

# Chapter 2: Particle Properties of Waves

- Particle: Mechanics
- Waves: Optics. Carry energy and momentum from one place to another.
- Microscopic world of atoms and molecules, electrons and nuclei: in this world, there are neither particles nor waves in our sense.
- We regard electrons as particles because they possess charge and mass and behave according to the laws of particle mechanics
  - However, that it is just as correct to interpret a moving electron as a wave as it is to interpret it as a particle.
- We regard electromagnetic waves as waves because under suitable circumstances they exhibit diffraction, interference, and polarization.
  - Similarly, we shall see that under other circumstances electromagnetic waves behave as though they consist of streams of particles.
- The wave-particle duality is central to an understanding of modern physics.

## 2.1 ELECTROMAGNETIC WAVES

- Coupled electric and magnetic oscillations that move with the speed of light and exhibit typical wave behavior.
- Accelerated electric charges generate linked electric and magnetic disturbances that can travel indefinitely through space.
- **If the charges oscillate periodically**, the disturbances are waves whose electric and magnetic components are perpendicular to each other and to the direction of propagation, as in Fig. 2.1.

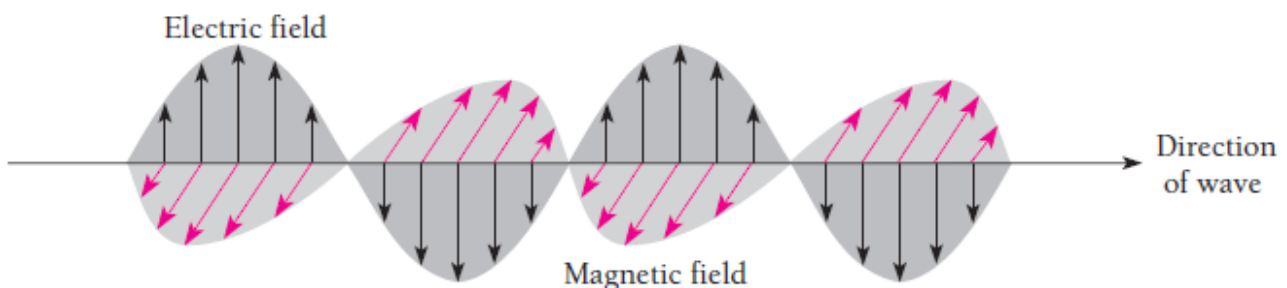


Figure 2.1 The electric and magnetic fields in an electromagnetic wave vary together. The fields are perpendicular to each other and to the direction of propagation of the wave.



James Clerk  
Maxwell  
(1831–1879)

Maxwell was able to show that the speed  $c$  of electromagnetic waves in free space is given by

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 2.998 \times 10^8 \text{ m/s}$$

where  $\epsilon_0$  the electric permittivity of free space and  $\mu_0$  is its magnetic permeability. This is the same as the speed of light waves.

- Many features of EM waves interaction with matter depend upon their frequencies. Light waves, which are em waves the eye responds to, span only a brief frequency interval, from about  $4.3 \times 10^{14}$  Hz for red light to about  $7.5 \times 10^{14}$  Hz for violet light.

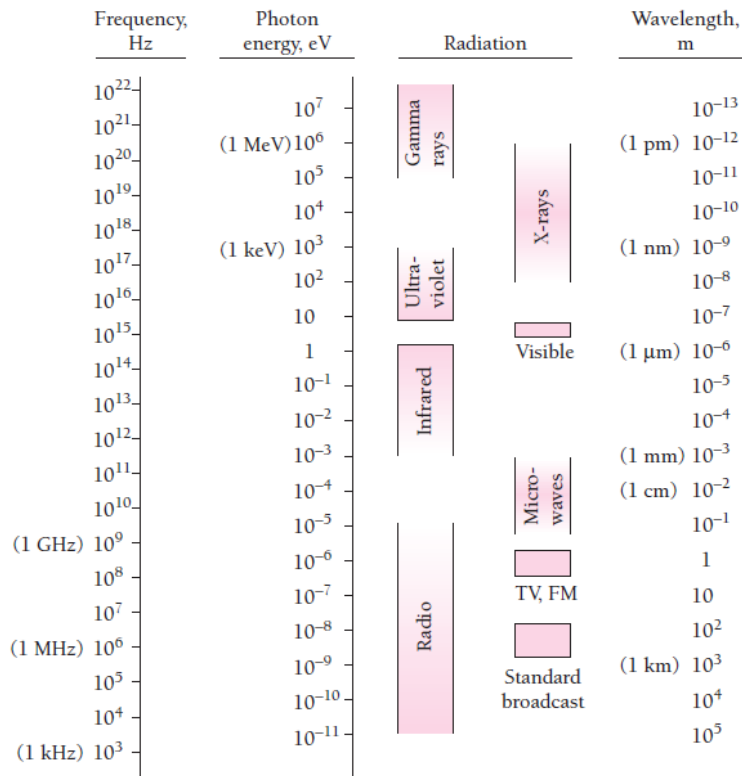


Figure 2.2 The spectrum of electromagnetic radiation.

- Principle of superposition. When two or more waves of the same nature travel past a point at the same time, the instantaneous amplitude there is the sum of the instantaneous amplitudes of the individual waves.
- Amplitude of a water wave is the height of the water surface relative to its normal level.
- Amplitude of a sound wave is the change in pressure relative to the normal pressure.
- Amplitude of a light wave ( $E=cB$ ) can be taken as either E or B. Usually E is used, since its interactions with matter give rise to nearly all common optical effects.
- When two or more trains of light waves meet in a region, they interfere to produce a new wave there whose instantaneous amplitude is the sum of those of the original waves.
  - Constructive interference refers to the reinforcement of waves with the same phase to produce a greater amplitude. Fig 2.3a
  - Destructive interference refers to the partial or complete cancellation of waves whose phases differ. Fig 2.3b
  - If the original waves have different frequencies, the result will be a mixture of constructive and destructive interference.



Figure 2.3 (a) In constructive interference, superposed waves in phase reinforce each other. (b) In destructive interference, waves out of phase partially or completely cancel each other.

- Young's Double Slit Experiment. Interference of light waves Fig. 2.4.
- From each slit secondary waves spread out as though originating at the slit. Diffraction.
- At those places on the screen where the path lengths from the two slits differ by an odd number of half wavelengths,  $\lambda (1/2, 3/2, 5/2, \dots)$ , destructive interference occurs and a dark line is the result.
- At those places where the path lengths are equal or differ by a whole number of wavelengths,  $\lambda (1, 2, 3, \dots)$ , constructive interference occurs and a bright line is the result.
- At intermediate places the interference is only partial, so the light intensity on the screen varies gradually between the bright and dark lines.



Thomas Young  
1773-1829

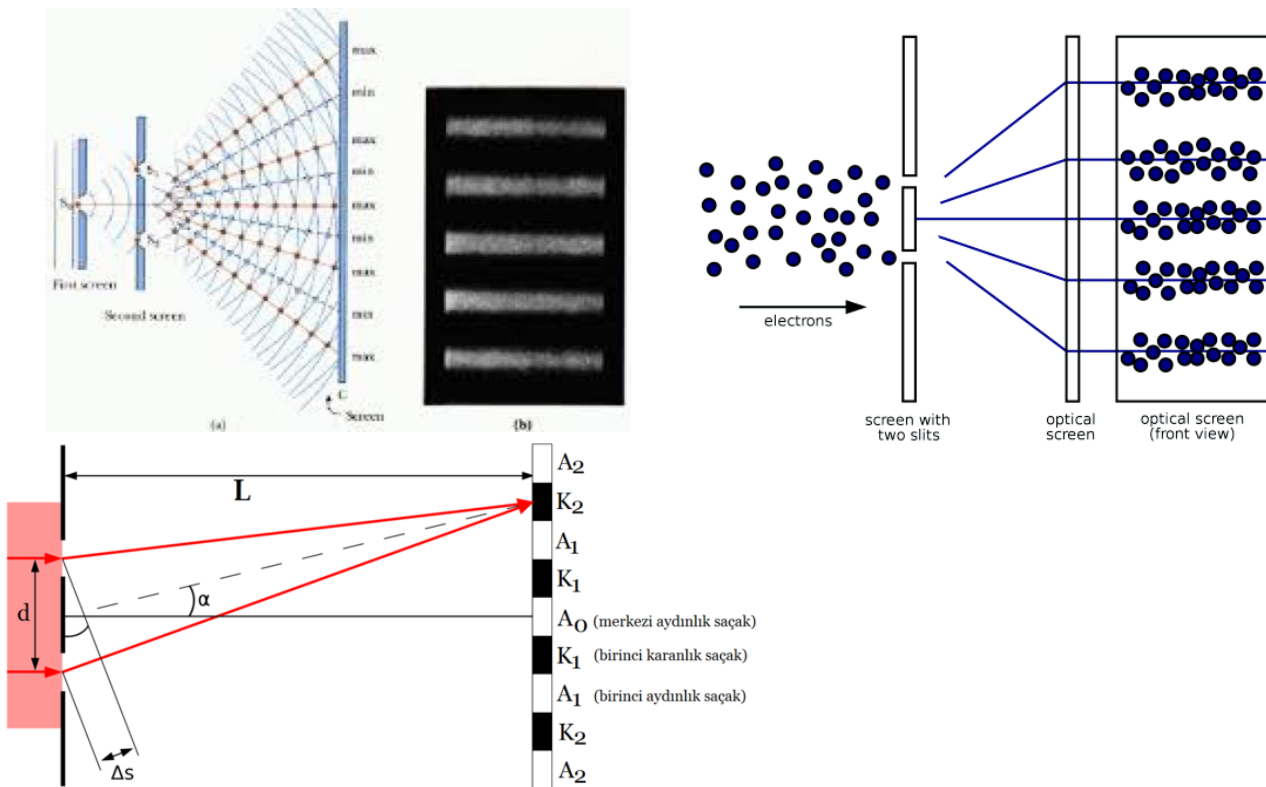


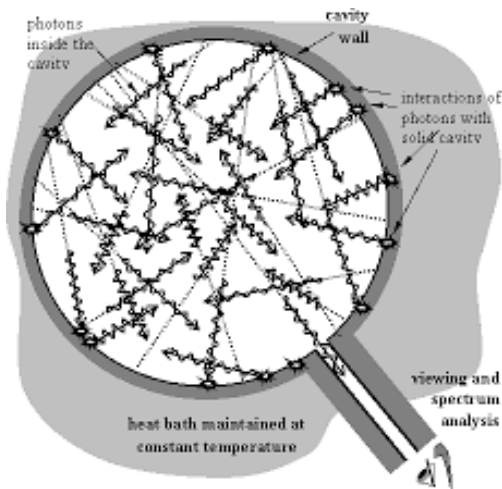
Figure 2.4 Origin of the interference pattern in Young's experiment.

- Interference and diffraction are found only in waves.
- If light consisted of a stream of classical particles, the entire screen would be dark.
- Thus, Young's experiment is proof that light consists of waves.

## 2.2 BLACKBODY RADIATION

- Only the quantum theory of light can explain its origin.
- Until the end of the nineteenth century the nature of light seemed settled forever.
- Attempts to understand the origin of the radiation emitted by bodies of matter.
- All objects radiate such energy continuously whatever their temperatures, though which frequencies predominate depends on the temperature. At room temperature, most of the radiation is in the infrared part of the spectrum and hence is invisible.
- The ability of a body to radiate is closely related to its ability to absorb radiation. This is to be expected, since a body at a constant temperature is in thermal equilibrium with its surroundings and must absorb energy from them at the same rate as it emits energy.

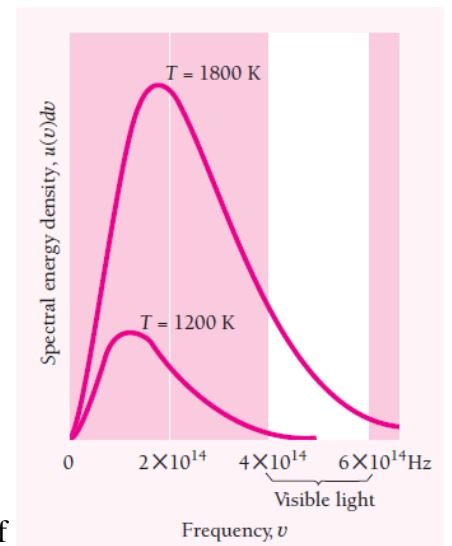
- Blackbody. Ideal body that absorbs all radiation incident upon it, regardless of frequency.
- In the laboratory, a blackbody can be approximated by a hollow object with a very small hole leading to its interior (Fig. 2.5).



- Any radiation striking the hole enters the cavity, where it is trapped by reflection back and forth until it is absorbed.
- The cavity walls are constantly emitting and absorbing radiation, and it is in the properties of this radiation (blackbody radiation).
- Sample blackbody radiation simply by inspecting what emerges from the hole in the cavity.
- A blackbody radiates more when it is hot than when it is cold, and the spectrum of a hot blackbody has its peak at a higher frequency than the peak in the spectrum of a cooler one.
- The spectrum of blackbody radiation is shown in Fig. 2.6 for two temperatures.

Figure 2.5 A hole in the wall of a hollow object is an excellent approximation of a blackbody.

Figure 2.6 Blackbody spectra. The spectral distribution of energy in the radiation depends only on the temperature of the body. The higher the temperature, the greater the amount of radiation and the higher the frequency at which the maximum emission occurs.



### The Ultraviolet Catastrophe

- Why does the blackbody spectrum have the shape shown in Fig. 2.6?
- Lord Rayleigh and James Jeans started by considering the radiation inside a cavity of absolute temperature to be a series of standing em waves (Fig. 2.7).
- The condition for standing waves in such a cavity is that the path length from wall to wall, whatever the direction, must be a whole number of half-wavelengths, so that a node occurs at each reflecting surface.
- The number of independent standing waves  $G(\nu)d\nu$  in the frequency interval per unit volume in the cavity turned out to be

Density of standing waves in cavity

$$G(\nu)d\nu = \frac{8\pi\nu^2 d\nu}{c^3} \quad (2.1)$$

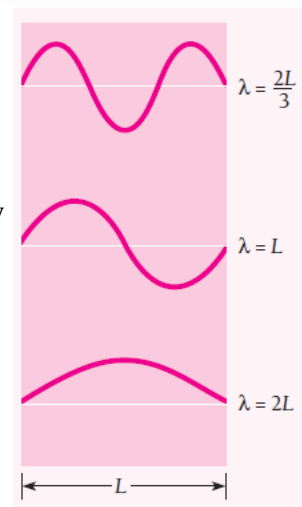


Figure 2.7 Standing waves that have nodes at the walls, which restricts their possible wavelengths. Shown are three possible wavelengths when the distance between opposite walls is L.

- Because each standing wave in a cavity originates in an oscillating electric charge in the cavity wall, two degrees of freedom are associated with the wave and it should have an average energy of

Classical average energy per standing wave 
$$\bar{\epsilon} = kT \quad (2.2)$$

- The total energy per unit volume in the cavity in the frequency interval is therefore

Rayleigh-Jeans formula 
$$u(\nu) d\nu = \bar{\epsilon}G(\nu) d\nu = \frac{8\pi kT}{c^3} \nu^2 d\nu \quad (2.3)$$

- The Rayleigh-Jeans formulae, contains everything that classical physics can say about the spectrum of blackbody radiation.
- Eq. (2.3) shows that it cannot possibly be correct.
- In reality, of course, the energy density (and radiation rate) falls to 0 as frequency goes to infinity (see Fig. 2.8). This discrepancy became known as the ultraviolet catastrophe of classical physics.
- Where did Rayleigh and Jeans go wrong?

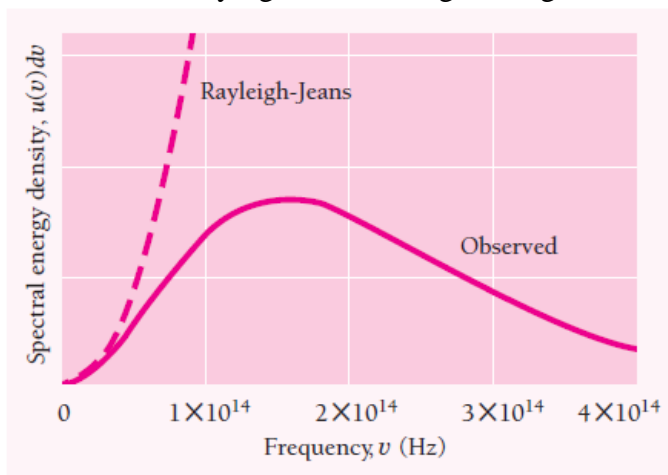
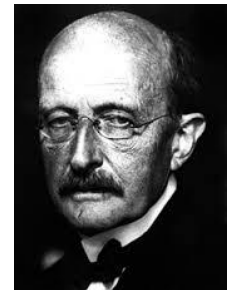


Figure 2.8 Comparison of the Rayleigh-Jeans formula for the spectrum of the radiation from a blackbody at 1500 K with the observed spectrum.



Max Planck (1858–1947)

Nobel Prize in Physics in 1918

- This failure of classical physics led Planck to the discovery that radiation is emitted in quanta whose energy is  $h\nu$ . Nobel Prize in 1918.

Planck's constant 
$$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$$

Planck radiation formula 
$$u(\nu) d\nu = \frac{8\pi h}{c^3} \frac{\nu^3 d\nu}{e^{h\nu/kT} - 1} \quad (2.4)$$

- Considered to mark the start of modern physics.
- At high frequencies and at low frequencies Planck's formula becomes Rayleigh-Jeans formula.
- The oscillators in the cavity walls could not have a continuous distribution of possible energies  $\epsilon$  but must have only the specific energies

Oscillator energies 
$$\epsilon_n = nh\nu \quad n = 0, 1, 2, \dots \quad (2.5)$$

- An oscillator emits radiation of frequency  $\nu$  when it drops from one energy state to the next lower one, and it jumps to the next higher state when it absorbs radiation of frequency  $\nu$ .
- Each discrete bundle of energy  $h\nu$  is called a quantum (plural quanta) from the Latin for "how much."
- With oscillator energies limited to  $nh\nu$ , the average energy per oscillator in the cavity walls

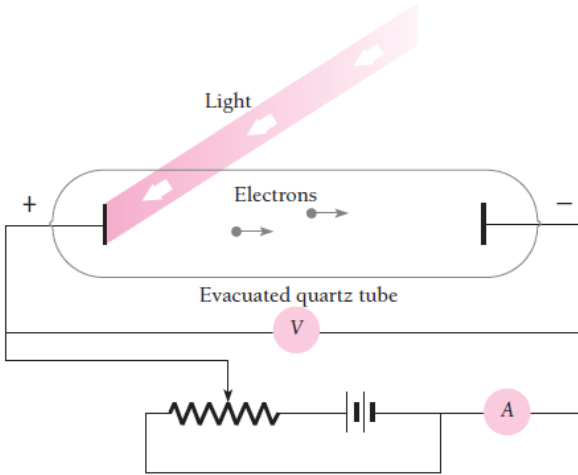
Actual average energy per standing wave 
$$\epsilon = \frac{h\nu}{e^{h\nu/kT} - 1} \quad (2.6)$$

Example 2.1

Assume that a certain 660-Hz tuning fork can be considered as a harmonic oscillator whose vibrational energy is 0.04 J. Compare the energy quanta of this tuning fork with those of an atomic oscillator that emits and absorbs orange light whose frequency is  $5.00 \times 10^{14}$  Hz.

2.3 PHOTOELECTRIC EFFECT

- The energies of electrons liberated by light depend on the frequency of the light.
- Electrons emitted when the frequency of the light was sufficiently high. This phenomenon is known as the photoelectric effect and the emitted electrons are called photoelectrons.

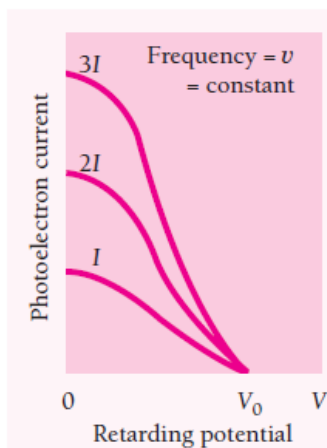


- Some of the photoelectrons that emerge from this surface have enough energy to reach the cathode despite its negative polarity, and they constitute the measured current.
- When the voltage is increased to a certain value  $V_0$ , of the order of several volts, no more photoelectrons arrive, as indicated by the current dropping to zero. Maximum photoelectron kinetic energy.

Figure 2.9 Experimental observation of the photoelectric effect.

Three experimental findings;

1. Within the limits of experimental accuracy (about  $10^{-9}$  s), there is no time interval between the arrival of light at a metal surface and the emission of photoelectrons. However, because the energy in an em wave is supposed to be spread across the wavefronts, a period of time should elapse before an individual electron accumulates enough energy (several eV) to leave the metal.



2. A bright light yields more photoelectrons than a dim one of the same frequency, but the electron energies remain the same (Fig. 2.10). The em theory of light, on the contrary, predicts that the more intense the light, the greater the energies of the electrons.

Figure 2.10 Photoelectron current is proportional to light intensity  $I$  for all retarding voltages. The stopping potential  $V_0$ , which corresponds to the maximum photoelectron energy, is the same for all intensities of light of the same frequency.



3. The higher the frequency of the light, the more energy the photoelectrons have (Fig. 2.11).
  - At frequencies below a certain critical frequency  $\nu_0$ , which is characteristic of each particular metal, no electrons are emitted.
  - Above  $\nu_0$  the photoelectrons range in energy from 0 to a maximum value that increases linearly with increasing frequency (Fig.2.12). This observation, also, cannot be explained by the em theory of light.

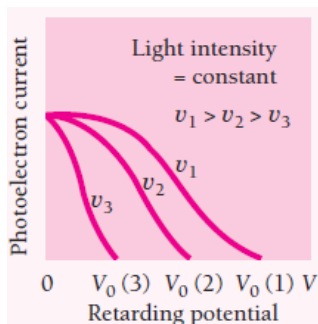


Figure 2.11 The stopping potential  $V_0$ , and hence the maximum photoelectron energy, depends on the frequency of the light. When the retarding potential is  $V=0$ , the photoelectron current is the same for light of a given intensity regardless of its frequency.

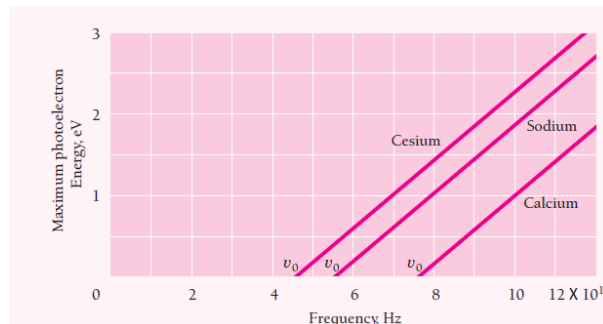


Figure 2.12 Maximum photoelectron kinetic energy  $KE_{\max}$  versus frequency of incident light for three metal surfaces.

### Quantum Theory of Light

- In 1905, Einstein realized that the photoelectric effect could be understood if the energy in light is not spread out over wavefronts but is concentrated in small packets, or photons. Nobel Prize in 1921.
- Each photon of light of frequency  $\nu$  has the energy  $h\nu$ , the same as Planck's quantum energy.



Albert Einstein 1879 – 1955  
Nobel Prize in Physics in 1921

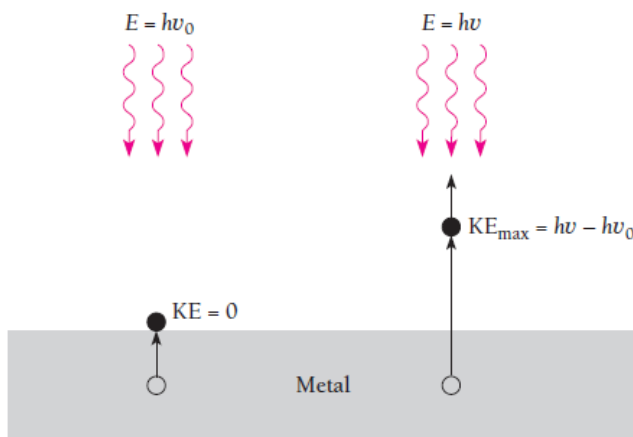
- Energy was not only given to em waves in separate quanta but was also carried by the waves in separate quanta.
- The three experimental observations listed above follow directly from Einstein's hypothesis.
  1. Because em wave energy is concentrated in photons and not spread out, there should be no delay in the emission of photoelectrons.
  2. All photons of frequency have the same energy, so changing the intensity of a monochromatic light beam will change the number of photoelectrons but not their energies.
  3. The higher the frequency  $\nu$ , the greater the photon energy  $h\nu$  and so the more energy the photoelectrons have.
- There must be a minimum energy  $\phi$  for an electron to escape from a particular metal surface or else electrons would pour out all the time. This energy is called the work function of the metal, and is related to  $\nu_0$  by the formula

$$\text{Work function} \quad \phi = h\nu_0 \quad (2.7)$$

- The greater the work function of a metal, the more energy is needed for an electron to leave its surface, and the higher the critical frequency for photoelectric emission to occur.

$$\text{Photoelectric effect} \quad h\nu = KE_{\max} + \phi \quad (2.8)$$

- where  $h\nu$  is the photon energy,  $KE_{\max}$  is the maximum photoelectron energy (which is proportional to the stopping potential), and  $\phi$  is the minimum energy needed for an electron to leave the metal.



$$h\nu = KE_{\max} + h\nu_0$$

$$KE_{\max} = h\nu - h\nu_0 = h(\nu - \nu_0) \quad (2.9)$$

Figure 2.13 If the energy  $h\nu_0$  (the work function of the surface) is needed to remove an electron from a metal surface, the maximum electron kinetic energy will be  $h\nu - h\nu_0$  when light of frequency is directed at the surface.

- In terms of electron volts, the formula  $E=h\nu$  for photon energy becomes

Photon energy

$$E = \left( \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{1.602 \times 10^{-19} \text{ J/eV}} \right) \nu = (4.136 \times 10^{-15}) \nu \text{ eV} \cdot \text{s} \quad (2.10)$$

### Example 2.2

Ultraviolet light of wavelength 350 nm and intensity 1.00 W/m<sup>2</sup> is directed at a potassium surface.

(a) Find the maximum KE of the photoelectrons.

(b) If 0.50 percent of the incident photons produce photoelectrons, how many are emitted per second if the potassium surface has an area of 1.00 cm<sup>2</sup>?

## 2.4 WHAT IS LIGHT?

- Both wave and particle.
  - The wave theory of light explains diffraction and interference, which the quantum theory cannot account for. According to the wave theory, light waves leave a source with their energy spread out continuously through the wave pattern.
  - The quantum theory explains the photoelectric effect, which the wave theory cannot account for. According to the quantum theory, light consists of individual photons, each small enough to be absorbed by a single electron.
- In double-slit interference pattern on a screen.
  - When it passes through the slits, light is behaving as a wave does. When it strikes the screen, light is behaving as a particle does. Apparently light travels as a wave but absorbs and gives off energy as a series of particles.
  - In the wave model, the light intensity at a place on the screen depends on  $E^2$ , the average over a complete cycle of the square of the instantaneous magnitude  $E$  of the em wave's electric field.
  - In the particle model, this intensity depends instead on  $Nh\nu$ , where  $N$  is the number of photons per second per unit area that reach the same place on the screen.
- See Fig 2.4. Number of photons.
- Think of light as having a dual character. The wave theory and the quantum theory complement each other. Either theory by itself is only part of the story and can explain only certain effects.



## 2.5 X-RAYS

- They consist of high-energy photons.
- The photoelectric effect provides convincing evidence that photons of light can transfer energy to electrons. Is the inverse process also possible?
- That is, can part or all of the kinetic energy of a moving electron be converted into a photon?
- In 1895 Wilhelm Roentgen found that a highly penetrating radiation of unknown nature is produced when fast electrons impinge on matter.
- The faster the original electrons, the more penetrating the resulting x-rays, and the greater the number of electrons, the greater the intensity of the x-ray beam.
- X-rays are em waves. Electromagnetic theory predicts that an accelerated electric charge will radiate em waves, and a rapidly moving electron suddenly brought to rest is certainly accelerated.
- Radiation produced under these circumstances is given the German name bremsstrahlung (“braking radiation”).
- Electromagnetic radiation with wavelengths from about 0.01 to about 10 nm falls into the category of x-rays.
- In 1912 a method was devised for measuring the wavelengths of x-rays. A diffraction experiment.



Wilhelm Konrad Roentgen (1845–1923) First Nobel Prize in Physics in 1902

- The spacing between adjacent lines on a diffraction grating must be of the same order of magnitude as the wavelength of the light for satisfactory results.
- Max von Laue realized that the wavelengths suggested for x-rays were comparable to the spacing between adjacent atoms in crystals.
- He therefore proposed that crystals be used to diffract x-rays, with their regular lattices acting as a kind of three-dimensional grating. Figure 2.15 is a diagram of an x-ray tube.
- Figure 2.15 is a diagram of an x-ray tube.

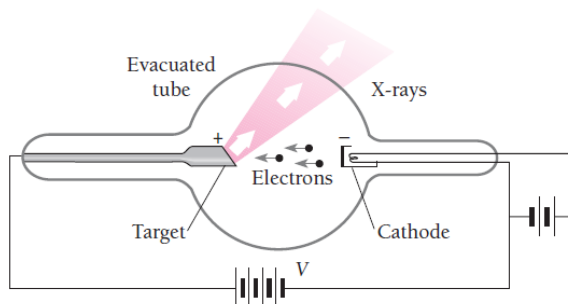


Figure 2.15 An x-ray tube. The higher the accelerating voltage  $V$ , the faster the electrons and the shorter the wavelengths of the x-rays.

- As we know, classical electromagnetic theory predicts bremsstrahlung when electrons are accelerated, which accounts in general for the x-rays produced by an x-ray tube.
- Figures 2.16 and 2.17 show the x-ray spectra that result when tungsten and molybdenum targets are bombarded by electrons at several different accelerating potentials.

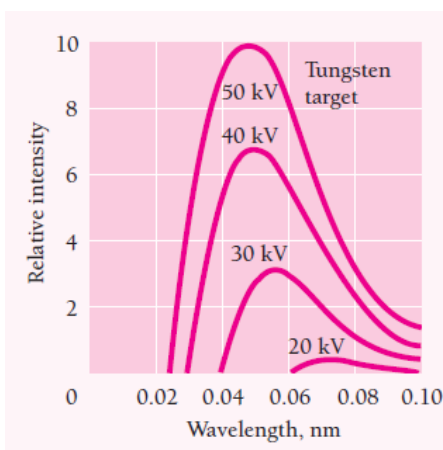
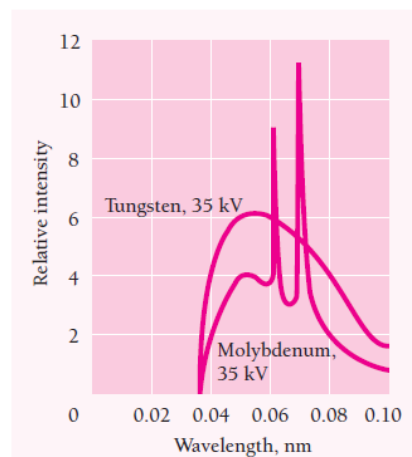


Figure 2.16 X-ray spectra of tungsten at various accelerating potentials.

Figure 2.17 X-ray spectra of tungsten and molybdenum at 35 kV accelerating potential.



The curves exhibit two features electromagnetic theory cannot explain:

- In the case of molybdenum,

intensity peaks occur that indicate the enhanced

production of x-rays at certain wavelengths. These peaks occur at specific wavelengths for each target material and originate in rearrangements of the electron structures of the target atoms after having been disturbed by the bombarding electrons.

- The presence of x-rays of specific wavelengths, a decidedly non-classical effect, in addition to a continuous x-ray spectrum.
- The x-rays produced at a given accelerating potential  $V$  vary in wavelength, but none has a wavelength shorter than a certain value  $\lambda_{\min}$ . Increasing  $V$  decreases  $\lambda_{\min}$ . Duane and Hunt found experimentally that

**X-ray production** 
$$\lambda_{\min} = \frac{1.24 \times 10^{-6}}{V} \text{ V} \cdot \text{m} \quad (2.12)$$

- The second observation fits in with the quantum theory of radiation. Most of the electrons that strike the target undergo numerous glancing collisions, with their energy going simply into heat.
- A few electrons, though, lose most or all of their energy in single collisions with target atoms. This is the energy that becomes x-rays.
- X-rays production, then, represents an inverse photoelectric effect. Instead of photon energy being transformed into electron KE, electron KE is being transformed into photon energy.
- Since work functions are only a few electron-volts whereas the accelerating potentials in x-ray tubes are typically tens or hundreds of thousands of volts, we can ignore the work function and interpret the short wavelength limit of Eq. (2.12) as corresponding to the case where the entire kinetic energy  $KE=Ve$  of a bombarding electron is given up to a single photon of energy  $h\nu_{\max}$ .

$$Ve = h\nu_{\max} = \frac{hc}{\lambda_{\min}}$$

$$\lambda_{\min} = \frac{hc}{Ve} = \frac{1.240 \times 10^{-6}}{V} \text{ V} \cdot \text{m}$$

which is the Duane-Hunt formula of Eq. (2.12).

#### Example 2.3

Find the shortest wavelength present in the radiation from an x-ray machine whose accelerating potential is 50,000 V.

## 2.6 X-RAY DIFFRACTION

- How x-ray wavelengths can be determined.
- A crystal consists of a regular array of atoms, each of which can scatter em waves. The mechanism of scattering is straightforward.
  - An atom in a constant electric field becomes polarized since its negatively charged electrons and positively charged nucleus experience forces in opposite directions. So, the result is a distorted charge distribution equivalent to an electric dipole.
  - In the presence of the alternating electric field of an em wave of frequency  $\nu$ , the polarization changes back and forth with the same frequency  $\nu$ .
  - An oscillating electric dipole is thus created at the expense of some of the energy of the incoming wave.
  - The oscillating dipole in turn radiates em waves of frequency  $\nu$ , and these secondary waves go out in all directions except along the dipole axis. (see Fig. 2.18)

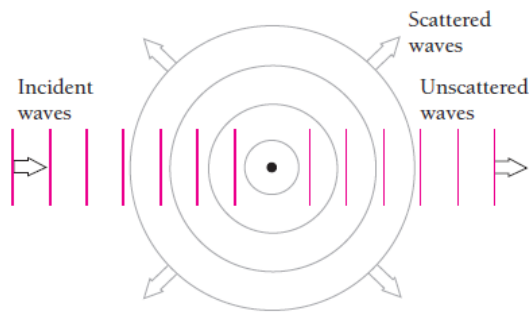
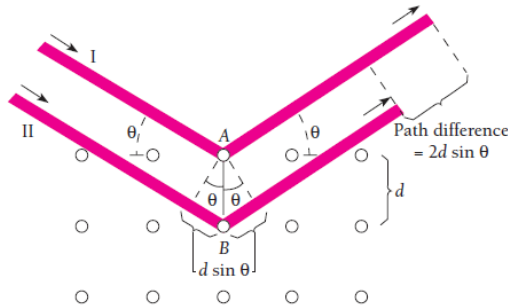


Figure 2.18 The scattering of electromagnetic radiation by a group of atoms. Incident plane waves are reemitted as spherical waves.

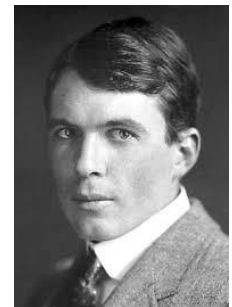
- A monochromatic beam of x-rays that falls upon a crystal will be scattered in all directions inside it.
- However, owing to the regular arrangement of the atoms, in certain directions the scattered waves will constructively interfere with one another while in others they will destructively

interfere.

- The atoms in a crystal may be thought of as defining families of parallel planes with each family having a characteristic separation between its component planes.
- This analysis was suggested in 1913 by W. L. Bragg, in honor of whom the above planes are called Bragg planes.



- The conditions that must be fulfilled for radiation scattered by crystal atoms to undergo constructive interference may be obtained from a diagram like that in Fig. 2.20.



William Lawrence Bragg (1890–1971) Nobel Prize in Physics in 1915

Figure 2.20 X-ray scattering from a cubic crystal.

- A beam containing x-rays of wavelength  $\lambda$  is incident upon a crystal at an angle  $\theta$  with a family of Bragg planes whose spacing is  $d$ .
- The beam goes past atom A in the first plane and atom B in the next, and each of them scatters part of the beam in random directions.
- Constructive interference takes place only between those scattered rays that are parallel and whose paths differ by exactly  $\lambda$ ,  $2\lambda$ ,  $3\lambda$ , and so on.
- That is, the path difference must be  $n\lambda$ , where  $n$  is an integer.
- Conditions;
  1. The first condition on I and II is that their common scattering angle be equal to the angle of incidence  $\theta$  of the original beam.
  2. The second condition is that

$$2d \sin \theta = n\lambda \quad n = 1, 2, 3, \dots \quad (2.13)$$

since ray II must travel the distance  $2d \sin \theta$  farther than ray I. The integer  $n$  is the order of the scattered beam.

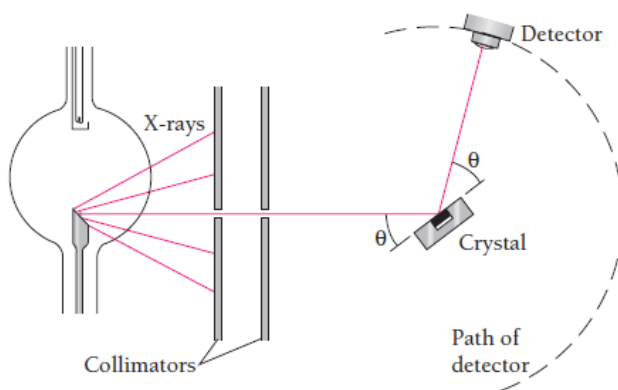


Figure 2.21 X-ray spectrometer.

- If the spacing  $d$  between adjacent Bragg planes in the crystal is known, the x-ray wavelength  $\lambda$  may be calculated.

## 2.7 COMPTON EFFECT

- Further confirmation of the photon model.
- According to the quantum theory of light, photons behave like particles except for their lack of rest mass.
- Figure 2.22 shows a collision: an x-ray photon strikes an electron (assumed to be initially at rest in the laboratory coordinate system).
- We can think of the photon as losing an amount of energy in the collision that is the same as the kinetic energy KE gained by the electron.

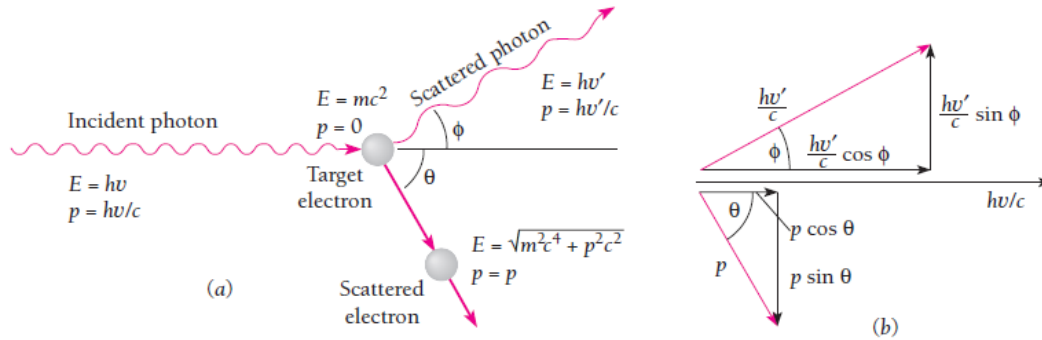


Figure 2.22 (a) The scattering of a photon by an electron is called the Compton effect. Energy and momentum are conserved in such an event, and as a result the scattered photon has less energy (longer wavelength) than the incident photon. (b) Vector diagram of the momenta and their components of the incident and scattered photons and the scattered electron.

$$\begin{aligned} \text{Loss in photon energy} &= \text{gain in electron energy} \\ h\nu - h\nu' &= \text{KE} \end{aligned} \quad (2.14)$$

the momentum of a massless particle is related to its energy by the formula

$$E = pc \quad (1.25)$$

Since the energy of a photon is  $h\nu$ , its momentum is

$$\text{Photon momentum} \quad p = \frac{E}{c} = \frac{h\nu}{c} \quad (2.15)$$

- In the collision momentum must be conserved in each of two mutually perpendicular directions.

◦

In the original photon direction

Initial momentum = final momentum

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c} \cos \phi + p \cos \theta \quad (2.16)$$

- and in perpendicular to original photon direction

Initial momentum = final momentum

$$0 = \frac{h\nu'}{c} \sin \phi - p \sin \theta \quad (2.17)$$

- The angle  $\phi$  is that between the directions of the initial and scattered photons, and  $\theta$  is that between the directions of the initial photon and the recoil electron.

- From Eqs. (2.14), (2.16), and (2.17) we can find a formula that relates the wavelength difference between initial and scattered photons with the angle  $\phi$  between their directions, both of which are readily measurable quantities (unlike the energy and momentum of the recoil electron).

**Compton effect**  $\lambda' - \lambda = \lambda_C(1 - \cos \phi)$  (2.23)

- Equation (2.23) was derived by Arthur H. Compton in the early 1920s, and the phenomenon it describes, which he was the first to observe, is known as the Compton effect.
- It constitutes very strong evidence in support of the quantum theory of radiation.
- Change in wavelength is independent of the wavelength  $\lambda$  of the incident photon.
- The quantity

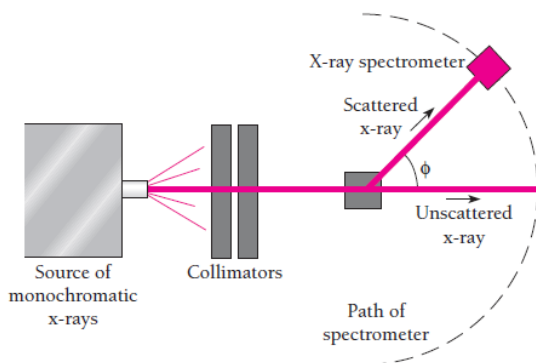


Arthur Holly Compton (1892–1962) Nobel Prize in Physics in 1927

**Compton wavelength**  $\lambda_C = \frac{h}{mc}$  (2.22)

is called the Compton wavelength of the scattering particle. For an electron  $\lambda_C=2.426 \times 10^{-12}$  m, which is 2.426 pm (1 pm=1 picometer= $10^{-12}$  m).

- The Compton wavelength gives the scale of the wavelength change of the incident photon. From (2.23) we note that the greatest wavelength change possible corresponds to  $\phi=180^\circ$ .
- Changes of this magnitude or less are readily observable only in x-rays.
- The Compton effect is the chief means by which x-rays lose energy when they pass through matter.
- The experimental demonstration of the Compton effect is straightforward. As in Fig. 2.23, a beam of x-rays of a single, known wavelength is directed at a target, and the wavelengths of the scattered x-rays are determined at various angles



- The results are shown in Fig. 2.24.

Figure 2.23 Experimental demonstration of the Compton effect.

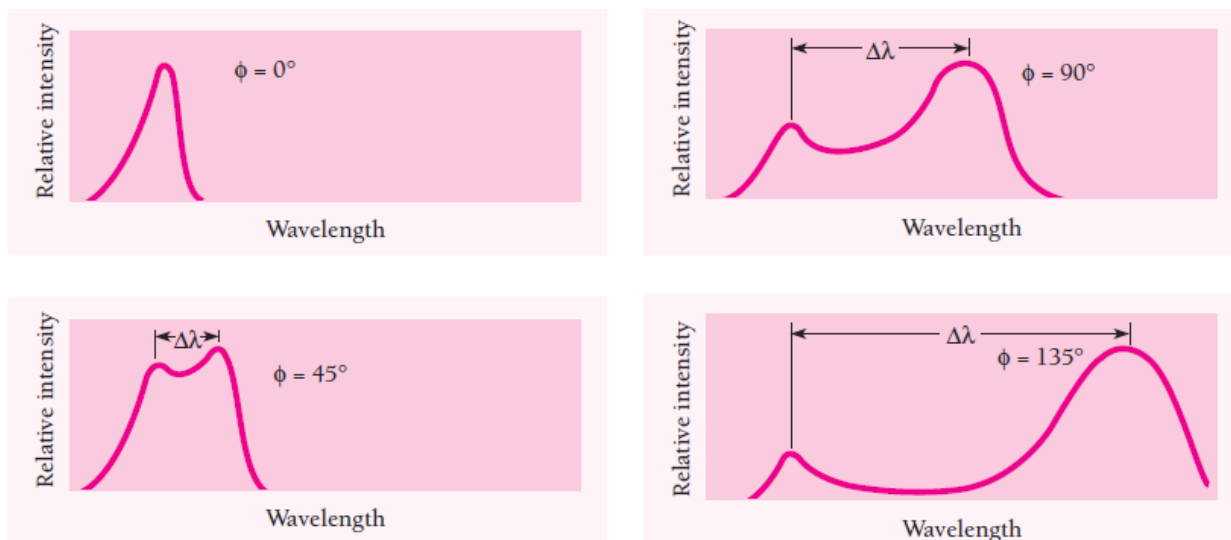


Figure 2.24 Experimental confirmation of Compton scattering. The greater the scattering angle, the greater the wavelength change, in accord with Eq. (2.23).

### Example 2.4

X-rays of wavelength 10.0 pm are scattered from a target. (a) Find the wavelength of the x-rays scattered through 45°. (b) Find the maximum wavelength present in the scattered x-rays. (c) Find the maximum kinetic energy of the recoil electrons.

## 2.8 PAIR PRODUCTION

1. Energy into matter.
2. As we have seen, in a collision a photon can give an electron all of its energy (the photoelectric effect) or only part (the Compton effect).
3. It is also possible for a photon to materialize into an electron and a positron, which is a positively charged electron. In this process, called pair production, electromagnetic energy is converted into matter.

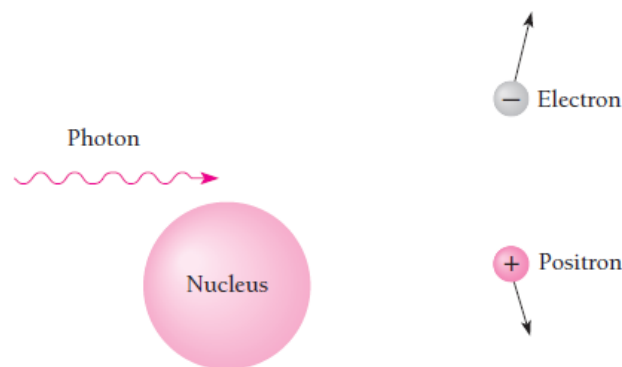


Figure 2.25 In the process of pair production, a photon of sufficient energy materializes into an electron and a positron.

- No conservation principles are violated when an electron-positron pair is created near an atomic nucleus (Fig. 2.25).
  - The sum of the charges of the electron ( $q=-e$ ) and of the positron ( $q=e$ ) is zero, as is the charge of the photon
  - the total energy, including rest energy, of the electron and positron equals the photon energy
  - linear momentum is conserved with the help of the nucleus, which carries away enough photon momentum for the process to occur.
- The rest energy  $mc^2$  of an electron or positron is 0.51 MeV, hence pair production requires a photon energy of at least 1.02 MeV.
- Any additional photon energy becomes kinetic energy of the electron and positron. The corresponding maximum photon wavelength is 1.2 pm. Electromagnetic waves with such wavelengths are called gamma rays, symbol  $\gamma$ , and are found in nature as one of the emissions from radioactive nuclei and in cosmic rays.
- The inverse of pair production occurs when a positron is near an electron and the two come together under the influence of their opposite electric charges. Both particles vanish simultaneously, with the lost mass becoming energy in the form of two gamma-ray photons:

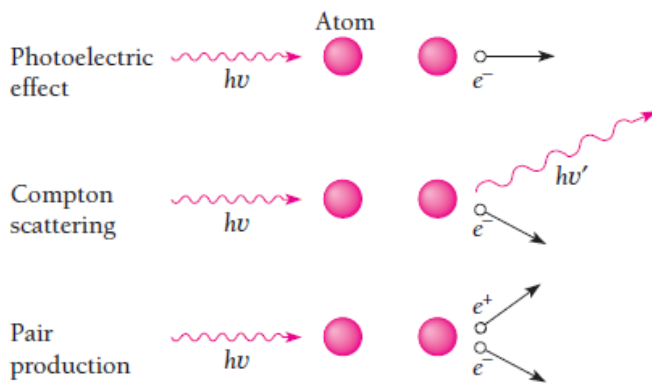


- The total mass of the positron and electron is equivalent to 1.02 MeV, and each photon has an energy  $h\nu$  of 0.51 MeV plus half the kinetic energy of the particles relative to their center of mass.
- The directions of the photons are such as to conserve both energy and linear momentum, and no nucleus or other particle is needed for this pair annihilation to take place.



## Photon Absorption

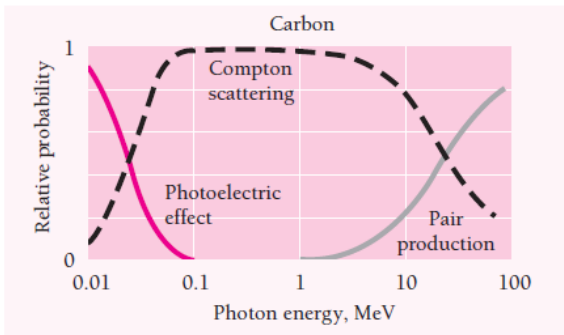
- The three chief ways in which photons of light, x-rays, and gamma rays interact with matter are summarized in Fig. 2.27.



- In all cases photon energy is transferred to electrons which in turn lose energy to atoms in the absorbing material.
- At low photon energies the photoelectric effect is the chief mechanism of energy loss.
- The importance of the photoelectric effect decreases with increasing energy, to be succeeded by Compton scattering.

Figure 2.27 X- and gamma rays interact with matter

chiefly through the photoelectric effect, Compton scattering, and pair production. Pair production requires a photon energy of at least 1.02 MeV.



- The greater the atomic number of the absorber, the higher the energy at which the photoelectric effect remains significant.
- In the lighter elements, Compton scattering becomes dominant at photon energies of a few tens of keV,
- whereas in the heavier ones this does not happen until photon energies of nearly 1 MeV are reached (Fig. 2.28).

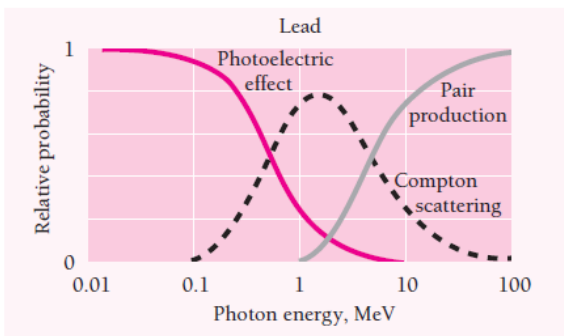


Figure 2.28 The relative probabilities of the photoelectric effect, Compton scattering, and pair production as functions of energy in carbon (a light element) and lead (a heavy element).