Ceng 375 Numerical Computing
Final
Jan 14, 2005 09.40–11.30
Good Luck!

1 (20 Pts)

I In Newton's method the approximation x_{n+1} to a root of $f(x) = 0$ is computed from the approximation x_n using the equation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Derive the above formula, using a Taylor series of $f(x)$.

II Consider the function:

$$f(x) = 5x - e^{-x}$$

i Show that this function has a simple root in the interval $0 < x < 1$

ii Estimate this root using two iterations of the Secant Method.

iii Estimate the error in your answer to part ii.

2 (20 Pts) Solve this system by Gaussian elimination with pivoting

$$\begin{bmatrix} 1 & -2 & 4 & 6 \\ 8 & -3 & 2 & 2 \\ -1 & 10 & 2 & 4 \end{bmatrix}$$

i How many row interchanges are needed?

ii Repeat without any row interchanges. Do you get the same results?

iii You could have saved the row multipliers and obtained a LU equivalent of the coefficient matrix. Use this LU to solve but with right-hand sides of $[1, -3, 5]^T$

3 (25 Pts)

i A function $f_{app}(x)$ is to be used as an approximation to a set of data (x_i, f_i) with $i = 0, 1, 2, \ldots, N$. Suppose further that the function $f_{app}(x)$ depends on two parameters a and b. Provide full details of how the parameters a and b can be determined by a Least Squares Method.

ii Using the result of the previous item, obtain the normal equations for the function $f_{app}(x) = a + b\sqrt{x}$. **Do not attempt to solve these equations.**

4 (20 Pts)

i Find the Fourier coefficients for $f(x) = x^3$ if it is periodic and one period extends from $x = -1$ to $x = 2$.

ii Write the Fourier series for this function.

5 (25 Pts) Consider the following table of data

x_i	f_i
0.0000	0.0000
0.2000	0.5879
0.4000	1.0637
0.6000	1.3927
0.8000	1.5573
1.0000	1.5575
1.2000	1.4091

i Approximate $\int_0^{1.2} f(x)\mathrm{d}x$ using the *Trapezoidal Rule* and a step size of $h = 0.6$.

ii Approximate $\int_0^{1.2} f(x)\mathrm{d}x$ using the *Trapezoidal Rule* and a step size of $h = 0.2$.

iii Estimate the *error* in your answer to previous item.
Hint: Use the procedure to estimate the proportionality factor, C.