1 Assignment I

1. (20 pts) Write a code that computes x_n,r_n,p_n,q_n, makes error analysis and tabulates the outcomes;
   $x_n = \frac{1}{3^n}$, approximated by (for n = 1, 2,)

$$r_0 = 1, r_n = \frac{1}{3} r_{n-1} \Longrightarrow \left(= \frac{A}{3^n} \right)$$

$$p_0 = 1, p_1 = \frac{1}{3}, p_n = \frac{4}{3} p_{n-1} - \frac{1}{3} p_{n-2} \Longrightarrow \left(A \frac{1}{3^n} + B \right)$$

$$q_0 = 1, q_1 = \frac{1}{3}, q_n = \frac{10}{3} q_{n-1} - q_{n-2} \Longrightarrow \left(A \frac{1}{3^n} + B 3^n \right)$$

   Generate a table for $x_n - r_n, x_n - p_n, x_n - q_n$, with errors introduced in the starting values: (should be similar to the Table 1)

$$r_0 = 0.99996, p_0 = q_0 = 1, p_1 = q_1 = 0.33332$$

   Comment the behaviors of the errors for the each case.

$x_n - r_n$	$x_n - p_n$	$x_n - q_n$
0.0000400000	0.0000000000	0.0000000000
0.0000133333	0.0000133333	0.0000013333
0.0000044444	0.0000177778	0.0000444444
0.0000014815	0.0000192593	0.0001348148
0.0000004938	0.0000197531	0.0004049383
0.0000001646	0.0000199177	0.0012149794
0.0000000549	0.0000199726	0.0036449931
0.0000000183	0.0000199909	0.0109349977
0.0000000061	0.0000199970	0.0328049992
0.0000000020	0.0000199990	0.0984149998
0.0000000007	0.0000199997	0.2952449999

Table 1: The Error Sequences

2. (20 pts) Write a code that computes values of this expression

$$z = \frac{(x+y)^2 - 2xy - y^2}{x^2}$$

   with different values of x and y. (Hint: use $y = 10000$ and change the x-value as $0.01, 0.001, 0.0001, , \ldots$)

```
x=0.01;   y=10000;z=((x+y)^2-2*x*y-y^2)/x^2;
x=0.001;   y=10000;z=((x+y)^2-2*x*y-y^2)/x^2
x=0.0001;   y=10000;z=((x+y)^2-2*x*y-y^2)/x^2
x=0.00001;   y=10000;z=((x+y)^2-2*x*y-y^2)/x^2
x=0.000001;   y=10000;z=((x+y)^2-2*x*y-y^2)/x^2
x=0.0000001;   y=10000;z=((x+y)^2-2*x*y-y^2)/x^2
```

3. (60 pts) The function $h(x) = xsin(x)$ occurs in the study of undamped forced oscillations. Write a MATLAB program to solve $h(x) = 1$ in [0,2] by:

   (a) Halving the Interval (Bisection) Method

   (b) The Method of False Position (regula falsi)

   (c) Newton's Method

   (d) Muller's method

   (e) Fixed-point Iteration; $x = g(x)$ Method

   - Tabulate the actual error values as the following; (See Table 2. The number of iterations is not limited to or defined as 15.)
   - Plot the behaviours of the errors (use ratios) for the all cases. Compare and discuss the rate of convergence.

n	Bisection $(x_n - r)$	Regula Falsi $(x_n - r)$	Newton $(x_n - r)$	Muller $(x_n - r)$	Fixed-point $(x_n - r)$
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
12					
13					
14					
15					
n	Bisection $f(x_n)$	Regula Falsi $f(x_n)$	Newton $f(x_n)$	Muller $f(x_n)$	Fixed-point $f(x_n)$
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
12					
13					
14					
15					

Table 2: The Error Sequences