MSE228 Engineering Quantum Mechanics
Quiz 5 Duration: 30 minutes Open Book Quiz

1. An electron in a hydrogen atom is in a 3d state. What is the most probable radius at which to find it? (Hint: use P(r) not P(r)dr and then to find the maximum set the derivative dP(r)/dr to zero.))

From Table 6.1; $R_{3,2}(r) = \frac{4}{81\sqrt{30}\,a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}$

$\rightarrow P(r) = r^2 R_{3,2}^2(r) = r^2 \frac{16}{81^2 30\, a_0^3} \frac{r^4}{a_0^4} e^{-2r/3a_0} \Rightarrow \frac{d}{dr}P(r) = 0$

$\Rightarrow \frac{16}{81^2 30\, a_0^7}\left(\frac{d}{dr}\left(r^6 e^{-2r/3a_0}\right)\right) = \frac{16}{81^2 30\, a_0^7}\left(6r^5 e^{-2r/3a_0} - \frac{2}{3a_0} r^6 e^{-2r/3a_0}\right)$

$= \frac{16}{81^2 30\, a_0^7}\underbrace{\left(6 - \frac{2}{3a_0} r\right)}_{0} r^5 e^{-2r/3a_0} = 0 \Rightarrow 6 - \frac{2}{3a_0} r = 0 \Rightarrow \underline{\underline{r = 9a_0}}$

2. What is the expectation value of the radius of an electron in hydrogen atom in a 3d state? Just write the expression without evaluating the integral.

$\langle r \rangle = \int \psi^* r \psi\, dr = \int r \psi^* \psi\, dr = \int r P(r)\, dr \;\}$ where $P(r) = r^2 R_{3,2}^2(r)$

$\rightarrow \langle r \rangle = \int_0^\infty r\left(r^2 \frac{16}{81^2 30\, a_0^3} \frac{r^4}{a_0^4} e^{-2r/3a_0}\right) dr = \underline{\underline{\frac{16}{81^2 30\, a_0^7} \int_0^\infty r^7 e^{-2r/3a_0}\, dr}}$

From Table; $\int_0^\infty x^m e^{-bx}\, dx = m!/b^{m+1} \;\{$ where $m = 7$ & $b = \frac{2}{3a_0}$

$\Rightarrow \frac{16}{81^2 30\, a_0^7}\left(\frac{7!}{\left(\frac{2}{3a_0}\right)^8}\right) = \frac{2^4}{3^8 (2*3*5)\, a_0^7}\left(\frac{3^8 a_0^8}{2^8} 1\times2\times3\times4\times5\times6\times7\right)$

$\Rightarrow \frac{1}{1} \frac{a_0}{2^4} 4\times6\times7 = \langle r \rangle \Rightarrow \langle r \rangle = 10.5 a_0$