Ceng 375 Numerical Computing
Midterm
Nov 10, 2010 14.40–16.30
Good Luck!

1. (10 pts) A three digit, decimal machine which rounds all intermediate calculations, calculates the value of

$$f(x) = x^2 - 6x + 8 \; for \; x = 1.99 \; as \; \overline{f}(1.99) = 0.0600$$

   What are the forward and backward errors error associated with this calculation?

2. (10 pts) Derive the Newton's method formula using a Taylor series of $f(x)$.

3. (20 pts) Use Muller's method to find the root of

$$f(x) = x^3 - x - 2$$

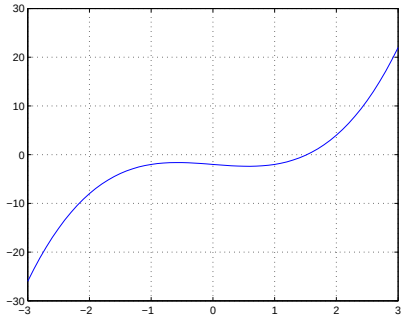

Figure 1: Plot of the function, $x^3 - x - 2$.

Start with $x_2 = 1.0$, $x_0 = 1.2$, and $x_1 = 1.4$ and find x_3 and x_4 (two iterations).

4. (30 pts) Consider the function:

$$f(x) = sin(x) - 4 * x + 2$$

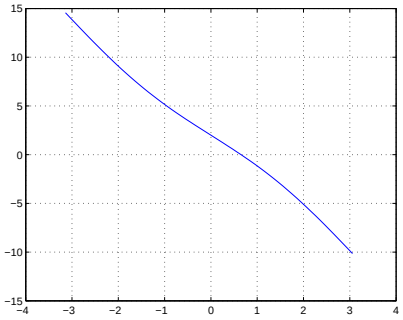

Figure 2: Plot of the function, $sin(x) - 4 * x + 2$.

i Use two iterations of Newton s method to estimate the root of this function between $x = 0.0$ and $x = 1.0$ (Use four significant figures)

ii Estimate the error in your answer to part i (Use more than four significant figures).

iii Approximately how many iterations of the bisection method would have been required to achieve the same error of part ii? (Hint: if the value in part ii is negative, take absolute value of it.)

5. (30 pts) Consider the linear system ($Ax = b$);

$$A = \begin{bmatrix} 1 & 3 & 1 & 1 \\ 2 & 5 & 2 & 2 \\ -1 & -3 & -3 & 5 \\ 1 & 3 & 2 & 2 \end{bmatrix} \quad and \quad b = \begin{bmatrix} 6 \\ 2 \\ 4 \\ 3 \end{bmatrix}$$

i Solve this system by Gaussian elimination with pivoting. How many row interchanges are needed?

ii What is the value of determinant?

iii Obtain the LU decomposition of the system.

iv Repeat without any row interchanges (only for the first item). Do you get the same results? Why?