İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy101 Physics I
Final Examination
May 22, 2019 10:30 – 12:30
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

INSTRUCTOR:

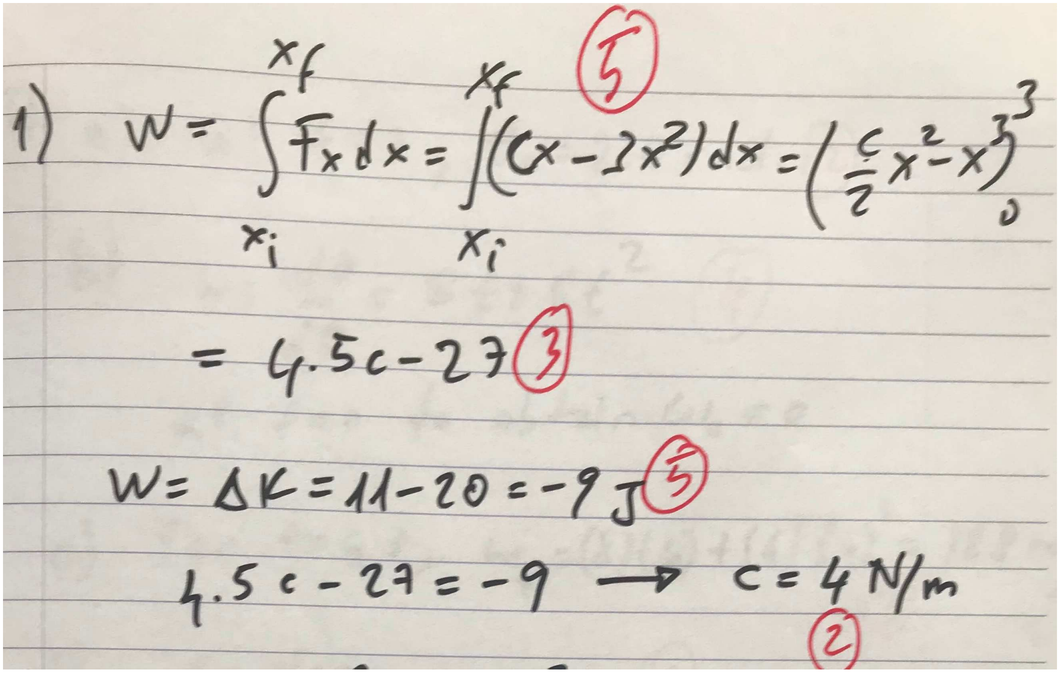
DURATION: 120 minutes

◇ **Answer all the questions.**
◇ **Write the solutions explicitly and clearly. Use the physical terminology.**
◇ **You are allowed to use Formulae Sheet.**
◇ **Calculator is allowed.**
◇ **You are not allowed to use any other electronic equipment in the exam.**
◇ **I declare hereby that I fulfilled the requirements for the attendance according to the University regulations and I accept that my examination will not be valid otherwise.**

Question	Grade	Out of
1A		15
1B		15
2		20
3		20
4		20
5		20
TOTAL		110

This page is intentionally left blank. Use the space if needed.

1. A) A force $\vec{F} = (cx - 3x^2)\hat{x}$ acts on a particle as the particle moves along an x axis, with $\vec{F}$ in newtons, x in meters, and c a constant. At $x = 0$, the particle's kinetic energy is 20.0 J; at $x = 3.00\ m$, it is 11.0 J. Find c.



1) $W = \int_{x_i}^{x_f} F_x\,dx = \int_{x_i}^{x_f} (cx - 3x^2)\,dx = \left(\frac{c}{2}x^2 - x^3\right)\Big|_0^3$ ⑤

$= 4.5c - 27$ ③

$W = \Delta K = 11 - 20 = -9\ \text{J}$ ⑤

$4.5c - 27 = -9 \longrightarrow c = 4\ \text{N/m}$ ②

B) You drop a 2.00 kg physics book to a friend who stands on the ground at distance D=10.0 m below. If your friend's outstretched hands are at distance d=1.50 m above the ground (see Figure),

i How much work Wg does the gravitational force do on the book as it drops to her hands?

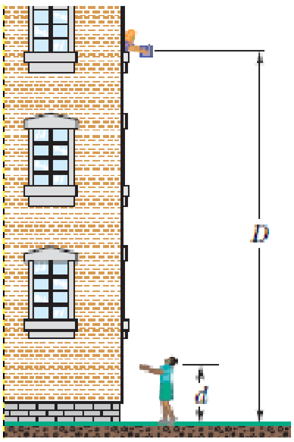
ii What is the change ΔU in the gravitational potential energy of the book–Earth system during the drop?

If the gravitational potential energy U of that system is taken to be zero at ground level,

iii what is U when the book is released?

iv what is U when it reaches her hands?

v Now take U to be 100 J at ground level and again find Wg, ΔU, U at the release point, and U at her hands.



i) $W_g = \vec{F}_g \cdot \vec{d} = mgd\cos\theta$ $\Big\}$ $d = 10m - 1.5m = 8.5m$, $\theta = 0°$

$= (2kg)(9.8m/s^2)8.5m\cos 0°$

$= \underline{\underline{167J}}$

ii) $\Delta U = U_f - U_i = mg(y_f - y_i) = (2kg)(9.8m/s^2)(1.5m - 10m)$

$= \underline{\underline{-167J}}$

iii) $U_i = mgy_i = (2kg)(9.8m/s^2)10m = \underline{\underline{196J}}$ iv) $\underline{\underline{29J}}$

v) $W_g = \underline{\underline{167J}}$, $\Delta U = \underline{\underline{-167J}}$, $U_i = mgy_i + 100J$

$= \underline{\underline{296J}}$

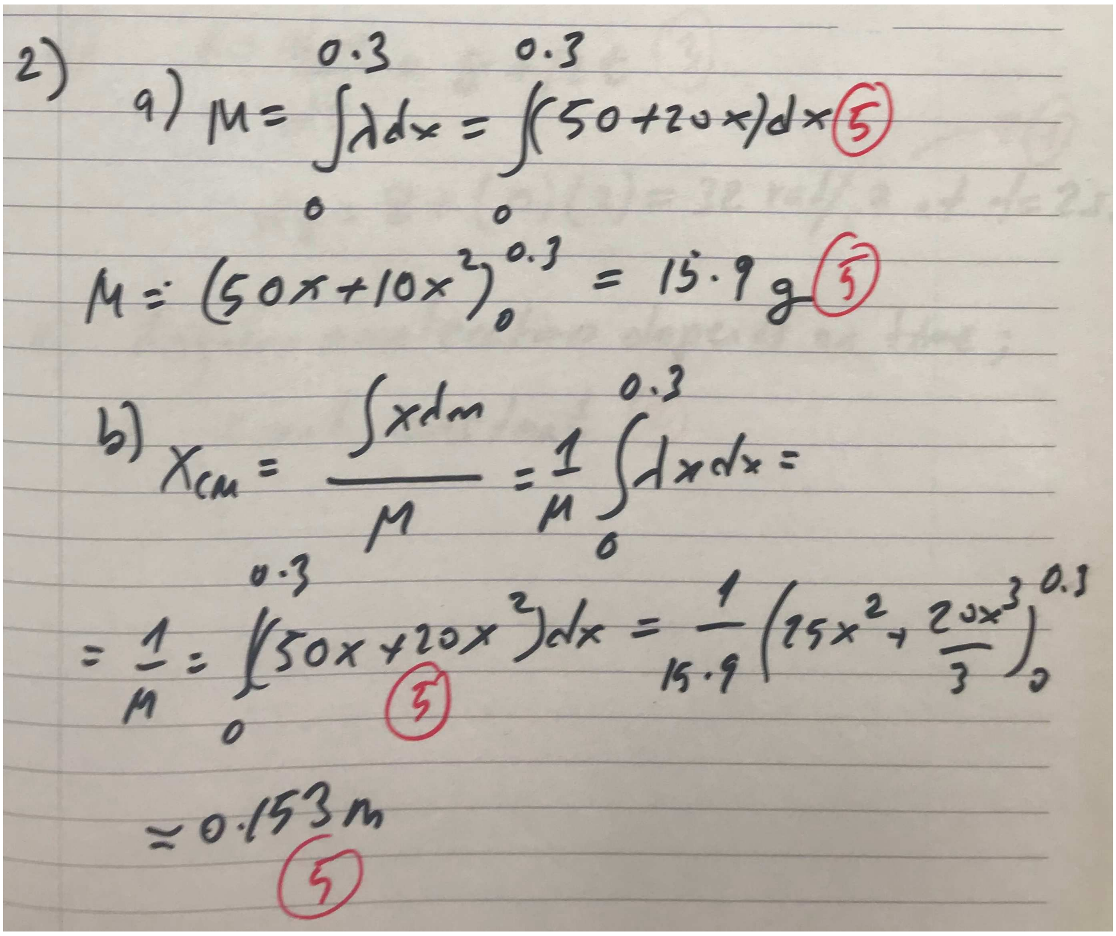
$U_f = mgy_f + 100J$

$= \underline{\underline{129J}}$

2. A rod of length 30.0 cm has linear density (mass per length) given by $\lambda = 50.0\frac{g}{m} + 20.0x\frac{g}{m}$ where x is the distance from one end, measured in meters.

   i What is the mass of the rod?

   ii How far from the $x = 0$ end is its center of mass?



2)

a) $M = \int_0^{0.3} \lambda dx = \int_0^{0.3} (50+20x)dx$ ⑤

$M = (50x+10x^2)_0^{0.3} = 15.9 g$ ⑤

b) $X_{CM} = \frac{\int x dm}{M} = \frac{1}{M}\int_0^{0.3} \lambda x dx =$

$= \frac{1}{M} = \int_0^{0.3} (50x+20x^2)dx = \frac{1}{15.9}\left(25x^2 + \frac{20x^3}{3}\right)_0^{0.3}$ ⑤

$\simeq 0.153 m$ ⑤

3. A soccer player kicks a soccer ball of mass 0.40 kg that is initially at rest. The foot of the player is in contact with the ball for 2.0×10^{-3} s, and the force of the kick is given by

$$F(t) = [(12.0 \times 10^{9})t^{2} - (4.0 \times 10^{12})t^{3}]\ N$$

for $0 \leq t \leq 2.0 \times 10^{-3}$ s, where t is in seconds. Find the magnitudes of

- i the impulse on the ball due to the kick,
- ii the average force on the ball from the player's foot during the period of contact,
- iii the maximum force on the ball from the player's foot during the period of contact,
- iv the ball's velocity immediately after it loses contact with the player's foot.

$m = 0.40$ kg
Initially at rest
Contact time = 2.0×10^{-3} s

i) $J = ?$ impulse $\vec{F}_N = \dfrac{\vec{J}}{\Delta t}$ OR $J = \int F(t)dt$

$$= \int_0^{2\times10^{-3}} (12\times10^{9})t^2 - (4\times10^{12})t^3 dt$$

$$\Rightarrow J = 4\times10^{9}t^3 - 1\times10^{12}t^4 \Big|_0^{2\times10^{-3}}$$

$$= (4\times10^{9})(8\times10^{-9}) - (1\times10^{12})(16\times10^{-12}) = \underline{\underline{16 Ns}}$$

ii) $F_{av} = \dfrac{J}{\Delta t} = \dfrac{16 Ns}{2.0\times10^{-3} s} = \underline{\underline{8\times10^{3} N}}$

iii) $F_{max} = ?$ during the period of contact $\left\{ \text{at maximum; } \dfrac{dF(t)}{dt} = 0 \right.$

$\rightarrow 24\times10^{9}t - 12\times10^{12}t^2 = 0 \rightsquigarrow t = 2\times10^{-3} s$

$\Rightarrow F(t = 2\times10^{-3}s) = F_{max} = (12\times10^{9})(2\times10^{-3}s)^2 - (4\times10^{12})(2\times10^{-3}s)^3 = \underline{\underline{16\times10^{3} N}}$

iv) $v = ?$ when contact is lost

$\Delta\vec{p} = \vec{P}_f - \vec{P}_i = m\vec{v}_f - \underset{0}{\cancel{m\vec{v}_i}} \rightsquigarrow \Delta p = mv = J \rightsquigarrow v = \dfrac{J}{m} = \dfrac{16 Ns}{0.40 kg} = \underline{\underline{40 m/s}}$

4. The angular position of a point on a rotating wheel is given by $\theta(t) = 2.0 + 4.0t^2 + 2.0t^3$, where θ is in radians and t is in seconds. At t = 0,

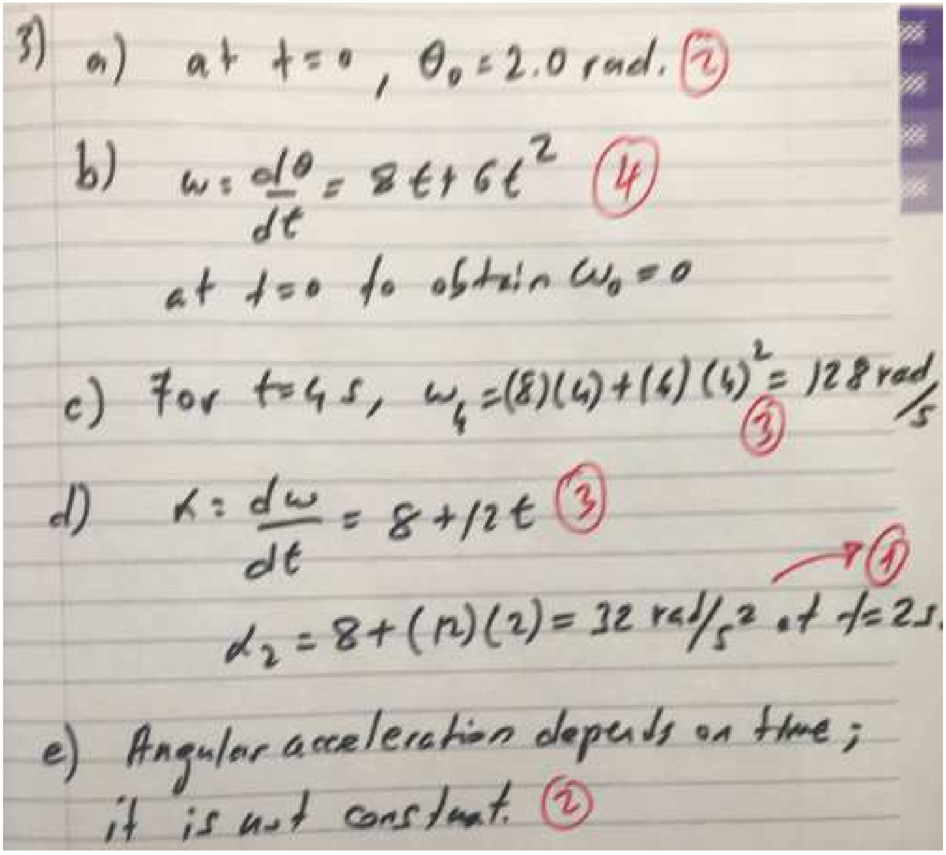
   i what is the point's angular position?

   ii what is its angular velocity?

   iii what is its angular velocity at t = 4.0 s?

   iv Calculate its angular acceleration at t = 2.0 s.

   v Is its angular acceleration constant?



3) a) at $t=0$, $\theta_0 = 2.0$ rad. ②

b) $\omega = \frac{d\theta}{dt} = 8t + 6t^2$ ④

at $t=0$ to obtain $\omega_0 = 0$

c) For $t=4$ s, $\omega_4 = (8)(4) + (6)(4)^2 = 128$ rad/s ③

d) $\alpha = \frac{d\omega}{dt} = 8 + 12t$ ③

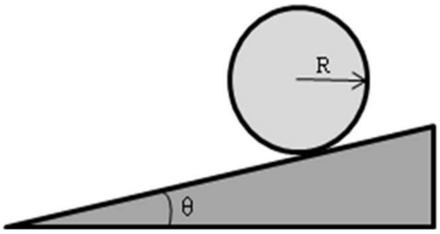
$\alpha_2 = 8 + (12)(2) = 32$ rad/s^2 at $t = 2$s. ①

e) Angular acceleration depends on time; it is not constant. ②

5. A uniform ball, of mass $M = 6.0$ kg and radius R, rolls smoothly from rest down a ramp at angle $\theta = 30.0°$ (see Figure, $I = \frac{2}{5}MR^2$)

i The ball descends a vertical height $h = 1.20$ m to reach the bottom of the ramp. What is its speed at the bottom?

ii What are the magnitude and direction of the frictional force (f_s) on the ball as it rolls down the ramp?



$M = 6\,kg$
$\theta = 30°$
$I_{com} = \frac{2}{5}MR^2$
$h = 1.2\,m$

i) Mechanical Energy is conserved for the ball-earth system

$\rightsquigarrow F_N$ & f_s does not work

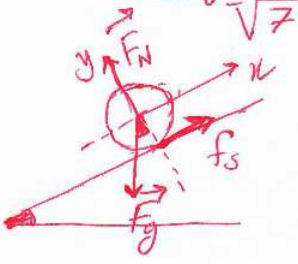
$$K_f + \underset{0}{\cancel{U_f}} = \underset{0}{\cancel{K_i}} + U_i \rightsquigarrow K_f = U_i \rightsquigarrow \frac{1}{2}I_{com}\omega^2 + \frac{1}{2}Mv_{com}^2 = Mgh$$

$$\rightsquigarrow \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v_{com}}{R}\right)^2 + \frac{1}{2}Mv_{com}^2 = Mgh$$

$$\frac{7}{10}\cancel{M}v_{com}^2 = \cancel{M}gh \rightarrow v = \sqrt{\frac{10}{7}gh}$$

$$\rightsquigarrow v = \sqrt{\frac{10}{7}(9.8\,m/s^2)(1.2\,m)} = \underline{\underline{4.1\,m/s}}$$

ii)



Newton's 2nd law in x-direction

$$-Mg\sin 30° + f_s = Ma_{com,x}$$

Newton's 2nd law in angular form

$$\tau_{net} = I_{com}\alpha \rightsquigarrow f_s R = \frac{2}{5}MR^2\alpha$$

$$a_{com,x} = -\alpha R \rightsquigarrow \frac{5}{2}f_s = -Ma_{com,x}$$

$$\Rightarrow -Mg\sin 30° + f_s = -\frac{5}{2}f_s \rightsquigarrow f_s = \frac{2}{7}Mg\sin 30°$$

$$= \frac{2}{7}(6\,kg)(9.8\,m/s^2)\frac{1}{2} = \underline{\underline{8.4\,N}}$$