Take-home ~~Open Book~~ Quiz - Ch1 Measurements
(~~Duration: 30 minutes~~)

1. The radius of a solid sphere is measured to be (6.50 ±0.20) cm, and its mass is measured to be (1.85 ± 0.02) kg. Determine the density of the sphere in kilograms per cubic meter and the uncertainty in the density.

$$R = (6.50 \pm 0.20)\,cm = (6.50 \pm 0.20) \times 10^{-2}\,m$$

$$m = (1.85 \pm 0.02)\,kg \quad \boxed{3 \text{ sig figs}}$$

$$\rho = \frac{m}{V} = \frac{m}{\frac{4\pi R^3}{3}}$$ Three steps

1st step: Raised to a power $C = A^n$ & $\Delta C = C\,|n|\,\frac{\Delta A}{A}$

$$C = R^3 \rightarrow \Delta C = R^3 |3| \frac{\Delta R}{R} = (2.75 \times 10^{-4}\,m^3)\,|3|\,\frac{0.20 \times 10^{-2}\,m}{6.50 \times 10^{-2}\,m}$$

$$= (6.50 \times 10^{-2}\,m)^3 \qquad = 2.54 \times 10^{-5}\,m^3$$

$$= 2.75 \times 10^{-4}\,m^3$$

$$\Longrightarrow (2.75 \times 10^{-4} \pm 2.54 \times 10^{-5})\,m^3$$

2nd step: Multiplication with a scalar

$$\frac{4}{3}\pi\,(2.75 \times 10^{-4} \pm 2.54 \times 10^{-5})\,m^3 = (1.15 \times 10^{-3} \pm 1.06 \times 10^{-4})\,m^3$$

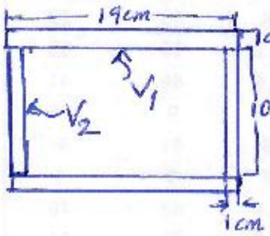
3rd step: Multiplication/Division $C = \frac{A}{B}$ & $\Delta C = |C| \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$

$$\frac{(1.85 \pm 0.02)\,kg}{(1.15 \times 10^{-3} \pm 1.06 \times 10^{-4})\,m^3} \Longrightarrow C = \frac{1.85\,kg}{1.15 \times 10^{-3}\,m^3} \;\&\; \Delta C = |1.61 \times 10^{3}\,kg/m^3| \sqrt{\left(\frac{0.02}{1.85}\right)^2 + \left(\frac{1.06 \times 10^{-4}}{1.15 \times 10^{-3}}\right)^2}$$

$$= 1.61 \times 10^{3}\,kg/m^3 \qquad = 0.149 \times 10^{3}\,kg/m^3$$

$$\Longrightarrow \boxed{(1.61 \pm 0.15) \times 10^{3}\,kg/m^3} \rightsquigarrow (1.6 \pm 0.2) \times 10^{3}\,kg/m^3$$

2. A sidewalk is to be constructed around a swimming pool that measures (10.0 ± 0.1) m by (17.0 ± 0.1) m. If the sidewalk is to measure (1.00 ± 0.01) m wide by (9.0 ± 0.1) cm thick, what volume of concrete is needed, and what is the approximate uncertainty of this volume?



Thickness: $(9.0 \pm 0.1)\,cm = (9.0 \pm 0.1) \times 10^{-2}\,m$

$V_{total} = 2(V_1 + V_2)$ & $(17.0 \pm 0.1)\,m$ & $(10.0 \pm 0.1)\,m$

① ② sidewalk: $(1.00 \pm 0.01)\,m$

①: Summation/Substraction

$\underset{A\ \ \Delta A}{(17.0 \pm 0.1)}\,m + \underset{B\ \ \Delta B}{(1.00 \pm 0.01)}\,m + \underset{C\ \ \Delta C}{(1.00 \pm 0.01)}\,m$

$\sim 19.0\,m \pm \sqrt{(0.1)^2 + (0.01)^2 + (0.01)^2} = (19.0 \pm 0.1)\,m$

$\Rightarrow V_1 = \underset{A\ \ \Delta A}{(19.0 \pm 0.1)}\,m\,\underset{B\ \ \Delta B}{(1.00 \pm 0.01)}\,m\,\underset{C\ \ \Delta C}{(9.0 \pm 0.1)} \times 10^{-2}\,m$; $(19.0)(1.00)(9.0 \times 10^{-2}) = 1.71\,m^3 = V_1$

$\Delta V_1 = |1.71\,m^3| \sqrt{\left(\frac{0.1}{19.0}\right)^2 + \left(\frac{0.01}{1.00}\right)^2 + \left(\frac{0.1 \times 10^{-2}}{9.0 \times 10^{-2}}\right)^2} = 0.0269\,m^3 \sim 0.03\,m^3$

$\Rightarrow V_1 \pm \Delta V_1 = (1.71 \pm 0.03)\,m^3$

② $V_2 = \underset{A\ \ \Delta A}{(10.0 \pm 0.1)}\,m\,\underset{B\ \ \Delta B}{(1.00 \pm 0.01)}\,m\,\underset{C\ \ \Delta C}{(9.0 \pm 0.1)} \times 10^{-2}\,m$; $(10.0)(1.00)(9.0 \times 10^{-2}) = 0.90\,m^3 = V_2$

$\Delta V_2 = |0.90\,m^3| \sqrt{\left(\frac{0.1}{10.0}\right)^2 + \left(\frac{0.01}{1.00}\right)^2 + \left(\frac{0.1 \times 10^{-2}}{9.0 \times 10^{-2}}\right)^2} = 0.0162\,m^3 \sim 0.02\,m^3$

$V_{total} = 2(V_1 + V_2) = 2\left(\underset{A\ \ \Delta A}{(1.71 \pm 0.03)}\,m^3 + \underset{B\ \ \Delta B}{(0.90 \pm 0.02)}\,m^3\right)$

$= \boxed{(5.22 \pm 0.08)\,m^3} \rightarrow \boxed{1.5\%} = \frac{0.08}{5.22} \times 100$

Open Book Quiz – Ch2 Motion Along a Straight Line
(Duration: 30 minutes)

1. Compute your average velocity in the following two cases:
    1. You walk 73.2 m at a speed of 1.22 m/s and then run 73.2 m at a speed of 3.05 m/s along a straight track.
    2. You walk for 1.00 min at a speed of 1.22 m/s and then run for 1.00 min at 3.05 m/s along a straight track.
    3. Graph x versus t for both cases and indicate how the average velocity is found on the graph.

a) 73.2 m & 1.22 m/s $\leadsto t_1 = \frac{73.2\,m}{1.22\,m/s}$

73.2 m & 3.05 m/s $\leadsto t_2 = \frac{73.2\,m}{3.05\,m/s}$

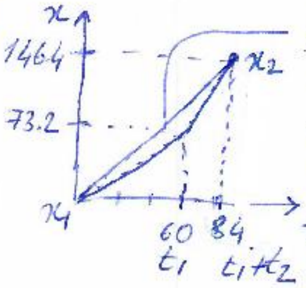
$\left.\right\} v_{avg} = \frac{\Delta x}{\Delta t} = \frac{73.2m + 73.2m}{t_1 + t_2} = 1.74\,m/s$

b) 60 s & 122 m/s $\sim x_1 = (60s)(122\,m/s)$

60s & 3.05 m/s $\sim x_2 = (60s)\,3.05\,m/s)$

$\left.\right\} v_{avg} = \frac{x_1 + x_2}{120s} = 2.14\,m/s$

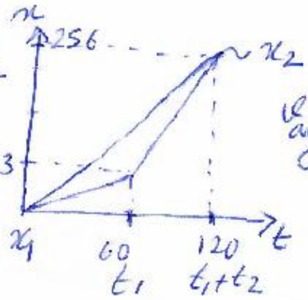
c)



$\Delta x / \Delta t$

slope: $\frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{84s}$

$v_{avg} = 1.74\,m/s$

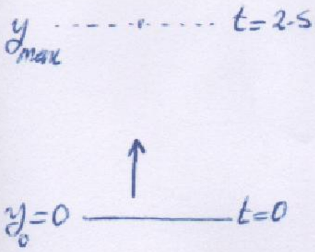


$x_2 - x_1$

$v_{avg} = \frac{256.2 - 0}{120}$

$= 2.14\,m/s$

2. A rock is thrown vertically upward from ground level at time t=0s. At t=1.5s it passes the top of a tall tower, and 1.0 s later it reaches its maximum height. What is the height of the tower?



at $y_{max} \rightarrow v_y = 0$

$\rightarrow 0 = v_{0y} - 9.8\,m/s^2 (2.5\,s)$

$v_{0y} = 24.5\,m/s$

$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$

$v_y = v_{0y} - g t$

$\Rightarrow y(t = 1.5s)$: height of the tower

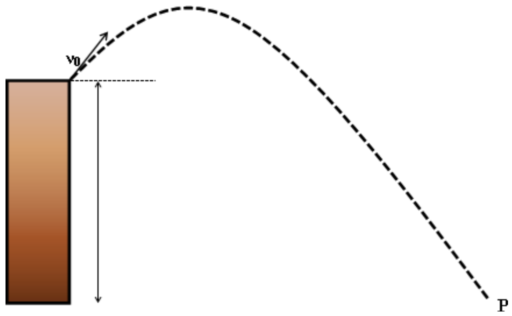
$y(t=1.5s) = y_0 + v_{0y}(1.5s) - \frac{1}{2} g (1.5s)^2$

$= 0 + (24.5\,m/s)(1.5s) - (4.9\,m/s^2)(1.5s)^2$

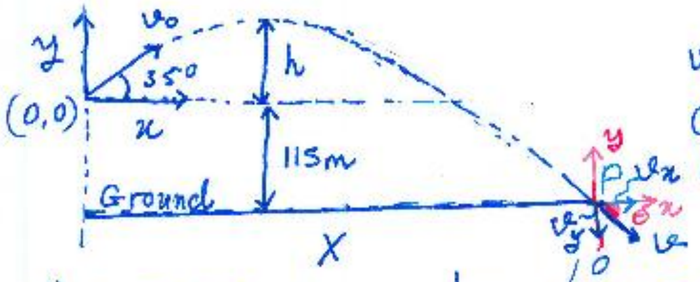
$= 25.725\,m \leadsto \boxed{height \cong 26m}$

Open Book Quiz – Ch4 Motion in Two and Three Dimensions (Duration: 30 minutes)

1. A projectile is shot from the edge cliff 115 m above ground level with an initial speed of 65 m/s at an angle of 35° with the horizontal.



   a. Determine the distance X of point P from the base of the vertical cliff.
   b. What is the velocity v at point P in magnitude-angle notation and in unit-vector notation?
   c. Find the maximum height reached by projectile **<u>above ground</u>.**



$v_0 = 65\ m/s$
$\theta = 35°$
$h_0 = 115\ m$

$v_{0x} = v_0 \cos\theta$
$v_{0y} = v_0 \sin\theta$
$\sin 35 = 0.57$
$\cos 35 = 0.82$

i) $x = x_0 + v_{0x} t$

$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$

$\Rightarrow x - x_0 = v_0 \cos\theta\, t$

$X = (65\,m/s) \cos 35 (9.97\,s)$

$\boxed{\cong 531\,m}$

$y - y_0 = v_0 \sin\theta t - \frac{1}{2} g t^2$

$-115\,m = (65\,m/s) \sin 35° t - \frac{1}{2}\, 9.8\,m/s^2\, t^2$

$\Rightarrow 4.9 t^2 - 37.3 t - 115 = 0$

$t_{1,2} = \frac{-(-37.3) \pm \sqrt{(-37.3)^2 - 4(4.9)(-115)}}{2(4.9)}$

$t_1 = -2.36\,s \rightsquigarrow$ not physical $\leftarrow$ minus sign

$t_2 = 9.97\,s = t$

ii) At point P

$v_x = v_{0x} = v_0 \cos\theta$

$v_y = v_{0y} - g t = v_0 \sin\theta - g t$

$v_x = (65\,m/s) \cos 35°$

$v_y = (65\,m/s) \sin 35° - (9.8\,m/s^2)(9.97\,s)$

$v_x = 53.2\,m/s$ & $v_y = -60.7\,m/s$

$\vec{v} = v_x \hat{i} + v_y \hat{j} = \boxed{(53.2\hat{i} - 60.7\hat{j})\,m/s}$

$\rightarrow |\vec{v}| = \sqrt{(53.2\,m/s)^2 + (-60.7\,m/s)^2}$

$\boxed{= 80.7\,m/s}$ $\theta = \tan^{-1} \frac{-60.7}{53.2}$

$\boxed{\cong -49°}$

iii) $h_{max} = h_0 + h \rightsquigarrow$ What is h?

$v_y = v_{0y} - g t = 0$ at max height

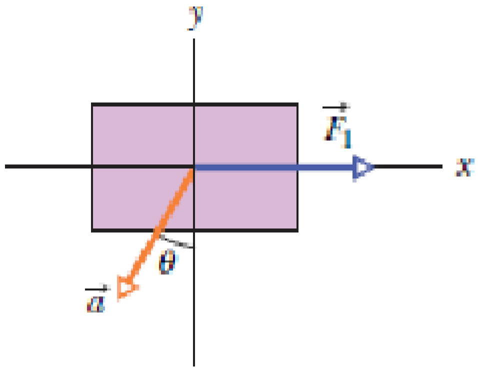
$t_f = \frac{v_{0y}}{g} \rightarrow h = v_{0y} \frac{v_{0y}}{g} - \frac{1}{2} g \left(\frac{v_{0y}}{g}\right)^2$

$h = \frac{1}{2} \frac{v_0^2 \sin^2\theta}{g} = \frac{1}{2} \frac{(65\,m/s)^2 \sin^2 35°}{9.8\,m/s^2}$

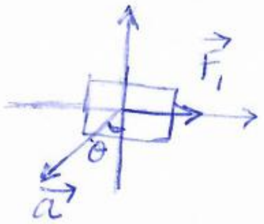
$\boxed{h \cong 71\,m} \Rightarrow h_{max} = 115\,m + 71\,m = \boxed{186\,m}$

Open Book Quiz – Ch5 Force and Motion I

(Duration: 30 minutes)



1. There are two forces on the 2.00 kg box in the overhead view of figure below, but only one is shown. For F_1=20.0 N, a=12.0 m/s^2, θ=30.0^o, find the second force
   a. in unit-vector notation
   b. as a magnitude
   c. an angle relative to the positive direction of the x-axis.



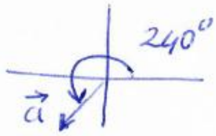
$m = 2\ kg$

$|\vec{F}_1| = 20\ N$

$|\vec{a}| = 12\ m/s^2$

$\theta = 30$

$\vec{F}_1 + \vec{F}_2 = m\vec{a}$



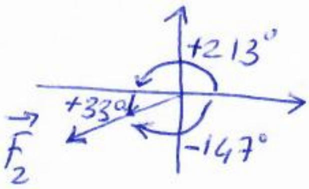
$\vec{F}_2 = (2kg)\left(|\vec{a}|\cos 240^\circ \hat{i} + |\vec{a}|\sin 240\, \hat{j}\right) - (20\,N\,\hat{i})$

i) unit vector notation $= 24(-0.5\,\hat{i}) + 24(-0.866)\hat{j} - 20\,\hat{i}$

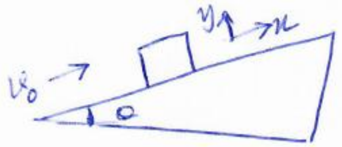
$= \boxed{-32\,\hat{i} - 20.8\,\hat{j}}$

ii) magnitude $\sim |\vec{F}_2| = \sqrt{(-32)^2 + (-20.8)^2} = 38.2N$

iii) $\theta = \tan^{-1}\dfrac{-20.8}{-32} = 33^\circ \quad \{\tan\theta = 0.656\}$



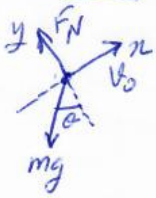
2. A block is projected up a frictionless inclined plane with initial speed v_0=3.50 m/s. The angle of incline is θ=32.0°. (a) How far up the plane does the block go? (b) How long does it take to get there? (c) What is its speed when it gets back to the bottom?



$\theta = 32°$

$v_0 = 3.5\,m/s$

FBD



$-mg\sin\theta = ma$

$F_N - mg\cos\theta = 0$

$\rightarrow a = -g\sin\theta$

$F_N = mg\cos\theta$

assuming constant acceleration

$x - x_0 = v_0 t + \frac{1}{2}at^2$

$v = v_0 + at$

$v^2 = v_0^2 + 2a(x - x_0)$

i) v = 0 at highest point

$\rightarrow 0 = v_0^2 + 2a(x - x_0)$

$0 = (3.5\,m/s)^2 - 2 \times 9.8\,m/s^2 \sin 32 (x - \phi)$

$x = \frac{(3.5\,m/s)^2}{2 \times 9.8\,m/s^2 \sin 32} = \underline{\underline{1.18\,m}}$

ii) $0 = v_0 + at$

$\rightarrow t = \frac{-3.5\,m/s}{-9.8\,m/s^2 \sin 32}$

$= 0.674\,s$

iii) $x - x_0 = v_0 t + \frac{1}{2}at^2$

$\phi - \phi = v_0 t + \frac{1}{2}at^2$

$-v_0 \cancel{t} = \frac{1}{2}at^{\cancel{2}}$

$-3.5\,m/s = \frac{1}{2}(-9.8\,m/s^2 \sin 32°)t$

$\rightarrow t = \underline{1.35\,s}$

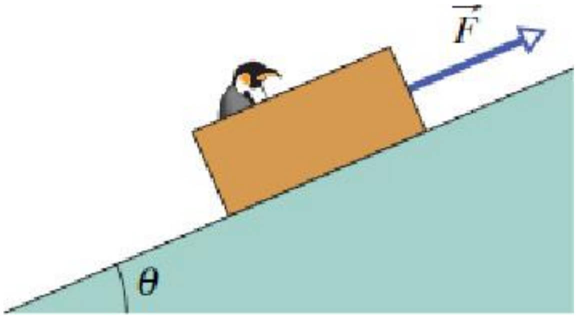
$\rightarrow v = 3.5\,m/s + (-9.8\,m/s^2 \sin 32°)(1.35\,s)$

$= -3.50\,m/s$

$\rightarrow \boxed{\vec{v} = 3.5\,m/s\,(-\hat{i})}$

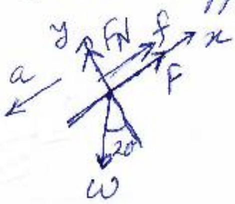
Open Book Quiz – Ch6 Force and Motion II
(Duration: 30 minutes)

1. A loaded penguin sled weighing 80 N rests on a plane inclined at angle θ=20.0^o to the horizontal (see Figure). Between the sled and the plane, the coefficient of static friction is 0.25, and the coefficient of kinetic friction is 0.15.
   a. What is the least magnitude of the force parallel to the plane, that will prevent the sled from slipping down the plane?
   b. What is the minimum magnitude F that will start the sled moving up the plane?
   c. What value of F is required to move the sled up the plane at constant velocity?



$W = 80\ N$
$\theta = 20°$
$\mu_s = 0.25$
$\mu_k = 0.15$

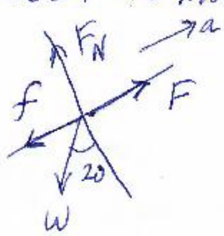
i) prevent slipping down ⇝ no motion. $F_{min} = ?$



$x:\ F + f - W\sin 20 = ma_x = 0$
$y:\ F_N - W\cos\theta = ma_y = 0$
where $f = f_{s,max}$

$F_N = W\cos\theta\ ;\ f_{s,max} = \mu_s F_N = \mu_s W\cos\theta$
$\rightarrow F = W\sin 20 - \mu_s W\cos 20$
$= 80N(\sin 20 - 0.25\cos 20) = \boxed{8.57N}$

ii) start to moving up ⇝ no motion. $F_{min} = ?$



$x:\ F - f - W\sin 20 = 0$
$y:\ F_N - W\cos 20 = 0$
$f = f_{s,max} = \mu_s F_N$

$F = f + W\sin 20 = \mu_s W\cos 20 + W\sin 20$
$= W(\mu_s\cos 20 + \sin 20) = 80N(0.25\cos 20 + \sin 20)$
$= \boxed{46.16N}$

iii) move up, $\underbrace{\text{constant velocity}}_{a=0}$ ⇝ motion. $F = ?$

$F - \mu_k F_N - W\sin 20 = ma_x = 0$
$F - \mu_k W\cos 20 - W\sin 20 = 0$

$F = W(\mu_k\cos 20 + \sin 20)$
$= 80\ N(0.15\cos 20 + \sin 20)$
$= \boxed{38.63}$

Open Book Quiz – Ch7 Kinetic Energy and Work
(Duration: 30 minutes)

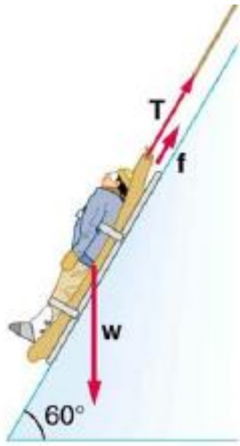
1. Suppose the ski patrol lowers a rescue sled and victim, having a total mass of 90.0 kg, down a $60.0°$ slope at **constant speed**, as shown in Figure. The coefficient of friction between the sled and the snow is 0.100.

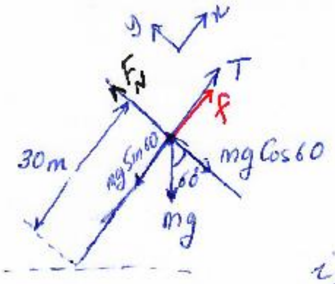
   (a) How much work is done by friction as the sled moves 30.0 m along the hill?

   (b) How much work is done by the rope on the sled in this distance?

   (c) What is the work done by the gravitational force on the sled?

   (d) What is the total work done?





downward motion

$T + f - mg\,Sin60 = ma_x = 0$ (constant speed)

$F_N - mg\,Cos60 = 0$

$f = \mu F_N$

i) Work done by friction; $W = \vec{F}\cdot\vec{d}$

$W = \vec{f}\cdot\vec{d} = fd\,Cos180° = \mu F_N d\,Cos180$

$= \mu mg\,Cos60\,d\,Cos180$

$= (0.1)(90\,kg)(9.8\,m/s^2)\,Cos60\,(30m)\,Cos180$

$= -1.323 \times 10^3\,J$

ii) Work done by rope;

$W = \vec{T}\cdot\vec{d} = Td\,Cos180° = (mg\,Sin60 - f)\,d\,Cos180$

$= (mg\,Sin60 - \mu mg\,Cos60)\,d\,Cos180$

$= (90\,kg)(9.80\,m/s^2)\,(Sin60 - (0.1)Cos60)\,(30m)\,Cos180$

$= -2.1592 \times 10^4\,J$

iii) Word done by gravitational force;

$W = \vec{F_g}\cdot\vec{d} = mg\,Sin60\,d\,Cos\phi$

$= (90\,kg)(9.8\,m/s^2)\,Sin60\,(30m)\,Cos\phi$

$= 2.2915 \times 10^4\,J$

iv) Total work; $W = -1.323 \times 10^3\,J - 2.1592 \times 10^4\,J + 2.2915 \times 10^4\,J$

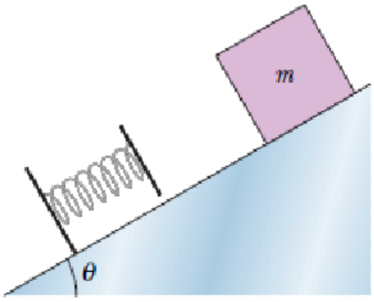
$= \underline{\underline{0}}$

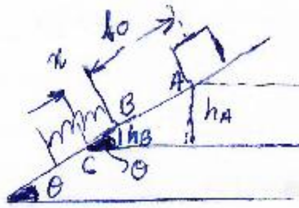
Open Book Quiz – Ch8 Potential Energy and Conservation of Energy (Duration: 30 minutes)

1. A block of mass m = 2 kg is released from rest on a frictionless incline of angle $\theta = 30^{\circ}$ as given in the figure. Below the block is a spring that can be compressed 2.0 cm by a force of 270 N. The block momentarily stops when it compresses the spring by 5.5 cm.

   (a) How far does the block move down the incline from its rest position to this stopping point?

   (b) What is the speed of the block just as it touches the spring?





$U = mgh_A$

$U = 0$ at point C

$\sin\theta = \dfrac{h_A}{x + l_0}$

$\vec{F} = -k\vec{x}$

$270N = k\, 2\times10^{-2}\,m$

$\Rightarrow k = 13500\, N/m = 1.35\times10^{4}\, N/m$

$m = 2\, kg$

i) $x = 5.5\times10^{-2}\,m$ (Point C)

$K_f + U_f = K_i + U_i$

$0 + (U_g + U_s)_f = 0 + (U_g)_i$

$0 + (0 + \frac{1}{2}kx^2) = 0 + mgh_A$

$\frac{1}{2}(1.35\times10^{4}\,N/m)(5.5\times10^{-2}\,m)^2 = (2kg)(9.8\,m/s^2)\,h_A$

$h_A = \dfrac{20.4\,Nm}{19.6\,N} = 1.04\,m = (\sin 30)(x + l_0)$

$\rightarrow l_0 = \dfrac{1.04m}{\sin 30} - x = 2.08 - 5.5\times10^{-2} = \underline{2.03\,m}$

$\boxed{l_0 + x = 2.08\,m}$

ii) Point B; $\Delta h = h_A - h_B$

$K_f + U_f = K_i + U_i$

$\frac{1}{2}mv_B^2 + mgh_B = 0 + mgh_A$

$\frac{1}{2}v_B^2 = g(h_A - h_B)$ $\quad \left|\ \begin{array}{l} h_A = (x + l_0)\sin 30^{\circ} \\ h_B = x \sin 30 \end{array} \right\} h_A - h_B = l_0 \sin 30^{\circ}$

$v_B^2 = 2g\, l_0 \sin 30 = 2 * 9.8\,m/s^2\,(2.03m)\sin 30$

$\Rightarrow \boxed{v_B = 4.46\,m/s}$

Open Book Quiz – Ch9 Center of Mass and Linear Momentum (Duration: 30 minutes)

4. A soccer player kicks a soccer ball of mass 0.45 kg that is initially at rest. The foot of the player is in contact with the ball for $3.0\text{x}10^{-3}$ s, and the force of the kick is given by $\mathbf{F(t)=[(6.0x10^{6})t-(2.0x10^{9})t^{2}]\ N}$ for $0 \leq t \leq 3.0\text{x}10^{-3}$ s, where t is in seconds. Find the magnitudes of
   1. the impulse on the ball due to the kick,
   2. the average force on the ball from the player's foot during the period of contact,
   3. the maximum force on the ball from the player's foot during the period of contact,
   4. the ball's velocity immediately after it loses contact with the player's foot.

$m = 0.45\,kg$
Initially at rest
Contact time $= 3.0\times10^{-3}s$
$F(t) = [6.0\times10^{6}t - 2.0\times10^{9}t^{2}]N$
$0 \leq t \leq 3.0\times10^{-3}s$

i) $J = ?$ impulse $\vec{F}_{av} = \frac{\vec{J}}{\Delta t}$ OR $J = \int F(t)\,dt = \int_0^{3\times10^{-3}} (6\times10^{6}t - 2\times10^{9}t^{2})\,dt$

$\rightarrow J = 3\times10^{6}t^{2} - \frac{2\times10^{9}}{3}t^{3}\Big|_0^{3\times10^{-3}} = 3\times10^{6}(9\times10^{-6}) - \frac{2}{3}\times10^{9}(27\times10^{-9})$ $\boxed{= 9\ Ns}$

ii) $F_{av} = \frac{J}{\Delta t} = \frac{9Ns}{3.0\times10^{-3}s}$ $\boxed{= 3\times10^{3}N}$

iii) $F_{max} = ?$ during the period of contact

$\frac{dF(t)}{dt} = 0 \rightarrow 6\times10^{6} - 4\times10^{9}t = 0 \Rightarrow t = 1.5\times10^{-3}s$

$\Rightarrow F(t = 1.5\times10^{-3}s) = F_{max} = 6\times10^{6}(1.5\times10^{-3}) - 2\times10^{9}(1.5\times10^{-3})^{2}$

$\boxed{F_{max} = 4.5\times10^{3}N}$

iv) $v = ?$ when contact is lost.

$\Delta\vec{p} = \vec{P}_f - \vec{P}_i = m\vec{v}_f - \cancel{m\vec{v}_i} \rightarrow \Delta p = mv = J \Rightarrow v = \frac{J}{m} = \frac{9Ns}{0.45\,kg} = 20\ kg\frac{m}{s^2}\frac{s}{kg}$

$\boxed{v = 20\ m/s}$