



İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy101 Physics I
Final Examination
May 30, 2022 11:00-12:30
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

INSTRUCTOR:

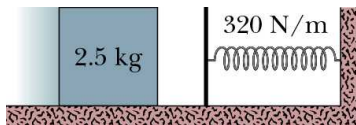
DURATION: 90 minutes

- ◇ Answer all the questions.
- ◇ Write the solutions explicitly and clearly.
Use the physical terminology.
- ◇ You are allowed to use Formulae Sheet.
- ◇ Calculator is allowed.
- ◇ You are not allowed to use any other electronic equipment in the exam.
- ◇ I declare hereby that I fulfilled the requirements for the attendance according to the University regulations and I accept that my examination will not be valid otherwise.

| Question | Grade | Out of |
|--------------|-------|--------|
| 1A | | 15 |
| 1B | | 10 |
| 1C | | 15 |
| 2 | | 15 |
| 3 | | 20 |
| 4 | | 20 |
| 5 | | 20 |
| TOTAL | | 115 |

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1. A) A 2.5 kg block slides into a spring with a spring constant of 320 N/m. When block stops, it has compressed the spring by 7.5 cm. The coefficient friction between the block and the horizontal surface is 0.25.



What is the block's speed just as the block reaches the spring?

Work-Energy Theorem $\Delta K + \Delta U + \Delta E_{th} = W = 0$ (no external force)

$$\Delta K = K_f - K_i \quad \left\{ \begin{array}{l} K_f = 0 \text{ since } v_f = 0 \\ \Delta E_{mech} \end{array} \right. \quad \text{with } m = 2.5 \text{ kg} \quad (3)$$

$$\Delta U = -W_s = \frac{1}{2} kx^2 \quad \left\{ \begin{array}{l} k = 320 \text{ N/m} \\ x = 7.5 \times 10^{-2} \text{ m} \end{array} \right. \quad (3)$$

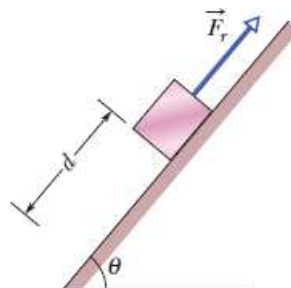
$$\Delta E_{th} = f_k x = (\mu_k mg) x \quad \left\{ \mu_k = 0.25 \right. \quad (3)$$

$$\Rightarrow -\frac{1}{2} m v_i^2 + \frac{1}{2} kx^2 + \mu_k mg x = 0 \quad (2)$$

$$v_i = \frac{2}{m} \sqrt{\frac{1}{2} kx^2 + \mu_k mg x} = \frac{2}{2.5 \text{ kg}} \sqrt{\frac{1}{2} \frac{320 \text{ N}}{\text{m}} (7.5 \times 10^{-2} \text{ m})^2 + (0.25 \cdot 2.5 \text{ kg} \cdot 9.8 \text{ m/s}^2) (7.5 \times 10^{-2} \text{ m})}$$

$$= 0.93 \text{ m/s} \quad (1)$$

B) In Figure, a block of ice slides down a frictionless ramp at angle $\theta = 50^\circ$ while an ice worker pulls on the block (via a rope) with a force \vec{F}_r that has a magnitude of 50 N and is directed up the ramp. As the block slides through distance $d = 0.50 \text{ m}$ along the ramp, its kinetic energy increases by 80 J. How much greater would its kinetic energy have been if the rope had not been attached to the block?



$$\Delta K + \Delta U + W = 0$$

⑤ $\vec{F}_r \cdot \vec{d}$
 $F_r d \cos 180^\circ$

$$|W| = (50 \text{ N})(0.5 \text{ m}) = 25 \text{ J}$$

$(\Delta K + 25 \text{ J}) + \Delta U = 0$
 increase does not change

$\Delta U = mgh$
 $h = d \sin 50$

K_i, U_i
 K_f, U_f

$\boxed{25 \text{ J}}$ ⑤

C) A force $\vec{F} = (cx - 3x^2)\hat{x}$ acts on a particle as the particle moves along an x axis, with \vec{F} in newtons, x in meters, and c a constant. At $x = 0$, the particle's kinetic energy is 20.0 J; at $x = 3.00 \text{ m}$, it is 11.0 J. Find c .

$$W = \int \vec{F} \cdot d\vec{x} = \int_{x_i}^{x_f} F_x dx = \int_{x_i}^{x_f} (cx - 3x^2) dx$$

②

$$= \frac{c}{2}x^2 - x^3 \Big|_0^3 = 4.5c - 27$$

③

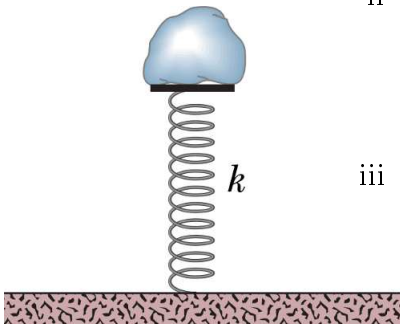
$$W = \Delta K = K_f - K_i = 11 - 20 = -9 \text{ J}$$

⑤

$$\Rightarrow 4.5c - 27 = -9 \rightarrow c = 4 \text{ N/m}$$

① ①

2. Figure below shows an 8.00 kg stone at rest on a spring. The spring is compressed 10.0 cm by the stone.



- i What is the spring constant?
- ii The stone is pushed down an additional 30.0 cm and released. What is the elastic potential energy of the compressed spring just before that release?
- iii What is the change in the gravitational potential energy of the stone-Earth system when the stone moves from the release point to its maximum height?
- iv What is that maximum height, measured from the release point?

y ↑
 x →
 0.1 m

$m = 8 \text{ kg}$
 $y = 0.1 \text{ m}$
 at rest

F_s ↑
 F_g ↓
 FBD

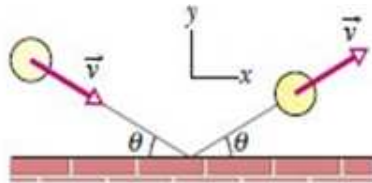
i) $F_s - F_g = ma = 0$
 at rest
 $-ky - mg = 0$
 $-k(-0.1 \text{ m}) = (8 \text{ kg})(9.8 \text{ m/s}^2)$
 $\Rightarrow k = 784 \text{ N/m}$

ii) pushed down 0.3 m further
 $\Delta K + \Delta U = 0 \rightarrow K_f + U_f = K_i + U_i$
 $\rightarrow U_f = \frac{1}{2} k x^2 = \frac{1}{2} (784 \text{ N/m}) (-0.4 \text{ m})^2 = 62.7 \text{ J}$ just before release

iii) at maximum height $K_f + U_f = K_i + U_i$

iv) $U_f = mgh = 62.7 \text{ J}$
 $h = \frac{62.7 \text{ J}}{mg} = \frac{62.7 \text{ J}}{(8 \text{ kg})(9.8 \text{ m/s}^2)} = 0.8 \text{ m}$ change in gravitational potential

3. In Figure, a 300 g ball with a speed v of 6.0 m/s strikes a wall at an angle θ of 30° and then rebounds with the same speed and angle. It is in contact with the wall for 10 ms. In unit vector notation, what are



- i the impulse on the ball from the wall,
ii the average force on the wall from the ball?

$$\vec{v}_i = v \cos \theta \hat{i} - v \sin \theta \hat{j} = 5.2 \hat{i} - 3.0 \hat{j} \quad (3)$$

rebounds with same speed $|\vec{v}_i| = |\vec{v}_f|$
& angle

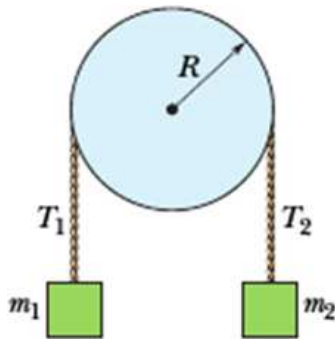
$$\vec{v}_f = v \cos \theta \hat{i} + v \sin \theta \hat{j} = 5.2 \hat{i} + 3.0 \hat{j} \quad (3)$$

i) $\vec{J} = \Delta \vec{p} = m \vec{v}_f - m \vec{v}_i = 2(0.30 \text{ kg})(3.0 \text{ m/s}) \hat{j}$
 $\quad \quad \quad (3) \quad \quad \quad = (1.8 \text{ N s}) \hat{j} \text{ upward}$

ii) $\frac{\vec{J}}{\Delta t} = \vec{F} = \frac{1.8 \hat{j}}{0.010} = (180 \text{ N}) \hat{j}$ average force on the
 $\quad \quad \quad (3) \quad \quad \quad (2) \quad (1)$ ball from the wall

Newton's third law: $(-180 \text{ N}) \hat{j}$ average force on the
 $\quad \quad \quad (2) \quad (1)$ wall from the ball

4. In Figure, block 1 has mass $m_1 = 460 \text{ g}$, block 2 has mass $m_2 = 500 \text{ g}$, and the pulley, which is mounted on a horizontal axle with negligible friction, has radius $R = 5.00 \text{ cm}$. When released from rest, block 2 falls 75.0 cm in 5.00 s without the cord slipping on the pulley.



In unit-vector notation, what are

- What is the magnitude of the acceleration of the blocks?
- What are tension T_2 and tension T_1 ?
- What is the magnitude of the pulley's angular acceleration?
- What is its rotational inertia?

$m_1 = 460 \text{ g}$
 $m_2 = 500 \text{ g}$
 pulley $R = 5.00 \text{ cm}$
 released from rest
 block 2 falls 75.0 cm in 5.00 s

i) $m_2 g - T_2 = m_2 a$
 $m_1 g - T_1 = m_1 a$

Unknowns T_1, T_2, a ?
 need one more equation

$y - y_0 = v_0 t + \frac{1}{2} a t^2$
 $0.75 \text{ m} = 0 \text{ m} + \frac{1}{2} a (5.0 \text{ s})^2$
 $\rightarrow a = 6 \times 10^{-2} \text{ m/s}^2$

of the system

ii) $T_2 = m_2 (g - a)$
 $= 0.5 \text{ kg} (9.8 - 6 \times 10^{-2}) \text{ m/s}^2$
 $= 4.87 \text{ N}$

$T_1 = m_1 (g + a)$
 $= 0.46 \text{ kg} (9.8 + 6 \times 10^{-2}) \text{ m/s}^2$
 $= 4.54 \text{ N}$

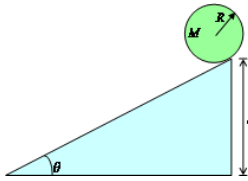
$a_t = \alpha R$
 without the cord slipping $\rightarrow a_t = a$

iii) $\alpha = ?$ $\alpha = \frac{a_t}{R} = \frac{a}{R}$
 $\alpha = \frac{6 \times 10^{-2} \text{ m/s}^2}{5 \times 10^{-2} \text{ m}} = 1.20 \text{ rad/s}^2$

iv) $I = ?$ $\tau = I \alpha \rightarrow (T_1 - T_2) R = I \alpha$

$I = \frac{(4.87 - 4.54) \text{ N} \cdot 0.05 \text{ m}}{1.20 \text{ rad/s}^2} = 1.38 \times 10^{-2} \text{ kg m}^2$

5. A solid ball of radius $R = 0.2 \text{ m}$ and mass $M = 3 \text{ kg}$ is placed at the top of a ramp of height $h = 1.2 \text{ m}$ and $\theta = 37^\circ$. (Hint: $I = \frac{2}{5}mR^2$)



i If the ramp surface is frictionless, calculate the velocity of the ball's center of mass (v_{com}) and its angular velocity (ω) at the bottom of the ramp.

ii Calculate the minimum value of the coefficient of static friction (μ_s) that would cause smooth rolling (no slipping) of the ball down the ramp. Calculate v_{com} and ω at the bottom of the ramp for this case.

iii If the coefficient of kinetic friction (μ_k) between the ball and the ramp surface is 0.1 and it is known that the ball does not roll smoothly down the ramp (there is sliding), calculate v_{com} and ω at the bottom of the ramp.

i) frictionless \rightarrow no rolling $\rightarrow v_{com} = v$ & $\omega = 0$

$$W_f + K_f = U_i + K_i \rightarrow \frac{1}{2}mv^2 = mgh \rightarrow v = \sqrt{2gh} \quad (1)$$

at the bottom $\rightarrow v = \sqrt{2 \cdot (9.8 \text{ m/s}^2) \cdot (1.2 \text{ m})} = \boxed{4.85 \text{ m/s}} \quad (1)$

ii) smooth rolling

$$a_{com} = \frac{g \sin \theta}{1 + I_{com}/MR^2} \quad \& \quad f_s = -I_{com} \frac{a_{com}}{R^2}$$

$$a_{com} = \frac{(9.8 \text{ m/s}^2) \sin 37^\circ}{1 + \frac{2}{5} \frac{MR^2}{MR^2}} = -4.21 \text{ m/s}^2 \quad (2)$$

at the bottom $\rightarrow f_s = -\frac{2}{5} MR^2 (-4.21 \text{ m/s}^2) = -5.05 \text{ N} \quad (1)$

$$\Rightarrow f_s = \mu_s F_N \rightarrow \mu_s = \frac{f_s}{F_N} = \frac{5.05 \text{ N}}{(3 \text{ kg}) \cdot (9.8 \text{ m/s}^2) \cos 37^\circ} = \boxed{0.22} \quad (1)$$

$$W_f + K_f = U_i + K_i \rightarrow \frac{1}{2}mv_{com}^2 + \frac{1}{2}I\omega^2 = mgh \quad | \quad v_{com} = \omega R$$

$$\frac{1}{2}mv_{com}^2 + \frac{1}{2} \cdot \frac{2}{5} MR^2 \left(\frac{v_{com}}{R}\right)^2 = mgh \rightarrow v_{com} = \sqrt{\frac{10}{7}gh} = \boxed{4.1 \text{ m/s}} \quad (1)$$

$$\omega = \frac{v_{com}}{R} = \frac{4.1 \text{ m/s}}{0.2 \text{ m}} = \boxed{20.5 \text{ rad/s}} \quad (1)$$

iii) friction & motion & sliding

$$f_k - mg \sin \theta = ma_{com} \quad | \quad f_k R = \tau = I\alpha \rightarrow \alpha = \frac{f_k R}{I}$$

$$\mu_k mg \cos \theta - mg \sin \theta = ma_{com} \quad | \quad \alpha = \frac{(0.1) \cdot (3 \text{ kg}) \cdot (9.8 \text{ m/s}^2) \cos 37^\circ R}{\frac{2}{5} MR^2} = \frac{9.78 \text{ rad}}{\text{s}^2} \quad (1)$$

$$a_{com} = 9.8 \text{ m/s}^2 (0.1 \cos 37^\circ - \sin 37^\circ) = -5.12 \text{ m/s}^2 \quad (1)$$

$$L = \frac{1}{2} a_{com} t^2 \quad | \quad v_{com} = a_{com} t$$

$$\frac{-1.2 \text{ m}}{\sin 37^\circ} = \frac{1}{2} (-5.12) t^2 \quad | \quad \omega = \alpha t = (9.78 \text{ rad/s}^2) (0.88 \text{ s}) = \boxed{8.6 \text{ rad/s}} \quad (1)$$

$$t = 0.88 \text{ s} \quad | \quad \Rightarrow v_{com} = 4.52 \text{ m/s} \quad (1)$$