İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy101 Physics I
Midterm Examination
April 04, 2022 08:30 – 10:00
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

INSTRUCTOR:

DURATION: 90 minutes

◇ **Answer all the questions.**
◇ **Write the solutions explicitly and clearly. Use the physical terminology.**
◇ **You are allowed to use Formulae Sheet.**
◇ **Calculator is allowed.**
◇ **You are not allowed to use any other electronic equipment in the exam.**

Question	Grade	Out of
1A		15
1B		10
1C		10
2		20
3		20
4		20
5		20
TOTAL		115

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1. A) The side of a cube of metal is measured to be (1.60±0.05) cm and its mass is measured to be (30.1±0.4) g

   i Find the perimeter of one face of the cube with the uncertainty.

   ii Find the volume and uncertainty in the volume.

   iii Determine the density of the solid in kilograms per cubic meter and the uncertainty in the density.

   You should be using the correct number of significant figures in your result.

$a = (1.60 \pm 0.05)\,cm = (1.60 \pm 0.05) \times 10^{-2}\,m$

$m = (30.1 \pm 0.4)\,g = (30.1 \pm 0.4) \times 10^{-3}\,kg$

$\rightsquigarrow$ 3 sig figs //

i) perimeter: $4a \overset{②}{=} 4(1.60 \pm 0.05) \times 10^{-2}\,m = (\overset{①}{6.40} \pm \overset{①}{0.20}) \times 10^{-2}\,m$

ii) volume: $V \overset{①}{=} a^3 \rightsquigarrow C = A^n,\ \Delta C = C|n|\frac{\Delta A}{A}$ ②

$\Rightarrow V = a^3 = (1.60 \times 10^{-2}\,m)^3 = 4.10 \times 10^{-6}\,m^3$.

$\Delta V = a^3 |3| \frac{\Delta a}{a} = 4.10 \times 10^{-6} |3| \frac{0.05}{1.60} = 0.38 \times 10^{-6}\,m^3$

$\Rightarrow$ Volume: $(\overset{①}{4.10} \pm \overset{①}{0.38}) \times 10^{-6}\,m^3$

iii) density: $\rho \overset{①}{=} m/V \rightsquigarrow C = \frac{A}{B},\ \Delta C = |C|\sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$

$\rho = \frac{m}{V} = \frac{30.1 \times 10^{-3}\,kg}{4.10 \times 10^{-6}\,m^3} = 7341\,kg/m^3 \sim \overset{①}{7.34} \times 10^{3}\,kg/m^3$

$\Delta\rho = \left|\frac{30.1 \times 10^{-3}\,kg}{4.10 \times 10^{-6}\,m^3}\right| \sqrt{\left(\frac{0.4}{30.1}\right)^2 + \left(\frac{0.38}{4.10}\right)^2} = 687\,kg/m^3$

$\Rightarrow$ density: $\rho = (\overset{①}{7.34} \pm \overset{①}{0.69}) \times 10^{3}\,kg/m^3$ ② Results in 3 sig figs

B) The position of a particle moving along an x-axis is given by $x(t) = 12t^2 - 2t^3$, where x is in meters and t is in seconds.

i Determine the acceleration of the particle at $t = 3.0\ s$.

ii What are the maximum positive coordinate reached by the particle and the acceleration of the particle at that instant?

i) $x(t) = 12t^2 - 2t^3$

$v(t) = \frac{dx}{dt} = 24t - 6t^2$ (1)

$a(t) = \frac{dv}{dt} = 24 - 12t$ (1) $\rightarrow a(t=3s) = 24 - 12\times3 = -12\ m/s^2$ (1) (1) (1)

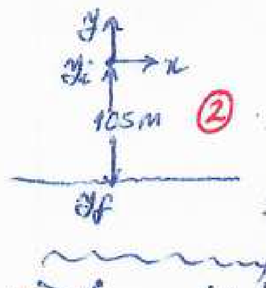
ii) maximum positive coordinate $\Rightarrow v(t) = \frac{dx}{dt} = 0$ (2)

$24t - 6t^2 = 0 = 6t(4-t) = 0 \Rightarrow t = 4s$ (1)

$\rightarrow x(t=4s) = 12(4)^2 - 2(4)^3 = 192 - 128 = 64m$ (1)

$a(t=4s) = 24 - 12\times4 = -24\ m/s^2$ (1)

C) A helicopter is ascending (move upward) vertically with a speed of 5.40 m/s. At a height of 105 m above the Earth, a package is dropped from the helicopter. How much time does it take for the package to reach the ground? [Hint: What is v_0 for the package?]



(2)

with our chosen coordinate system

$v_0 = 5.40\ m/s$ as upward & $y_i = 0$, $y_f = -105m$

$y - y_0 = v_0 t - \frac{1}{2} g t^2$ (2)

$-105 - 0 = 5.4t - \frac{1}{2}\,9.8\,t^2 \Rightarrow 4.9t^2 - 5.4t - 105 = 0$ (2)

Quadratic equation $at^2 + bt + c = 0$

$\Rightarrow t_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5.4) \pm \sqrt{(-5.4)^2 - 4\times4.9\times(-105)}}{2\times4.9}$ (2)

$= 5.21\,s$ ✓ (2)

$-4.11\,s$ ✗

2. Vectors $\overrightarrow{A}$ and $\overrightarrow{B}$ lie in an xy-plane. $\overrightarrow{A}$ has magnitude 8.0 and an angle 130°; B has components $B_x = -7.72$ and $B_y = -9.20$.

   i What are $5\overrightarrow{A} \cdot \overrightarrow{B}$ and $4\overrightarrow{A} \times 3\overrightarrow{B}$ in unit vector notation?

   ii What is $(3\hat{i} + 5\hat{j}) \times (4\overrightarrow{A} \times 3\overrightarrow{B})$? Find magnitude and angle of resultant vector.

$\vec{A}$ & $\vec{B}$ lie in xy-plane ⇒ only x & y components

$|\vec{A}| = 8$ with angle $130°$ & $B_x = -7.72$, $B_y = -9.20$

$$\Rightarrow \vec{A} = |\vec{A}|\cos 130\,\hat{i} + |\vec{A}|\sin 130\,\hat{j} = \underline{-5.14\,\hat{i} + 6.13\,\hat{j}}$$

$$\vec{B} = -7.72\,\hat{i} + 9.20(-\hat{j})$$

i) $$5\vec{A}\cdot\vec{B} = 5(-5.14\,\hat{i} + 6.13\,\hat{j})\cdot(-7.72\,\hat{i} - 9.20\,\hat{j})$$

$$= 5\left[(-5.14)\times(-7.72) + (6.13\times(-9.20))\right] = \underline{-83.58}$$

$$4\vec{A}\times 3\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -20.56 & 24.52 & 0 \\ -23.16 & -27.6 & 0 \end{vmatrix} = \begin{matrix} (24.52\times 0 - 0(-27.6))\,\hat{i} \\ +\,(0(-23.16) - (-20.56)0)\,\hat{j} \\ +\,((-20.56)(-27.6) - (24.52)(-23.16))\,\hat{k} \end{matrix}$$

$$= \underline{1135.4\,\hat{k}}$$

ii) $$(3\hat{i} + 5\hat{j})\times(1135.4\,\hat{k}) = (3\times 1135.4)(\hat{i}\times\hat{k}) + (5\times 1135.4)(\hat{j}\times\hat{k})$$

$\hat{i}\times\hat{k} = -\hat{j}$, $\hat{j}\times\hat{k} = \hat{i}$ ($\hat{i} \to \hat{j} \to \hat{k}$)

$$= \underline{5677\,\hat{i} + 3460.2(-\hat{j})}$$

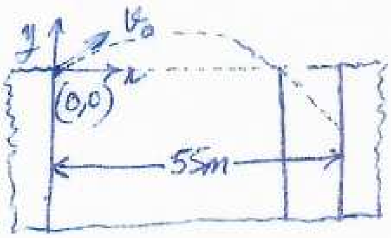
magnitude: $\sqrt{(5677)^2 + (-3460.2)^2} = \underline{6620.5}$

angle: $\theta = \tan^{-1}\dfrac{-3460.2}{5677} = \underline{-31° \text{ or } 329°}$

3. A ball is shot from the top of a building with an initial velocity of 18 m/s at an angle $\theta = 42°$ above the horizontal.

   i What are the horizontal and vertical components of the initial velocity?

   ii If a nearby building is the same height and 55 m away, how far below the top of the building will the ball strike the nearby building?



$v_0 = 18$ m/s

$\theta = 42°$

i) $v_x = v_0 \cos\theta = 18 \text{ m/s} \cos 42 = 13.38$ m/s

$v_y = v_0 \sin\theta = 18 \text{ m/s} \sin 42 = 12.04$ m/s

ii) $v_x = v_{0x}$ & $\Delta x = v_{0x} t \rightsquigarrow$ time of flight

$t = \frac{\Delta x}{v_{0x}} = \frac{55 m}{13.38 m/s} = 4.11 s$

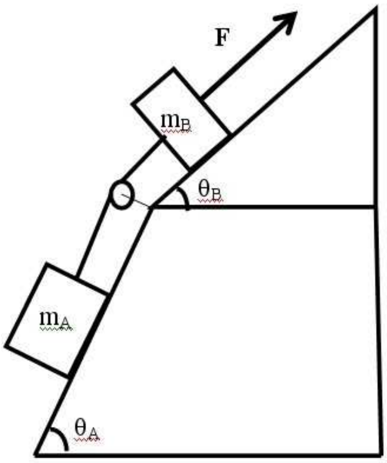
same time interval for y direction & $y_0 = 0$

$\Delta y = y - y_0 = v_{0y} t - \frac{1}{2} g t^2$

$= (12.04)(4.11) - \frac{1}{2} 9.8 (4.11)^2$

$= -33.3$ m { minus sign shows below the top of the building

4. Consider the system shown in figure with $m_A = 9.5\ kg$ and $m_B = 11.5\ kg$. The angles $\theta_A = 59°$ and $\theta_B = 32°$.



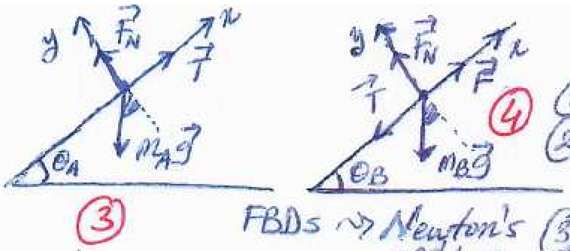
i Draw the free body diagrams for block A and block B.

ii In the absence of friction, what force F would be required to pull the masses at a **constant velocity** up?

iii The force F now is removed . What is the magnitude and direction of acceleration of the two blocks?

iv In the absence of F, what is the tension in the string?

i)



mass A

(1) x: $T - m_A g \sin\theta_A = m_A a_{Ax}$

(2) y: $F_N - m_A g \cos\theta_A = m_A a_{Ay}$

mass B

(3) x: $F - T - m_B g \sin\theta_B = m_B a_{Bx}$

(4) y: $F_N - m_B g \cos\theta_B = m_B a_{By}$

FBDs ⇝ Newton's 2nd law ⇝ equations of motion

$a_{Ay} = 0 = a_{By}$ & $a_{Ax} = a_{Bx} = a$ — no motion in y-direction

ii) constant velocity ⇒ $a = 0$ (1)&(3)

(1) $T - m_A g \sin\theta_A = 0$

(1) $F - T - m_B g \sin\theta_B = 0$

$F = T + m_B g \sin\theta_B$ (1)

$= m_A g \sin\theta_A + m_B g \sin\theta_B$

$= (9.5 \sin 59 + 11.5 \sin 32)\,9.8 = 79.8N + 59.7N \cong 140N$ (1)(1)

iii) now $a \neq 0$ (1)&(3) (1)

(1) $T - m_A g \sin\theta_A = m_A a$

(1) $F - T - m_B g \sin\theta_B = m_B a$

$F = 0 \Rightarrow m_A(g \sin\theta_A + a) - m_B g \sin\theta_B = m_B a$

$\Rightarrow a = \dfrac{-m_A g \sin\theta_A - m_B g \sin\theta_B}{m_A + m_B} = 6.7\, m/s^2$ & $(-\hat{i})$ (1) (1)

iv) $T - m_A g \sin\theta_A = m_A a$ (3) ~ $T = m_A(a + g \sin\theta_A)$ (1)

$= 9.5(-6.7 + 9.8 \sin 59°)$

$\cong 16.2N$

(1) (1)

5. In Figure, blocks A and B have weights of 44 N and 22 N, respectively.

i Determine the minimum weight of block C to keep A from sliding if μ_s between A and the table is 0.20.

ii Block C suddenly is lifted off A. What is the acceleration of block A if μ_k between A and the table is 0.15?