

MSE228 Engineering Quantum Mechanics
 Quiz 3 Duration: 30 minutes Open Book Quiz

1. The shortest wavelength of the hydrogen Lyman series is 91.13 nm. Find the three longest wavelengths in this series. (Hint: The shortest wavelength is the series limit)

Lyman Series $\frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{n^2} \right)$
 Shortest wavelength \Rightarrow series limit $\Rightarrow n \rightarrow \infty$ $\frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{\infty^2} \right)$
 $\frac{1}{\lambda} = R$, $\lambda_{\text{shortest}} = \frac{1}{R} = \frac{1}{1.097373 \times 10^7 \text{ m}^{-1}} = 91.127 \text{ nm}$
 other wavelengths; $\frac{1}{\lambda} = R \left(\frac{n^2 - 1}{n^2} \right) \rightarrow \lambda = \frac{1}{R} \left(\frac{n^2}{n^2 - 1} \right) = (91.127 \text{ nm}) \frac{n^2}{n^2 - 1}$
 1st longest wavelength; $n=2$ $\lambda = (91.127 \text{ nm}) \frac{2^2}{2^2 - 1} = 121.5 \text{ nm}$
 2nd longest wavelength; $n=3$ $\lambda = (91.127 \text{ nm}) \frac{3^2}{3^2 - 1} = 102.5 \text{ nm}$
 3rd longest wavelength; $n=4$ $\lambda = (91.127 \text{ nm}) \frac{4^2}{4^2 - 1} = 97.2 \text{ nm}$

2. In the $n = 3$ state of hydrogen (for Bohr model), find the electron's radius, velocity, kinetic energy, and potential energy.

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 $R_n = n^2 \left(\frac{\hbar^2 \epsilon_0}{\pi m e^2} \right) = n^2 a_0 = n^2 (0.53 \times 10^{-10} \text{ m})$; $v_n = \sqrt{\frac{1}{4\pi\epsilon_0} \frac{e^2}{m R_n}}$
 $KE_n = \frac{1}{2} m v_n^2 = \frac{1}{2} m \frac{1}{4\pi\epsilon_0} \frac{e^2}{m R_n}$; $PE_n = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{R_n}$
 $\Rightarrow R_{n=3} = 9a_0 = 9(0.53 \times 10^{-10} \text{ m}) = 0.477 \text{ nm}$
 $v_{n=3} = \sqrt{\frac{1}{4\pi(8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2)} \frac{(1.602 \times 10^{-19} \text{ C})^2}{(9.1 \times 10^{-31} \text{ kg})(0.477 \times 10^{-10} \text{ m})}} = 7.29 \times 10^5 \text{ m/s}$
 $KE_{n=3} = \frac{1}{8\pi(8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2)} \frac{(1.602 \times 10^{-19} \text{ C})^2}{0.477 \times 10^{-10} \text{ m}} = 2.42 \times 10^{-19} \text{ J} = 1.51 \text{ eV}$
 $PE_{n=3} = -\frac{1}{4\pi(8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2)} \frac{(1.602 \times 10^{-19} \text{ C})^2}{0.477 \times 10^{-10} \text{ m}} = -4.84 \times 10^{-19} \text{ J} = -3.02 \text{ eV}$