İZMİR KATİP ÇELEBİ UNIVERSITY	FACULTY OF ENG. & ARCH. PHY101, FINAL EXAM 6th January 2020, 13:30, DURATION: 120 MIN			
Student Name	**ID Number**	**Instructor Name**	**Department**	**Signature**

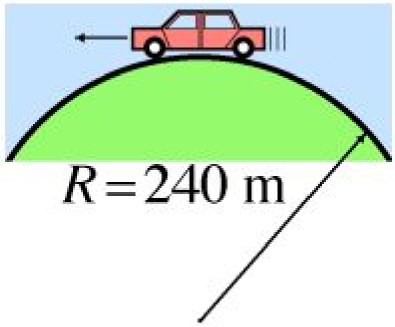
Please read the following directions carefully.

- You must show all your work to get credit; you will not be given any points unless you show the details of your work (this applies even if your final answer is correct).
- Write neatly and clearly; unreadable answers will not be given any credit. If you need more writing space, use the backs of the question pages and put down the appropriate pointer marks.
- Make sure that you include units in your results. Incomplete calculations will not be graded.
- Turn off your mobile phones, and put away. No notebooks or textbooks are allowed to use during the exam.
- You are not allowed to leave the class during the first 15 minutes, and last 15 minutes.
- Calculator is allowed to use. Calculator is assumed to be used only for simple arithmetics, other intentions will be considered as cheating. Everybody must use his/her own calculator. Do not exchange calculators during the exam!
- There are 5 questions with multiple parts. Each question's point value is written next to it.
- Before you begin, please check all pages.
- At the end of the exam, make sure that you turn in your exam paper to your proctor by yourself! Do not give your exam paper to others!

1 (20pts)	2 (20pts)	3 (20pts)	4 (20pts)	5 (20pts)	Total grade

QUESTIONS
(Put your solutions under each question!)

1. A 2000 kg car is driving at a constant speed over a hill, which is a circular dome of radius 240 m.
   a. Above what speed will the car leave the road at the top of the hill?
   b. What is the force applied by the road on the car if its speed is 45 m/s. Take g=9.8 m/s2.



a) N, a_c, mg

$\sum_r F = m a_c$ where $a_c = \frac{v^2}{R}$ } 5pts

$-N + mg = m\frac{v^2}{R}$ } 5pts

About to leave the road $\Rightarrow N = 0$ } 3pts

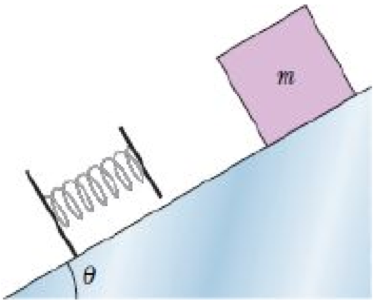
$\Rightarrow mg = m\frac{v^2}{R} \Rightarrow v = \sqrt{gR} = \sqrt{9{,}8 \times 240} = 48.5\,m/s$ } 2pts

b) $N = mg - m\frac{v^2}{R}$ } 3pts

$= 2000\left(9{,}8 - \frac{45^2}{240}\right)$

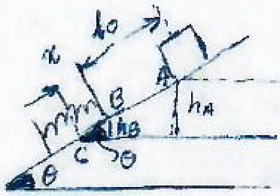
$= \underbrace{2725}_{1pt}\ \underbrace{\text{Newton}}_{1pt}$

2. A block of mass m= 2 kg is released from rest on a frictionless incline of angle Θ=30° as given in the figure. Below the block is a spring that can be compressed 2.0 cm by a force of 270 N. The block **momentarily stops** when it compresses the spring by 5.5 cm. Take g=9.8m/s2.



i. How far does the block move down the incline from **its rest position to this stopping point** ?

ii. What is the **speed** of the block **just as it touches the spring**?



$U = mgh_A$

$U = 0$ at point C

$\sin\theta = \dfrac{h_A}{x + l_0}$ (2)

$\vec{F} = -k\vec{x}$

$270N = k\, 2\times10^{-2} m$

$\Rightarrow k = 13500\, N/m = 1.35\times10^4\, N/m$

$m = 2\, kg$

i) $x = 5.5\times10^{-2} m$ (Point C)

$K_f + U_f = K_i + U_i$ (3)

$0 + (U_g + U_s)_f = 0 + (U_g)_i$

$0 + (0 + \frac{1}{2}kx^2) = 0 + mgh_A$ (2)

$\frac{1}{2}(1.35\times10^4 N/m)(5.5\times10^{-2} m)^2 = (2kg)(9.8 m/s^2)\, h_A$ (2)

$h_A = \dfrac{20.4\, Nm}{19.6\, N} = 1.04\, m = (\sin30)(x + l_0)$

$\rightarrow l_0 = \dfrac{1.04m}{\sin30} - x = 2.08 - 5.5\times10^{-2}$

$= 2.03m$ (2)

$\boxed{l_0 + x = 2.08m}$ (1)

ii) Point B; $\Delta h = h_A - h_B$

$K_f + U_f = K_i + U_i$ (2)

$\frac{1}{2}mv_B^2 + mgh_B = 0 + mgh_A$ (2)

$\frac{1}{2}v_B^2 = g(h_A - h_B)$ | $h_A = (x + l_0)\sin30°$, $h_B = x\sin30$ } $h_A - h_B = l_0 \sin30°$ (2)

$v_B^2 = 2g\, l_0 \sin30 = 2 * 9.8 m/s^2 (2.03m) \sin30$

$\Rightarrow \boxed{v_B = 4.46\, m/s}$ (2)

3. A 4.0 kg object sliding on a frictionless surface explodes into two 2.0 kg parts: 3.0 m/s, due north, and 5.0 m/s, 30° north of east. What is the original speed of the object?

+x direction → East
+y direction → North

$V_1 = 3\,m/s$
$V_2 = 5\,m/s$ and $\theta = 30°$

Linear momenta: $\vec{P}_1 = m\vec{V}_1\hat{j}$
for 2 parts
$\vec{P}_2 = m(V_{2x}\hat{i} + V_{2y}\hat{j})$ 1
$= mV_2(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$ 1

Combined linear momentum of both parts:

$$\vec{P} = \vec{P}_1 + \vec{P}_2 = (mV_2\cos\theta)\hat{i} + (mV_1 + mV_2\sin\theta)\hat{j}$$
$$= (8.66\,\hat{i} + 11\,\hat{j})\ kg \cdot m/s$$

→ 5pt

Conservation of linear momentum says

$$\boxed{\vec{P}_1 + \vec{P}_2 = M V_i} \Rightarrow \boxed{V_i = \frac{\sqrt{(8.66)^2 + (11)^2}}{4.0}} = 3.5\ m/s$$

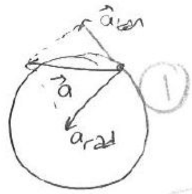
5pt (M = 4kg)

5pt

2pt 1pt

4. Diameter of a disk is 40.0 cm and its mass is 1.5 kg. It starts from rest and rotates with a constant angular acceleration of 3 rad/s2. At the instant the disk has completed its second revolution, compute the radial acceleration, tangential acceleration and resultant acceleration of a point on the disk. What is the linear velocity of the disk at that point? Calculate the rotational kinetic energy of the rotating disk. (the rotation axis is passing through the center of the mass of the disk. Idisk= $\frac{1}{2}\mathrm{m}\,r^2$) (Show the direction of all accelerations on the circular path.)

$2\ \text{rev} = 4\pi\ \text{rad} \qquad r = 0.2\,m$



$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$\omega = \sqrt{2\alpha(\theta-\theta_0)} = \sqrt{2(3\,rad/s^2)(4\pi\,rad)} = 8.68\ rad/s$

$a_{rad} = r\omega^2 = (0.2m)(8.68\,rad/s^2) = 15.1\ m/s^2$

$a_{tan} = r\alpha = (0.2m)(3\,rad/s^2) = 0.6\ m/s^2$

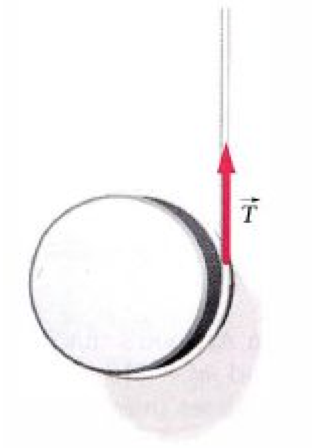
$a = \sqrt{a_{rad}^2 + a_{tan}^2} = \sqrt{(15.1\,m/s)^2 + (0.6\,m/s^2)^2} = 15.11\,m/s^2$

$v = \omega r = (8.68\,rad/s)(0.2m) = 1.74\ m/s$

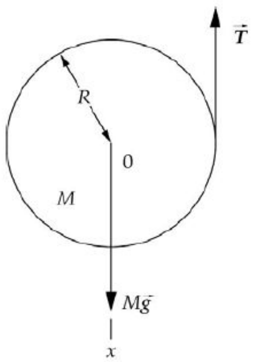
$K_{rotational} = \frac{1}{2} I \omega^2 \qquad I = \frac{1}{2}mr^2 = \frac{1}{2}(1.5\,kg)(0.2m)^2 = 0.03\,kg\,m^2$

$K = \frac{1}{2}(0.03\,kg\,m^2)(8.68\,rad/s)^2 = 1.13\ \text{Joule}.$

5. The string wrapped around the cylinder in Figure is held by a hand that is accelerated upward so that the center of mass of the cylinder does not move. Find (a) the tension in the string, (b) the angular acceleration of the cylinder, and (c) the acceleration of the hand.



$I_{cylinder}$=½ MR^2



a)

$$\sum \tau_0 = TR = I_0 \frac{a}{R}$$ 3pts

and

$$\sum F_x = Mg - T = 0$$ 3pts

$$T = \boxed{Mg}$$ 3pts

b)

$$TR = I_0 \alpha$$

$$\alpha = \frac{TR}{I_0}$$

$$\alpha = \frac{MgR}{\frac{1}{2}MR^2} = \boxed{\frac{2g}{R}}$$

5pts

c)

$$a = R\alpha$$ 3pts

$$a = R\left(\frac{2g}{R}\right) = \boxed{2g}$$ 3pts