MSE228 Engineering Quantum Mechanics

Quiz 4 Duration: 30 minutes Open Book Quiz

1. An electron is trapped in a one-dimensional region of length 1.00×10^{-10} m (a typical atomic diameter).
(a) Find the energies of the ground state and first two excited states.
(b) How much energy must be supplied to excite the electron from the ground state to the second excited state?
(c) From the second excited state, the electron drops down to the first excited state. How much energy is released in this process?

Particle in a box with $L = 1\times10^{-10}$ m

$E_n = \frac{n\pi^2\hbar^2}{2mL^2} = \frac{n\pi^2}{2mL^2}\left(\frac{h}{2\pi}\right)^2 = \frac{nh^2}{8mL^2}$, $n = 1,2,3,4 \ldots$

i) Ground state $n=1 \rightarrow E_1 = \frac{h^2}{8mL^2} = \frac{(6.626\times10^{-34}\ m^2 kg/s)^2}{8(9.31\times10^{-31}\ kg)(1\times10^{-10}\ m)^2} =$

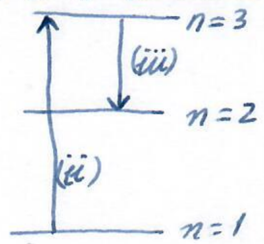
$= 6.03\times10^{-18}\ kg m^2/s^2 = 6.03\times10^{-18} J = \boxed{37.64 eV}$

1st excited state $n=2 \rightarrow E_2 = 4E_1 = \boxed{150.57 eV}$

2nd excited state $n=3 \rightarrow E_3 = 9E_1 = \boxed{338.78 eV}$

ii) $\Delta E = E_3 - E_1 = 338.78 eV - 37.64 eV = \boxed{301.13 eV}$

iii) $\Delta E = E_3 - E_2 = 338.78 eV - 150.57 eV = \boxed{188.21 eV}$



2. An electron is bound to a region of space by a springlike force with an effective spring constant of $k = 95.7\ eV/nm^2$.
(a) What is its ground-state energy?
(b) How much energy must be absorbed for the electron to jump from the ground state to the second excited state?

Harmonic oscillation $\omega = \sqrt{\frac{k}{m}}$, $E_n = (n+\frac{1}{2})h\nu$, $E_0 = \frac{h\nu}{2}$

$\omega = 2\pi\nu$

i) Ground state energy $n=0$ $E_0 = \frac{1}{2}h\frac{\omega}{2\pi} = \frac{1}{2}\hbar\sqrt{\frac{k}{m}}$

$E_0 = \frac{1}{2}\frac{6.626\times10^{-34}\ m^2 kg/s}{2\pi}\sqrt{\frac{95.7\ eV/nm^2}{9.31\times10^{-31}\ kg}} = \frac{1}{4\pi}\frac{6.626\times10^{-34} Js}{1.602\times10^{-19} J/eV}\sqrt{\frac{9.57\ eV/(10^{-10}m)^2}{9.31\times10^{-31}\ kg}}$

$= \frac{1}{4\pi} 6.626\times10^{-34} Js \sqrt{\frac{9.57(1.602\times10^{-19}\frac{J}{eV}\ eV)/(10^{-10}m)^2}{9.31\times10^{-31}\ kg}} = 2.14\times10^{-19} J = \boxed{1.34 eV}$

ii) second excited state $\rightsquigarrow n=2 \rightsquigarrow E_2 = \frac{5}{2}h\nu$

$\Delta E = \frac{5}{2}h\nu - \frac{1}{2}h\nu = 2h\nu = 4E_0 = \boxed{5.34 eV}$