i. Let $f(x)=x^3$ be periodic on the interval $\frac{-\pi}{5}, \frac{\pi}{5}$

- Find the Fourier coefficients
- Write Fourier series expansion

$$A_n = \frac{2}{P}\int_{-P/2}^{P/2} f(x)\cos\left(\frac{n\pi x}{P/2}\right)dx \quad , n=0,1,2\cdots$$

$$B_n = \frac{2}{P}\int_{-P/2}^{P/2} f(x)\sin\left(\frac{n\pi x}{P/2}\right)dx \quad , n=1,2,3,\cdots$$

$$f(x) \sim \frac{A_0}{2} + \sum_{n=1}^{\infty}\left[A_n\cos\left(\frac{n\pi x}{P/2}\right) + B_n\sin\left(\frac{n\pi x}{P/2}\right)\right]$$

$\rightarrow P = \frac{\pi}{5} - \left(\frac{-\pi}{5}\right) = \frac{2\pi}{5}$ & $f(x)=x^3 \rightarrow$ odd fnc

$\Rightarrow$
$$A_n = 0$$
$$B_n = \frac{2}{\frac{2\pi}{5}}\int_{-\pi/5}^{\pi/5} x^3\sin\left(\frac{n\pi x}{\pi/5}\right)dx$$

$$f(x)=x^3 \sim \sum_{n=1}^{\infty}\left(\frac{5}{\pi}\int_{-\pi/5}^{\pi/5} x^3\sin(5nx)dx\right)\sin(5nx)$$

ii. Let $f(x)=x^4$ be periodic on the interval $\left(\frac{-\pi}{4}, \frac{\pi}{4}\right)$

- Find the Fourier coefficients
- Write Fourie series expansion

$P = \frac{\pi}{4} - \left(\frac{-\pi}{4}\right) = \frac{\pi}{2}$ & $f(x)=x^4 \rightarrow$ even fnc

$$A_n = \frac{2}{\pi/2}\int_{-\pi/4}^{\pi/4} x^4\cos\left(\frac{n\pi x}{\pi/4}\right)dx \quad , \quad A_0 = \frac{1}{2}\frac{2}{\pi/2}\int_{-\pi/4}^{\pi/4} x^4dx$$

$$B_n = 0$$
$$f(x)=x^4 \sim \sum_{n=1}^{\infty}\left(\frac{4}{\pi}\int_{-\pi/4}^{\pi/4} x^4\cos(4nx)dx\right)\cos(4nx) + \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} x^4dx$$