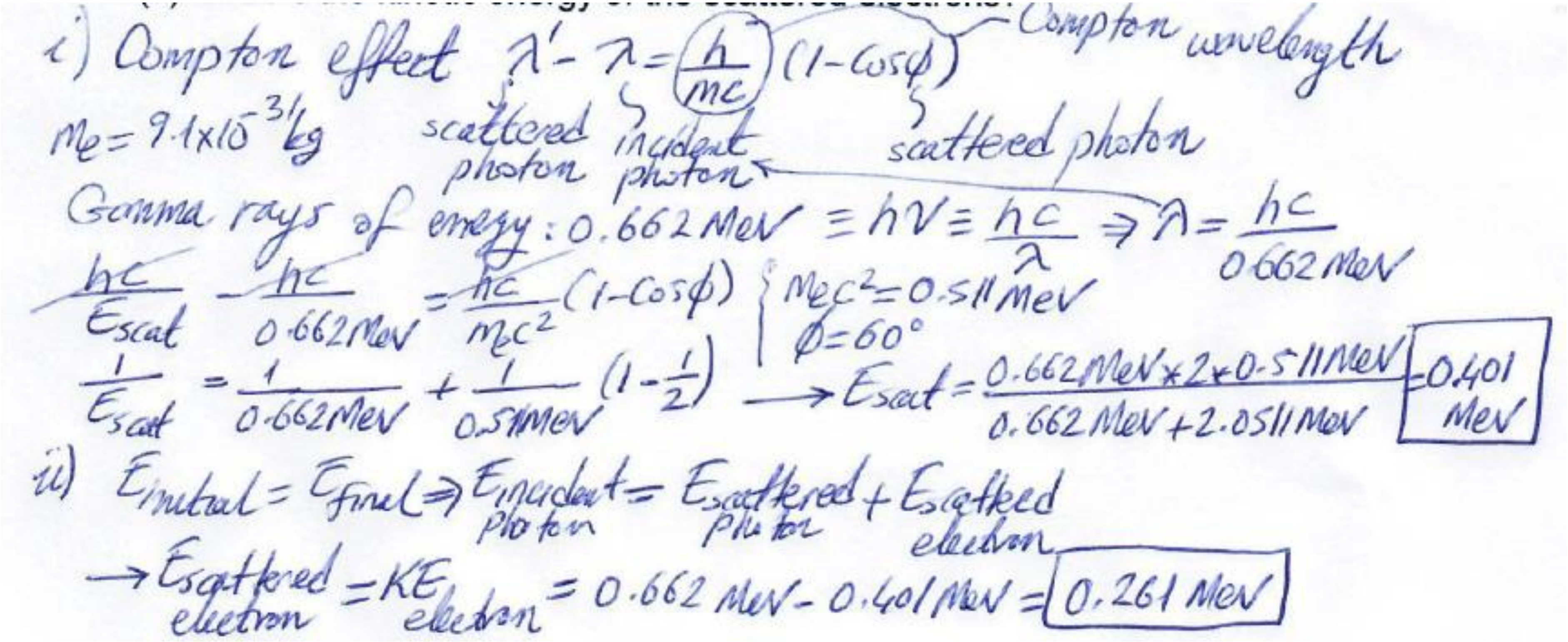
$\rightarrow \lambda' = 0.2400\times10^{-9}\,m + \frac{6.63\times10^{-34}\,J.s}{(9.1\times10^{-31}\,kg)(3\times10^{8}\,m)}(1-\cos\underset{\overset{5}{60^\circ}}{\phi}) = \underline{\underline{0.2412\ nm}}$

b) Energy of scattered photon $E = h\nu' = \frac{hc}{\lambda'} = 5141\ eV$

c) $KE_{electron} = h\nu - h\nu' = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = hc\left(\frac{1}{0.2400nm} - \frac{1}{0.2412nm}\right)$

$= \underline{\underline{26\ eV}}$

8. Gamma rays of energy 0.662 MeV are Compton scattered.
   a. What is the energy of the scattered photon observed at a scattering angle of $60^o$?
   b. What is the kinetic energy of the scattered electrons?



i) Compton effect $\underbrace{\lambda'}_{\text{scattered photon}} - \underbrace{\lambda}_{\text{incident photon}} = \overbrace{\boxed{\frac{h}{mc}}}^{\text{Compton wavelength}}(1-\cos\phi)$ scattered photon

$M_e = 9.1\times10^{-31}\text{ kg}$

Gamma rays of energy: $0.662\text{ MeV} \equiv h\nu \equiv \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{0.662\text{ MeV}}$

$\frac{\cancel{hc}}{E_{scat}} - \frac{\cancel{hc}}{0.662\text{ MeV}} = \frac{\cancel{hc}}{M_e c^2}(1-\cos\phi)$ $\left\{ \begin{array}{l} M_e c^2 = 0.511\text{ MeV} \\ \phi = 60^\circ \end{array} \right.$

$\frac{1}{E_{scat}} = \frac{1}{0.662\text{ MeV}} + \frac{1}{0.511\text{ MeV}}\left(1-\frac{1}{2}\right) \longrightarrow E_{scat} = \frac{0.662\text{ MeV}\times 2\times 0.511\text{ MeV}}{0.662\text{ MeV} + 2\cdot 0.511\text{ MeV}} = \boxed{0.401\text{ MeV}}$

ii) $E_{initial} = E_{final} \Rightarrow E_{\substack{\text{incident}\\\text{photon}}} = E_{\substack{\text{scattered}\\\text{photon}}} + E_{\substack{\text{scattered}\\\text{electron}}}$

$\rightarrow E_{\substack{\text{scattered}\\\text{electron}}} = KE_{\text{electron}} = 0.662\text{ MeV} - 0.401\text{ MeV} = \boxed{0.261\text{ MeV}}$

# Additional Materials

# 2.8 Pair Production

1. Energy into matter.
2. As we have seen, in a collision a photon can give an electron all of its energy (the photoelectric effect) or only part (the Compton effect).
3. It is also possible for a photon to materialize into an electron and a positron, which is a positively charged electron. In this process, called pair production, electromagnetic energy is converted into matter.

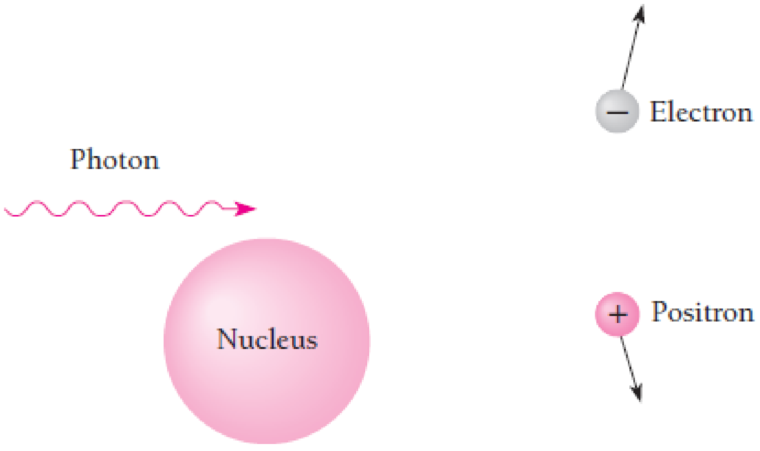


Figure 2.25 In the process of pair production, a photon of sufficient energy materializes into an electron and a positron.

- No conservation principles are violated when an electron-positron pair is created near an atomic nucleus (Fig. 2.25).
  - The sum of the charges of the electron (q=-e) and of the positron (q=e) is zero, as is the charge of the photon
  - The total energy, including rest energy, of the electron and positron equals the photon energy
  - Linear momentum is conserved with the help of the nucleus, which carries away enough photon momentum for the process to occur.

## 2.8 Pair Production

- The rest energy $mc^2$ of an electron or positron is 0.51 MeV, hence pair production requires a photon energy of at least 1.02 MeV.
- Any additional photon energy becomes kinetic energy of the electron and positron. The corresponding maximum photon wavelength is 1.2 pm. Electromagnetic waves with such wavelengths are called gamma rays, symbol γ, and are found in nature as one of the emissions from radioactive nuclei and in cosmic rays.
- The inverse of pair production occurs when a positron is near an electron and the two come together under the influence of their opposite electric charges. Both particles vanish simultaneously, with the lost mass becoming energy in the form of two gamma-ray photons:

$$e^+ + e^- \rightarrow \gamma + \gamma$$

(Pair annihilation)

- The total mass of the positron and electron is equivalent to 1.02 MeV, and each photon has an energy hν of 0.51 MeV plus half the kinetic energy of the particles relative to their center of mass.
- The directions of the photons are such as to conserve both energy and linear momentum, and no nucleus or other particle is needed for this pair annihilation to take place.

- The three chief ways in which photons of light, x-rays, and gamma rays interact with matter are summarized in Fig. 2.27.

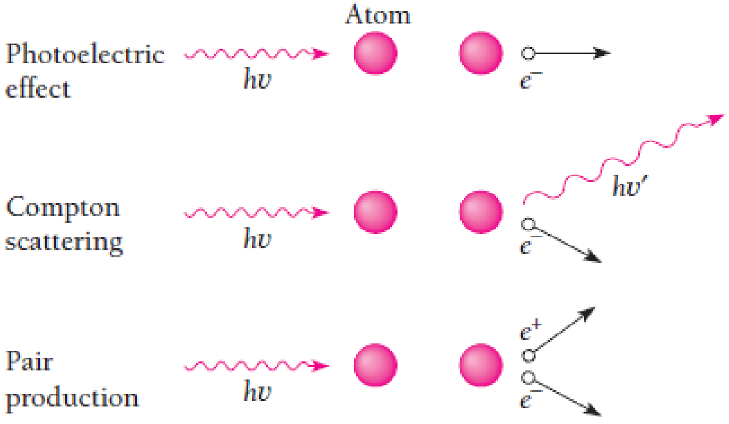


Figure 2.27 X- and gamma rays interact with matter chiefly through the photoelectric effect, Compton scattering, and pair production. Pair production requires a photon energy of at least 1.02 MeV.

- In all cases photon energy is transferred to electrons which in turn lose energy to atoms in the absorbing material.
- At low photon energies, the photoelectric effect is the main mechanism of energy loss.
- The importance of the photoelectric effect decreases with increasing energy, to be succeeded by Compton scattering.

# 2.8 Pair Production: Photon Absorption

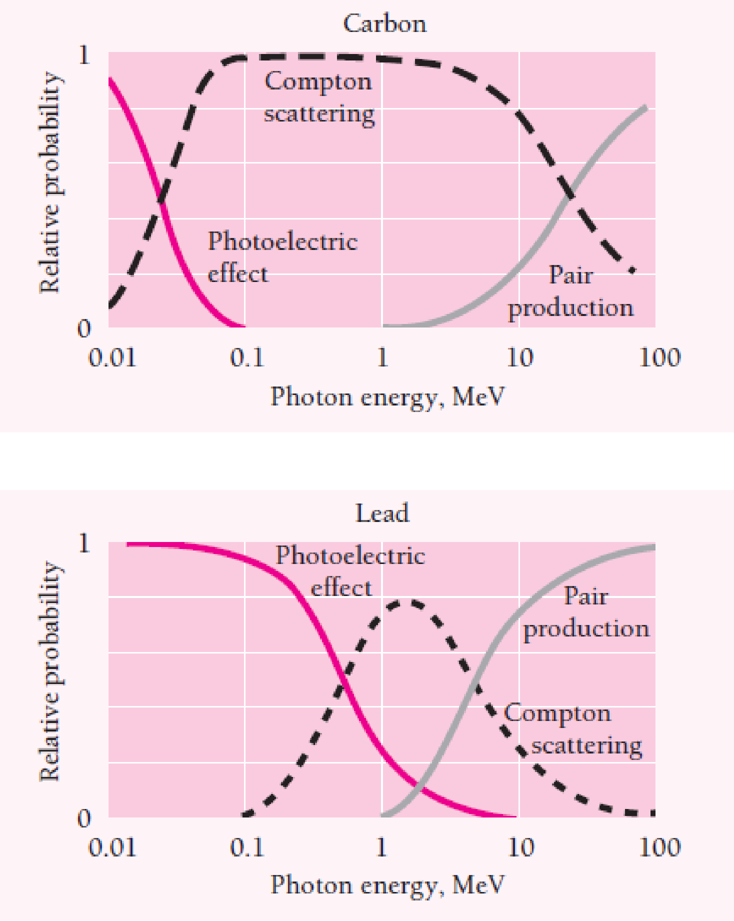


Figure 2.28 The relative probabilities of the photoelectric effect, Compton scattering, and pair production as functions of energy in carbon (a light element) and lead (a heavy element).

- The greater the atomic number of the absorber, the higher the energy at which the photoelectric effect remains significant.
- In the lighter elements, Compton scattering becomes dominant at photon energies of a few tens of keV,
- whereas in the heavier ones this does not happen until photon energies of nearly 1 MeV are reached (Fig. 2.28).