1 Assignment I

1. (20 pts) Write a code that computes x_n,r_n,p_n,q_n, makes error analysis and tabulates the outcomes;
   $x_n = \frac{1}{3^n}$, approximated by (for n = 1, 2,)

$$r_0 = 1, r_n = \frac{1}{3} r_{n-1} \Longrightarrow \left(= \frac{A}{3^n} \right)$$

$$p_0 = 1, p_1 = \frac{1}{3}, p_n = \frac{4}{3} p_{n-1} - \frac{1}{3} p_{n-2} \Longrightarrow \left(A \frac{1}{3^n} + B \right)$$

$$q_0 = 1, q_1 = \frac{1}{3}, q_n = \frac{10}{3} q_{n-1} - q_{n-2} \Longrightarrow \left(A \frac{1}{3^n} + B 3^n \right)$$

   Generate a table for $x_n - r_n, x_n - p_n, x_n - q_n$, with errors introduced in the starting values: (should be similar to the Table 1)

$$r_0 = 0.99996, p_0 = q_0 = 1, p_1 = q_1 = 0.33332$$

   Comment the behaviors of the errors for the each case.

$x_n - r_n$	$x_n - p_n$	$x_n - q_n$
0.0000400000	0.0000000000	0.0000000000
0.0000133333	0.0000133333	0.0000013333
0.0000044444	0.0000177778	0.0000444444
0.0000014815	0.0000192593	0.0001348148
0.0000004938	0.0000197531	0.0004049383
0.0000001646	0.0000199177	0.0012149794
0.0000000549	0.0000199726	0.0036449931
0.0000000183	0.0000199909	0.0109349977
0.0000000061	0.0000199970	0.0328049992
0.0000000020	0.0000199990	0.0984149998
0.0000000007	0.0000199997	0.2952449999

Table 1: The Error Sequences

2. (20 pts) Write a code that computes values of this expression

$$z = \frac{(x+y)^2 - 2xy - y^2}{x^2}$$

   with different values of x and y. (Hint: use $y = 10000$ and change the x-value as $0.01, 0.001, 0.0001, , \ldots$)

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x=0.01;  y=10000;z=((x+y)^2-2*x*y-y^2)/x^2;
x=0.001;  y=10000;z=((x+y)^2-2*x*y-y^2)/x^2
x=0.0001;  y=10000;z=((x+y)^2-2*x*y-y^2)/x^2
x=0.00001;  y=10000;z=((x+y)^2-2*x*y-y^2)/x^2
x=0.000001;  y=10000;z=((x+y)^2-2*x*y-y^2)/x^2
x=0.0000001;  y=10000;z=((x+y)^2-2*x*y-y^2)/x^2
```

3. (60 pts) Write a MATLAB program for the given function to solve by using bisection, regula falsi, and Newton's methods.

$$f(x) = 4x + cos(x) - e^{x^2}$$

Your program should

- tabulate the error values as the following;

n	Bisection $(x_n - r)$	...	Bisection $f(x_n)$	...
1				
$\vdots$				

Table 2: The Error Sequences

- plot the behaviours of the errors (may use ratios) for the all cases.

Compare and discuss the rate of convergence.