MSE228 Engineering Quantum Mechanics
Quiz 2 Duration: 30 minutes Open Book Quiz

1. Show that de Broglie wavelength of electron with having kinetic energy value of 100 keV is 0.0037 nm.

$KE = 100\ keV = 0.1\ MeV$ which is comparable to electron rest mass energy: $0.51\ MeV$ (E_0) $\Rightarrow$ Relativistic Equations

① $E = KE + mc^2 = KE + E_0$

② $E^2 = (mc^2)^2 + (pc)^2 = E_0^2 + (pc)^2$

$pc = h\nu = \frac{hc}{\lambda}$: $\lambda = ?$

①&② $(KE + E_0)^2 = E_0^2 + \left(\frac{hc}{\lambda}\right)^2$

$$(0.1\ MeV + 0.5\ MeV)^2 = (0.5\ MeV)^2 + \left(\frac{(6.626\times10^{-34}\ J\cdot s / 1.602\times10^{-19}\ J/eV)(3\times10^8\ m/s)}{\lambda^2}\right)^2$$

$$\Rightarrow \lambda^2 = \frac{[(4.14\times10^{-15}\ eV\cdot s)(3\times10^8\ m/s)]^2}{3.6\times10^{11}\ eV^2 - 2.5\times10^{11}\ eV^2} = \frac{[1.24\times10^{-6}\ eV\cdot m]^2}{1.1\times10^{11}\ eV^2} = 1.40\times10^{-23}\ m^2$$

$$\Rightarrow \lambda = \sqrt{1.40\times10^{-23}\ m^2} = 0.0037\ nm$$

2. An electron moves in the x direction with a speed of 3.6×10^6 m/s. We can measure its speed to a precision of 1%. With what precision can we simultaneously measure its x coordinate?

x-direction, speed of precision of 1% $\Rightarrow \Delta v_x = v_x \times \frac{1}{100}$

$\Delta p_x = m\,\Delta v_x = (9.11\times10^{-31}\ kg)(3.6\times10^6\ m/s) = 3.3\times10^{-26}\ kg\,m/s$

$\rightarrow \Delta x\,\Delta p_x \geq \hbar \rightarrow \Delta x \simeq \frac{\hbar}{\Delta p_x} = \frac{1.05\times10^{-34}\ J\cdot s}{3.3\times10^{-26}\ kg\,m/s} = 3.18\times10^{-9}\ m = 3.2\ nm$

($J = kg\,m^2/s^2$)

about 10 atoms in that length