

Ceng 375 Numerical Computing

Final

Jan 18, 2012 10.00–11.50

Good Luck!

1. **(10 pts)** An engineer runs *the same* FORTRAN program on two different computers, a PC and a UNIX Workstation. Neither system produces any error messages, but the resulting outputs differ by several orders of magnitude more than machine precision. What, if any, reasonable explanations are there for this phenomenon?

2. **(25 pts)** In Newton's method the approximation x_{n+1} to a root of $f(x) = 0$ is computed from the approximation x_n using the equation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- i **(10 pts)** Derive the above formula, using a Taylor series of $f(x)$.
- ii **(15 pts)** For $f(x) = x - 3^{-x}$, refine the approximation $x_0 = 0.54$ to the unique root of $f(x)$ by carrying out one iteration of Newton's method.

3. Choose only **three** questions. Each question is 25 points. **Circle/Mark the question number that you want to be graded.**

1 Consider the linear system

$$\begin{aligned}7x_1 - 3x_2 + 4x_3 &= 6 \\-3x_1 + 2x_2 + 6x_3 &= 2 \\2x_1 + 5x_2 + 3x_3 &= -5\end{aligned}$$

- i **(10 pts)** Solve this system with the Jacobi method. First rearrange to make it diagonally dominant if possible. Use $[0, 0, 0]$ as the starting vector.
- ii **(15 pts)** Repeat with Gauss-Seidel method. Compare with Jacobi method.

2 For the given data points;

x	y
1	1.06
2	1.12
3	1.34
5	1.78

- i **(15 pts)** construct the divided-difference table.
- ii **(5 pts)** interpolate for $x = 4$.
- iii **(5 pts)** extrapolate for $x = 5.5$.

3 Least Squares Method

- i **(12.5 pts)** A function $f_{app}(x)$ is to be used as an approximation to a set of data (x_i, f_i) with $i = 0, 1, 2, \dots, N$. Suppose further that the function $f_{app}(x)$ depends on two parameters a and b . Provide full details of how the parameters a and b can be determined.
- ii **(12.5 pts)** Using the result of the previous item, obtain the normal equations for the function $f_{app}(x) = a + b\sqrt{x}$. **Do not attempt to solve these equations.**

- 4 Write the expression to economize the the Maclaurin series for e^{2x} with the precision 0.008 by using Chebyshev polynomials. Do not perform the calculations.

5 Consider the following table of data

x_i	f_i
0.0000	0.0000
0.2000	0.5879
0.4000	1.0637
0.6000	1.3927
0.8000	1.5573
1.0000	1.5575
1.2000	1.4091

- i **(7.5 pts)** Approximate $\int_0^{1.2} f(x)dx$ using the *Trapezoidal Rule* and a step size of $h = 0.6$.
- ii **(7.5 pts)** Approximate $\int_0^{1.2} f(x)dx$ using the *Trapezoidal Rule* and a step size of $h = 0.2$.
- iii **(10 pts)** Estimate the *error* in your answer to previous item.
Hint: Use the procedure to estimate the proportionality factor, C .

$$\int_a^b f(x)dx = (h/2)(f_0 + 2f_1 + 2f_2 + \dots + 2f_{n-1} + f_n) + Ch^2$$

