



İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy101 Physics I
Final Examination
May 30, 2022 11:00-12:30
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

INSTRUCTOR:

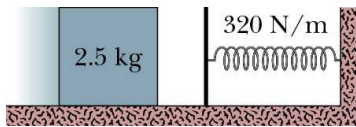
DURATION: 90 minutes

- ◇ Answer all the questions.
- ◇ Write the solutions explicitly and clearly.
Use the physical terminology.
- ◇ You are allowed to use Formulae Sheet.
- ◇ Calculator is allowed.
- ◇ You are not allowed to use any other electronic equipment in the exam.
- ◇ I declare hereby that I fulfilled the requirements for the attendance according to the University regulations and I accept that my examination will not be valid otherwise.

Question	Grade	Out of
1A		15
1B		10
1C		15
2		15
3		20
4		20
5		20
TOTAL		115

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1. A) A 2.5 kg block slides into a spring with a spring constant of 320 N/m. When block stops, it has compressed the spring by 7.5 cm. The coefficient friction between the block and the horizontal surface is 0.25.



What is the block's speed just as the block reaches the spring?

Work-Energy Theorem $\Delta K + \Delta U + \Delta E_{th} = W = 0$ (no external force)

$$\Delta K = K_f - K_i \quad \left\{ \begin{array}{l} K_f = 0 \text{ since } v_f = 0 \\ m = 2.5 \text{ kg} \end{array} \right. \quad \text{③}$$

$$\Delta U = -W_s = \frac{1}{2} kx^2 \quad \left\{ \begin{array}{l} k = 320 \text{ N/m} \\ x = 7.5 \times 10^{-2} \text{ m} \end{array} \right. \quad \text{③}$$

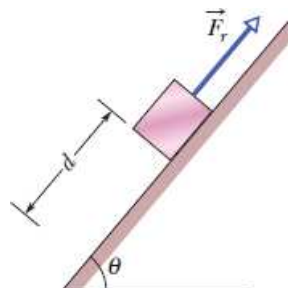
$$\Delta E_{th} = f_k x = (\mu_k mg) x \quad \left\{ \mu_k = 0.25 \right. \quad \text{③}$$

$$\Rightarrow -\frac{1}{2} m v_i^2 + \frac{1}{2} kx^2 + \mu_k mg x = 0 \quad \text{②}$$

$$v_i = \frac{2}{m} \sqrt{\frac{1}{2} kx^2 + \mu_k mg x} = \frac{2}{2.5 \text{ kg}} \sqrt{\frac{1}{2} \frac{320 \text{ N}}{\text{m}} (7.5 \times 10^{-2} \text{ m})^2 + (0.25 \cdot 2.5 \text{ kg} \cdot 9.8 \text{ m/s}^2) (7.5 \times 10^{-2} \text{ m})}$$

$$= 0.93 \text{ m/s} \quad \text{①}$$

B) In Figure, a block of ice slides down a frictionless ramp at angle $\theta = 50^\circ$ while an ice worker pulls on the block (via a rope) with a force \vec{F}_r that has a magnitude of 50 N and is directed up the ramp. As the block slides through distance $d = 0.50 \text{ m}$ along the ramp, its kinetic energy increases by 80 J. How much greater would its kinetic energy have been if the rope had not been attached to the block?



$$\Delta K + \Delta U + W = 0$$

⑤ $\vec{F}_r \cdot \vec{d}$
 $F_r d \cos 180^\circ$

$$|W| = (50 \text{ N})(0.5 \text{ m}) = 25 \text{ J}$$

$(\Delta K + 25 \text{ J}) + \Delta U = 0$
 increase ΔU does not change

$\Delta U = mgh$
 $h = d \sin 50$

K_i, U_i
 K_f, U_f

$\boxed{25 \text{ J}}$ ⑤

C) A force $\vec{F} = (cx - 3x^2)\hat{x}$ acts on a particle as the particle moves along an x axis, with \vec{F} in newtons, x in meters, and c a constant. At $x = 0$, the particle's kinetic energy is 20.0 J; at $x = 3.00 \text{ m}$, it is 11.0 J. Find c .

$$W = \int \vec{F} \cdot d\vec{x} = \int_{x_i}^{x_f} F_x dx = \int_{x_i}^{x_f} (cx - 3x^2) dx$$

②

$$= \frac{c}{2}x^2 - x^3 \Big|_0^3 = 4.5c - 27$$

③

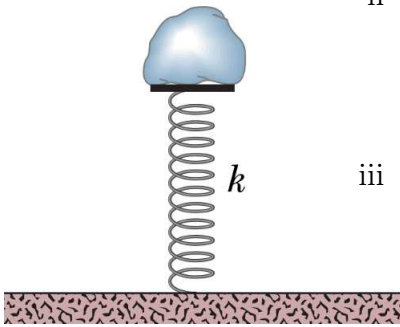
$$W = \Delta K = K_f - K_i = 11 - 20 = -9 \text{ J}$$

⑤

$$\Rightarrow 4.5c - 27 = -9 \rightarrow c = 4 \text{ N/m}$$

① ①

2. Figure below shows an 8.00 kg stone at rest on a spring. The spring is compressed 10.0 cm by the stone.



- i What is the spring constant?
- ii The stone is pushed down an additional 30.0 cm and released. What is the elastic potential energy of the compressed spring just before that release?
- iii What is the change in the gravitational potential energy of the stone-Earth system when the stone moves from the release point to its maximum height?
- iv What is that maximum height, measured from the release point?

y ↑
 x →
 0.1 m

$m = 8 \text{ kg}$
 $y = 0.1 \text{ m}$
 at rest

F_s ↑
 F_g ↓
 FBD

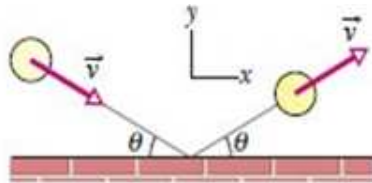
i) $F_s - F_g = ma = 0$
 at rest
 $-ky - mg = 0$
 $-k(-0.1 \text{ m}) = (8 \text{ kg})(9.8 \text{ m/s}^2)$
 $\Rightarrow k = 784 \text{ N/m}$

ii) pushed down 0.3 m further
 $\Delta K + \Delta U = 0 \rightarrow K_f + U_f = K_i + U_i$
 $\rightarrow U_f = \frac{1}{2} k x^2 = \frac{1}{2} (784 \text{ N/m}) (-0.4 \text{ m})^2 = 62.7 \text{ J}$ just before release

iii) at maximum height $K_f + U_f = K_i + U_i$

iv) $U_f = mgh = 62.7 \text{ J}$ change in gravitational potential
 $h = \frac{62.7 \text{ J}}{mg} = \frac{62.7 \text{ J}}{(8 \text{ kg})(9.8 \text{ m/s}^2)} = 0.8 \text{ m}$

3. In Figure, a 300 g ball with a speed v of 6.0 m/s strikes a wall at an angle θ of 30° and then rebounds with the same speed and angle. It is in contact with the wall for 10 ms. In unit vector notation, what are



- i the impulse on the ball from the wall,
ii the average force on the wall from the ball?

$$\vec{v}_i = v \cos \theta \hat{i} - v \sin \theta \hat{j} = 5.2 \hat{i} - 3.0 \hat{j} \quad (3)$$

rebounds with same speed $|\vec{v}_i| = |\vec{v}_f|$
& angle

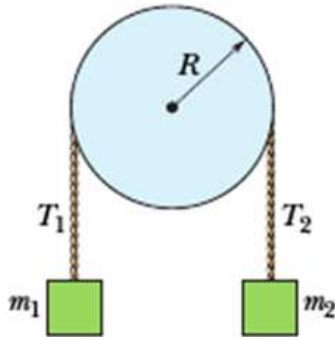
$$\vec{v}_f = v \cos \theta \hat{i} + v \sin \theta \hat{j} = 5.2 \hat{i} + 3.0 \hat{j} \quad (3)$$

i) $\vec{J} = \Delta \vec{p} = m \vec{v}_f - m \vec{v}_i = 2(0.30 \text{ kg})(3.0 \text{ m/s}) \hat{j}$
 $\quad \quad \quad (3) \quad \quad \quad = (1.8 \text{ N s}) \hat{j} \text{ upward}$

ii) $\frac{\vec{J}}{\Delta t} = \vec{F} = \frac{1.8 \hat{j}}{0.010} = (180 \text{ N}) \hat{j}$ average force on the
 $\quad \quad \quad (3) \quad \quad \quad (2) \quad (1)$ ball from the wall

Newton's third law: $(-180 \text{ N}) \hat{j}$ average force on the
 $\quad \quad \quad (2) \quad (1)$ wall from the ball

4. In Figure, block 1 has mass $m_1 = 460 \text{ g}$, block 2 has mass $m_2 = 500 \text{ g}$, and the pulley, which is mounted on a horizontal axle with negligible friction, has radius $R = 5.00 \text{ cm}$. When released from rest, block 2 falls 75.0 cm in 5.00 s without the cord slipping on the pulley.



In unit-vector notation, what are

- What is the magnitude of the acceleration of the blocks?
- What are tension T_2 and tension T_1 ?
- What is the magnitude of the pulley's angular acceleration?
- What is its rotational inertia?

$m_1 = 460 \text{ g}$
 $m_2 = 500 \text{ g}$
 pulley $R = 5.00 \text{ cm}$
 released from rest
 block 2 falls 75.0 cm in 5.00 s

i) $m_2 g - T_2 = m_2 a$
 $m_1 g - T_1 = m_1 a$

Unknowns T_1, T_2, a ?
 need one more equation

$y - y_0 = v_0 t + \frac{1}{2} a t^2$
 $0.75 \text{ m} = 0 \text{ m} + \frac{1}{2} a (5.0 \text{ s})^2$
 $\rightarrow a = 6 \times 10^{-2} \text{ m/s}^2$

of the system

ii) $T_2 = m_2 (g - a)$
 $= 0.5 \text{ kg} (9.8 - 6 \times 10^{-2}) \text{ m/s}^2$
 $= 4.87 \text{ N}$

$T_1 = m_1 (g + a)$
 $= 0.46 \text{ kg} (9.8 + 6 \times 10^{-2}) \text{ m/s}^2$
 $= 4.54 \text{ N}$

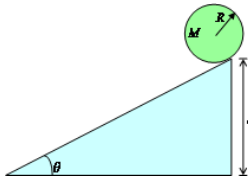
$a_t = \alpha R$
 without the cord slipping $\rightarrow a_t = a$

iii) $\alpha = ?$ $\alpha = \frac{a_t}{R} = \frac{a}{R}$
 $\alpha = \frac{6 \times 10^{-2} \text{ m/s}^2}{5 \times 10^{-2} \text{ m}} = 1.20 \text{ rad/s}^2$

iv) $I = ?$ $\tau = I \alpha \rightarrow (T_1 - T_2) R = I \alpha$

$I = \frac{(4.87 - 4.54) \text{ N} \cdot 0.05 \text{ m}}{1.20 \text{ rad/s}^2} = 1.38 \times 10^{-2} \text{ kg m}^2$

5. A solid ball of radius $R = 0.2 \text{ m}$ and mass $M = 3 \text{ kg}$ is placed at the top of a ramp of height $h = 1.2 \text{ m}$ and $\theta = 37^\circ$. (Hint: $I = \frac{2}{5}mR^2$)



i If the ramp surface is frictionless, calculate the velocity of the ball's center of mass (v_{com}) and its angular velocity (ω) at the bottom of the ramp.

ii Calculate the minimum value of the coefficient of static friction (μ_s) that would cause smooth rolling (no slipping) of the ball down the ramp. Calculate v_{com} and ω at the bottom of the ramp for this case.

iii If the coefficient of kinetic friction (μ_k) between the ball and the ramp surface is 0.1 and it is known that the ball does not roll smoothly down the ramp (there is sliding), calculate v_{com} and ω at the bottom of the ramp.

i) frictionless \rightarrow no rolling $\rightarrow v_{com} = v$ & $\omega = 0$

$$W_f + K_f = U_i + K_i \rightarrow \frac{1}{2}mv^2 = mgh \rightarrow v = \sqrt{2gh} \quad (1)$$

at the bottom $\rightarrow v = \sqrt{2 \cdot (9.8 \text{ m/s}^2) \cdot (1.2 \text{ m})} = \boxed{4.85 \text{ m/s}} \quad (1)$

ii) smooth rolling

$$a_{com} = \frac{g \sin \theta}{1 + I_{com}/MR^2} \quad \& \quad f_s = -I_{com} \frac{a_{com}}{R^2}$$

$$\rightarrow a_{com} = \frac{(9.8 \text{ m/s}^2) \sin 37^\circ}{1 + \frac{2}{5} \frac{MR^2}{MR^2}} = -4.21 \text{ m/s}^2 \quad (2)$$

at the bottom $\rightarrow f_s = -\frac{2}{5} MR^2 (-4.21 \text{ m/s}^2) = -5.05 \text{ N} \quad (1)$

$$\Rightarrow f_s = \mu_s F_N \rightarrow \mu_s = \frac{f_s}{F_N} = \frac{5.05 \text{ N}}{mg \cos 37^\circ} = \frac{5.05 \text{ N}}{(3 \text{ kg}) \cdot (9.8 \text{ m/s}^2) \cos 37^\circ} = \boxed{0.22} \quad (1)$$

$$W_f + K_f = U_i + K_i \rightarrow \frac{1}{2}mv_{com}^2 + \frac{1}{2}I\omega^2 = mgh \quad | \quad v_{com} = \omega R$$

$$\frac{1}{2}mv_{com}^2 + \frac{1}{2} \cdot \frac{2}{5} MR^2 \left(\frac{v_{com}}{R}\right)^2 = mgh \rightarrow v_{com} = \sqrt{\frac{10}{7}gh} = \boxed{4.1 \text{ m/s}} \quad (1)$$

$$\omega = \frac{v_{com}}{R} = \frac{4.1 \text{ m/s}}{0.2 \text{ m}} = \boxed{20.5 \text{ rad/s}} \quad (1)$$

iii) friction & motion & sliding

$$f_k - mg \sin \theta = ma_{com} \quad | \quad f_k R = \tau = I\alpha \rightarrow \alpha = \frac{f_k R}{I}$$

$$\mu_k mg \cos \theta - mg \sin \theta = ma_{com} \quad | \quad \alpha = \frac{(0.1) mg \cos 37^\circ R}{\frac{2}{5} MR^2} = \frac{9.78 \text{ rad}}{\text{s}^2} \quad (1)$$

$$a_{com} = 9.8 \text{ m/s}^2 (0.1 \cos 37^\circ - \sin 37^\circ) = -5.12 \text{ m/s}^2 \quad (1)$$

$$L = \frac{1}{2} a_{com} t^2 \quad | \quad v_{com} = a_{com} t$$

$$\frac{-1.2 \text{ m}}{\sin 37^\circ} = \frac{1}{2} (-5.12) t^2 \quad | \quad \omega = \alpha t = (9.78 \text{ rad/s}^2) (0.88 \text{ s}) = \boxed{8.6 \text{ rad/s}} \quad (1)$$

$$t = 0.88 \text{ s} \quad | \quad \Rightarrow v_{com} = 4.52 \text{ m/s} \quad (1)$$



Izmir Kâtip Çelebi University
Department of Engineering Sciences
Phy101 Physics I
Final Examination Take Home Assignment
Due date: June 05, 2020 10:30
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

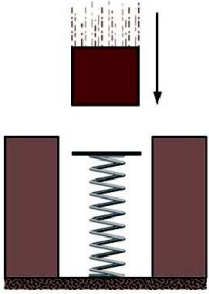
DEPARTMENT:

- ◇ Answer all the questions.
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Question	Grade	Out of
TOTAL		100

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1. (Kinetic Energy and Work) A 250 g block is dropped onto a relaxed vertical spring that has a spring constant of $k = 2.5 \text{ N/cm}$. The block becomes attached to the spring and compresses the spring 12 cm before momentarily stopping.



What is the speed of the block just before it hits the spring (assume that friction is negligible)?

Answer: $v = 3.5 \text{ m/s}$

initial final ④ $k = 2.5 \frac{\text{N}}{\text{cm}} = 250 \text{ N/m}$

$x = 0.12 \text{ m}$

Point A
Point B (reference point)

Conservation of mechanical energy:

④ $U_A + K_A = U_B + K_B$ 0 ($h_B = 0$) 0 ($U_B = 0$)

④ $mgh_A + \frac{1}{2} m v_A^2 = mgh_B + \frac{1}{2} kx^2 + \frac{1}{2} m v_B^2$

⑥ $(0.25)(9.8)(0.12) + \frac{1}{2} (0.25) v_A^2 = \frac{1}{2} (250) (0.12)^2 \Rightarrow v_A = 3.5 \text{ m/s}$ ②

$\approx 0 \text{ R} =$

Kinetic energy work theorem

$\Delta K = W_{\text{gravitational force}} + W_{\text{spring force}}$ ⑤

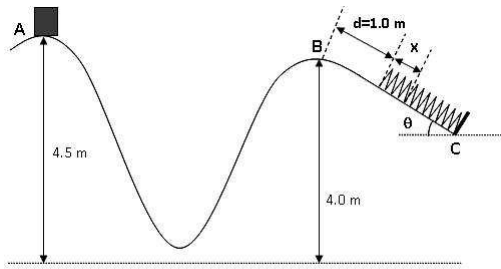
$W_g = -\Delta U = -(U_A - U_B) = U_B = mgh_B = (0.25)(9.8)(0.12) = 0.29 \text{ J}$ ⑤

$W_s = -\frac{1}{2} kx^2 = -\frac{1}{2} (250) (0.12)^2 = -1.8 \text{ J}$ ⑤

$\Delta K = K_B - K_A = \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 = W_g + W_s$

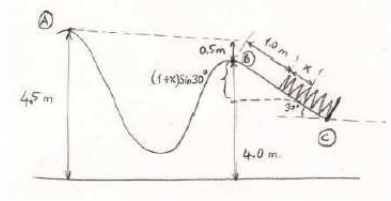
③ $-\frac{1}{2} (0.25) v_A^2 = (0.29 - 1.8) \Rightarrow v_A = 3.5 \text{ m/s}$ ②

2. (Kinetic Energy and Work) An object having a mass of 1 kg, which is initially at rest at point **A**, starts its motion along frictionless path of **A-C** and finally, stops momentarily after it compresses the spring ($k = 100 \text{ N/m}$) by x m. Take $g = 10 \text{ m/s}^2$.



- i Find the compression distance, x , if angle θ of the inclined plane is 30° and the distance travelled on inclined plane, d , just before hitting the spring is 1.0 m.
- ii The object starts moving in opposite direction after it stops momentarily. What is the speed of the object just after it starts its motion in opposite direction?
- iii Determine the final position of the object at the end of motion in opposite direction along path $C - A$.

Answer: i) $x = 0.5 \text{ m}$ ii) $v = 5 \text{ m/s}$ iii) A

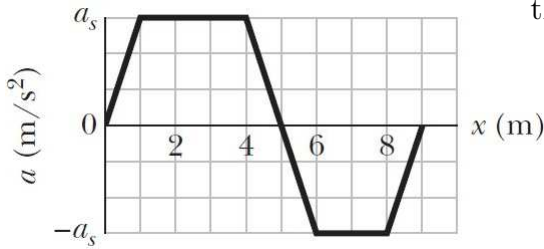


$$\begin{aligned}
 \text{a) } mg(0.5 + (1+x)\sin 30) &= \frac{1}{2} kx^2 \\
 0.5mg + 0.5mg(1+x) &= \frac{1}{2}(100)x^2 \Rightarrow 100x^2 = 2mg + mgx \\
 100x^2 &= 20 + 10x \Rightarrow x^2 - 0.1x - 0.2 = 0 \\
 (x+0.4)(x-0.5) &= 0 \\
 x &= 0.5 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } \frac{1}{2} kx^2 &= \frac{1}{2} m v_0^2 \Rightarrow 100 \left(\frac{1}{2}\right)^2 = (1) v_0^2 \\
 v_0 &= 5 \text{ m/s}
 \end{aligned}$$

c) Since there is no friction, no energy is lost so that all the elastic potential energy stored in the spring will transform into potential and kinetic energies and finally, the object will be at its initial point of A .

3. (Kinetic Energy and Work) Figure gives the acceleration of a 2.00 kg particle as an applied force F_a moves it from rest along an x -axis from $x = 0$ to $x = 9.0$ m. The scale of the figure's vertical axis is set by $a_s = 6.0$ m/s².



How much work has the force done on the particle when the particle reaches

- i $x = 4.0$ m,
- ii $x = 7.0$ m,
- iii What is the particle's speed and direction of travel when it reaches $x = 4.0$ m.

Answer: i) 42 J ii) 30 J iii) 6.5 m/s + x direction

ii What is the average and instantaneous angular acceleration for that time interval? (1.5)

i) a) $W = Fx = (mas)x$, area under the curve

$0 \leq x \leq 4$ (1.5) $6 + 36$
 $2 \times 6 \times 2 \text{ kg} \times 6 \text{ m/s}^2 (4 \text{ m}) + 2 \text{ kg} \times 6 \text{ m/s}^2 (4+16)$

i) 42 J

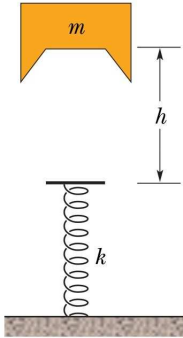
ii) $0 \leq x \leq 7$

iii) $\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + \Delta K = W$
 $v = 6.48 \rightarrow v = 6.5 \text{ m/s}$ + x direction

$6 + 36 + 6 - 6 - 12 = 30 \text{ J}$

(0.6) (0.6) (0.6) (0.6) (0.6)

4. (Kinetic Energy and Work - 40%) A block of mass $m = 2.0$ kg is dropped from height $h = 40$ cm onto a spring of spring constant $k = 1960$ N/m (See the figure below).



Find the maximum distance the spring is compressed.

System: Block + spring + Earth
 No external forces \Rightarrow Mechanical energy is conserved

$$K_i + U_i = K_f + U_f$$

$K_i = 0$, block is dropped.
 (1) $K_f = 0$ to have maximum distance of compression.

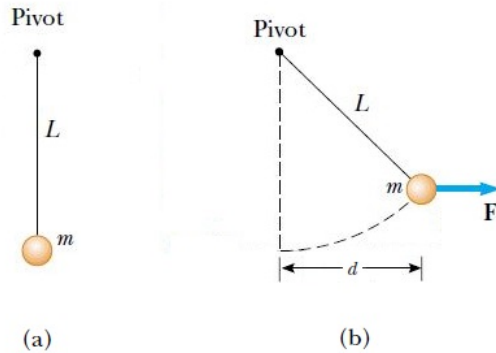
$$(2) \quad mgh = (3) \quad mgy + \frac{(4) \quad (5)}{2} ky^2$$

$$(2) \quad \left\{ \begin{aligned} (2.0 \text{ kg})(9.8 \text{ m/s}^2)(0.40 \text{ m}) &= (2.0 \text{ kg})(9.8 \text{ m/s}^2)y + \frac{1}{2}(1960 \text{ N/m})y^2 \\ 0.40 \text{ m} &= y + (50 \frac{1}{\text{m}})y^2 \\ 0 &= 50y^2 + 10y - 4 \end{aligned} \right.$$

Roots: $y = -0.1 \text{ m}$, $y = 0.08 \text{ m}$

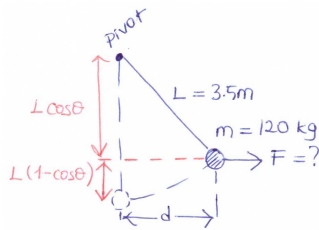
Since y should be negative, $\boxed{y_{\text{max}} = -0.10 \text{ m}}$

5. (Kinetic Energy and Work- 40%) A 120 kg mass hangs by vertical rope $L = 3.5 \text{ m}$ long as in Figure a. A person then displaces the mass to a position $d = 2.0 \text{ m}$ sideways from its original position, always keeping the rope taut, and holds it in this new position as in Figure b.



As the mass is moved to this position, how much work is done by the person?

Answer: 738 J



$$d = 2.0 \text{ m.}$$

$$W_{\text{tot}} = \cancel{K_2} - \cancel{K_1} \quad (\text{Since the mass starts and ends at rest})$$

$$W_{\text{grav.}} + W_T + W_F = 0$$

0 (The tension in the rope is radial and displacement is tangential so there is no component of T in the direction in the displacement during the motion and the tension in the rope does not work.)

$$W_F = -W_{\text{grav.}}, \quad W_{\text{grav.}} = -\Delta U_{\text{grav.}} = U_1 - U_2 = mg(y_1 - y_2)$$

$$= mg [0 - L(1 - \cos\theta)], \quad \theta = \sin^{-1}\left(\frac{d}{L}\right) = \sin^{-1}\left(\frac{2.0 \text{ m}}{3.5 \text{ m}}\right)$$

$$\theta = 34.85^\circ$$

$$= -(120 \text{ kg})(9.80 \text{ m/s}^2) [(3.5 \text{ m})(1 - \cos 34.85^\circ)]$$

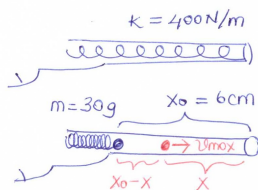
$$= -738 \text{ J}$$

$$W_F = +738 \text{ J}$$

6. (Kinetic Energy and Work- 40%) The spring of a spring gun has force constant $k = 400 \text{ N/m}$ and negligible mass. The spring is compressed 6.00 cm , and a ball with mass 30.0 g is placed in the horizontal barrel against the compressed spring. The spring is then released, and the ball is propelled out the barrel of the gun. The barrel is 6.00 cm long, so the ball leaves the barrel at the same point that it loses contact with the spring. The gun is held so the barrel is horizontal.

- i) Calculate the speed of the ball as it leaves the barrel if a constant resisting force of 6.00 N acts on the ball as it moves along the barrel.
- ii) What is the greatest speed the ball has along the barrel if a constant resisting force of 6.00 N acts on the ball as it moves along the barrel?

Answer: i) 4.90 m/s ii) 5.20 m/s



$$i) \quad f \leftarrow \bullet \rightarrow F_{\text{spring}}, \quad f = 6.00 \text{ N}$$

$$W_{\text{tot}} = K_f - K_i \quad \text{with } K_i = 0$$

$$W_f + W_{\text{spring}} = \frac{1}{2} m v^2, \quad W_f = \vec{f} \cdot \vec{x}_0 = -f x_0$$

$$W_{\text{spring}} = \frac{1}{2} k x_i^2 - \frac{1}{2} k x_f^2 = \frac{1}{2} k x_0^2 - 0$$

$$-f x_0 + \frac{1}{2} k x_0^2 = \frac{1}{2} m v^2$$

$$-(6.00 \text{ N})(0.06 \text{ m}) + \frac{1}{2} (400 \text{ N/m})(0.06 \text{ m})^2 = \frac{1}{2} (30 \times 10^{-3} \text{ kg}) v^2$$

$$v = 4.90 \text{ m/s}$$

ii) The greatest speed occurs when the acceleration (and the net force) are zero.

$$f \leftarrow \bullet \rightarrow F_{\text{spring}} \quad \sum \vec{F} = 0 \rightarrow kx = f$$

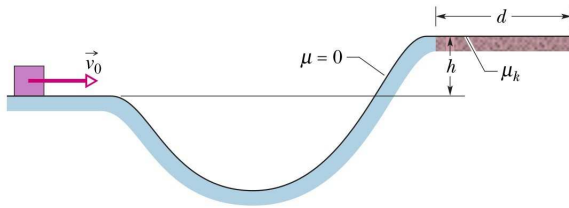
$$x = f/k$$

$$W_f + W_{\text{spring}} = K_f - K_i \quad \text{with } K_i = 0$$

$$-f(x - x_0) + \frac{1}{2} k x_0^2 - \frac{1}{2} k x^2 = \frac{1}{2} m v_{\text{max}}^2$$

$$-(6.00 \text{ N}) \left(\frac{6.00 \text{ N}}{400 \text{ N/m}} - 0.06 \text{ m} \right) + \frac{1}{2} (400 \text{ N/m}) \left(\frac{6.00 \text{ N}}{400 \text{ N/m}} \right)^2 = \frac{1}{2} (30 \times 10^{-3} \text{ kg}) v_{\text{max}}^2 \rightarrow v_{\text{max}} = 5.20 \text{ m/s}$$

7. (Potential Energy and Conservation of Energy) In the figure below, a block slides along a track from one level to a higher level after passing through an intermediate valley. The track is frictionless until the block reaches the higher level. There a frictional force stops the block in a distance d .



The block's initial speed v_0 is 6.0 m/s, the height difference h is 1.1 m, and μ_k is 0.60. Find d .

Answer: $d = 1.2 \text{ m}$

System: Block + Earth + track
 No external force, then

$$0 = \Delta E_{\text{mech}} + \Delta E_{\text{th}}$$

$$0 = \left(\frac{1}{2} m v_f^2 + mgh \right) - \frac{1}{2} m v_i^2 + f_k d$$

$$0 = \overset{+6}{mgh} - \frac{1}{2} m v_i^2 + \overset{+6}{\mu_k (mg) d} \quad \left(F_N = mg \text{ at the higher level} \right) \quad \textcircled{+1}$$

(OR you can reach same equation by taking block + Earth as a system and having f_k as the external force;

$$W_{\text{fric}} = \Delta E_{\text{mech}}$$

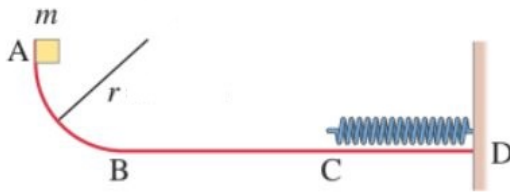
Then, solving for d ;

$$\textcircled{+2} \quad d = \frac{1}{\mu_k} \left(\frac{v_i^2}{2g} - h \right) = \frac{1}{0.60} \left[\frac{(6.0 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} - (1.1 \text{ m}) \right]$$

$$\Rightarrow \boxed{d = 1.2 \text{ m}}$$

3

8. (Potential Energy and Conservation of Energy - 40%) Consider the track shown in Figure below. The section AB is one quadrant of a circle of radius 2.0 m and is frictionless. B to C is a horizontal span 3.0 m long with a coefficient of kinetic friction $\mu_k = 0.25$. The section CD under the spring is frictionless. A block of mass 1.0 kg is released from rest at A. After sliding on the track, it compresses the spring by 0.20 m .



Determine the stiffness constant k for the spring.

Answer: 612 N/m

$\Delta U_{\text{int}} = -W_{\text{other}}$ ΔU_{int} : Change in internal energy.
 W_{other} : The work done by forces other than conservative forces (gravitational force, elastic force, ...)

$\Delta K + \Delta U + \Delta U_{\text{int}} = 0$ (law of conservation of energy)

$(\cancel{K_f} - \cancel{K_i}) + (U_f - U_i) + (-W_{\text{fric.}}) = 0$, $U_f = \frac{1}{2} k x_{\text{max}}^2$

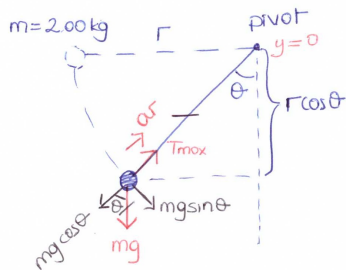
$U_i = mg r$
 $W_{\text{fric.}} = \vec{F}_{\text{fric.}} \cdot \vec{\Delta x} = \mu_k mg \Delta x \underbrace{\cos 180^\circ}_{-1}$
 \downarrow
 $|BC|$

$\frac{1}{2} k x_{\text{max}}^2 - mg r + \mu_k mg |BC| = 0$

$\frac{1}{2} k (0.20\text{m})^2 - (1.0\text{kg})(9.80\text{ m/s}^2)(2.0\text{m}) + (0.25)(1.0\text{kg})(9.80\text{ m/s}^2)(3.0\text{m}) \rightarrow \boxed{k = 612.5\text{ N/m}}$

9. (Potential Energy and Conservation of Energy - 40%) A 2.00 kg ball is attached to the bottom end of a length of rope with a breaking strength of 44.5 N. The top end of the rope is held stationary. The ball is released from rest with the line taut and horizontal ($\theta = 90.0^\circ$). At what angle (measured from the vertical) will the rope break?

Answer: 40.8°



$$\sum F_r = m a_r$$

$$T_{\max} - mg \cos \theta = m \frac{v^2}{r} \Rightarrow v^2 = \frac{r}{m} (T_{\max} - mg \cos \theta) \quad (\text{at the breaking point})$$

$$E_i = E_f$$

$$K_i + U_i = K_f + U_f$$

$$0 = \frac{1}{2} m v^2 + m g y, \quad y = -r \cos \theta$$

$$0 = \frac{1}{2} m \left(\frac{r}{m} (T_{\max} - mg \cos \theta) \right) - m g r \cos \theta$$

$$0 = \frac{T_{\max}}{2} - \frac{3}{2} mg \cos \theta \Rightarrow \theta = \cos^{-1} \left(\frac{T_{\max}}{3mg} \right)$$

$$\frac{T_{\max}}{2}$$

$$\theta = \cos^{-1} \left(\frac{44.5 \text{ N}}{3 \cdot 2.00 \text{ kg} \cdot 9.80 \text{ m/s}^2} \right)$$

$$\theta = 40.8^\circ$$

10. (Center of Mass and Linear Momentum) A system containing three objects, which is initially at rest, accelerates by the application of a net force. Masses and time dependent position vectors of objects are given as;

Find;

$$m_1 = 1 \text{ kg}, m_2 = 3 \text{ kg}, m_3 = 8 \text{ kg}$$

$$\vec{r}_1 = (t^2 - 4)\hat{i} + 2\hat{j}$$

$$\vec{r}_2 = (4 - t)\hat{i} + 2\hat{j}$$

$$\vec{r}_3 = 2\hat{i} + (t - 3)\hat{j}$$

i Position vector of system's center of mass, r_{com} , as a function of time,

ii Acceleration of the center of mass of the system, a_{com} ,

iii Net force acting on the system, F_{net} .

Answer: i) $\vec{r}_{com} = [(t^2 - 3t + 24)\hat{i} + (8t - 16)\hat{j}] / 12$ ii) $\vec{a}_{com} = \hat{i}/6 \text{ m/s}^2$
iii) $\vec{F}_{net} = 2\hat{i} \text{ N}$

$$4) \quad b) \quad i) \quad \vec{r}_{com} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3}{m_1 + m_2 + m_3}$$

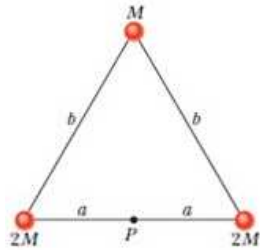
$$\vec{r}_{com} = \frac{(1)[(t^2 - 4)\hat{i} + 2\hat{j}] + (3)[(4 - t)\hat{i} + 2\hat{j}] + (8)[2\hat{i} + (t - 3)\hat{j}]}{1 + 3 + 8}$$

$$\vec{r}_{com} = \frac{(t^2 - 3t + 24)\hat{i} + (8t - 16)\hat{j}}{12} //$$

$$ii) \quad \vec{a}_{com} = \frac{d^2 \vec{r}_{com}}{dt^2} = \frac{2\hat{i}}{12} = \frac{\hat{i}}{6} \text{ m/s}^2 //$$

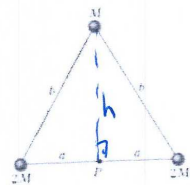
$$iii) \quad \vec{F}_{net} = M \cdot \vec{a}_{com} \Rightarrow \vec{F}_{net} = (1 + 3 + 8) \cdot \frac{\hat{i}}{6} \Rightarrow \vec{F}_{net} = 2\hat{i} \text{ N} //$$

11. (Center of Mass and Linear Momentum) The rigid body shown in figure given below consists of three particles connected by **massless** rods. It is to be rotated about an axis perpendicular to its plane through point P .



If $M = 0.40 \text{ kg}$, $a = 30 \text{ cm}$, and $b = 50 \text{ cm}$,
What is the rotational inertia about point P ?

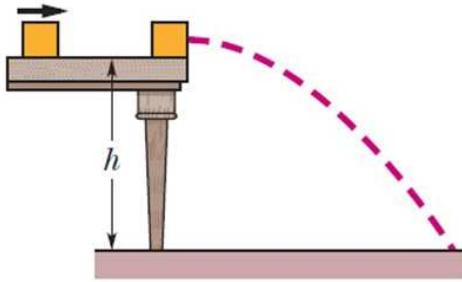
Answer: $I = 0.208 \text{ kgm}^2$



If $M = 0.40 \text{ kg}$, $a = 30 \text{ cm}$, and $b = 50 \text{ cm}$,
What is the rotational inertia about point P ?

$$\begin{aligned}
 \bar{I} &= \sum_{i=1}^3 m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 \\
 &= 2M(a^2) + (2M)a^2 + M(h^2) \\
 &= 4Ma^2 + Mb^2 - Ma^2 = 3Ma^2 + Mb^2 \\
 &= M(3a^2 + b^2) = (0.4 \text{ kg}) (3(0.3 \text{ m})^2 + (0.5 \text{ m})^2) \\
 \bar{I} &= 0.208 \text{ kg m}^2
 \end{aligned}$$

12. (Center of Mass and Linear Momentum) A 3.2 kg box slides on a horizontal frictionless table and collides with a 2.0 kg box initially at rest on the edge of the table, at height $h = 0.40$ m. The speed of the 3.2 kg box is 3.0 m/s just before the collision.



If the two boxes stick together because of packing tape on their sides, what is their kinetic energy just before they strike the floor?

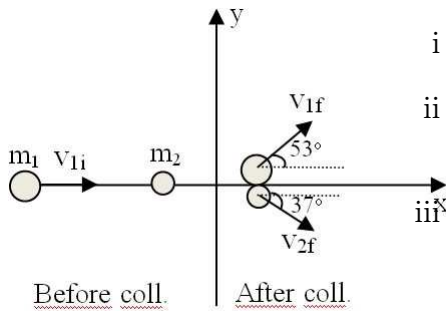
Answer: $KE = 29$ J

$m_1 = 3.2$ kg v_{1i}
 $m_2 = 2.0$ kg $v_{2i} = 0$
 $h = 0.40$ m
 $v_{1i} = 3.0$ m/s
 stick together
 \Rightarrow completely
 inelastic
 collision
 $K = ?$ before impact
 to floor

Conservation of momentum $\vec{p}_i = \vec{p}_f$ in x -direction $\vec{p}_x = \vec{p}_x \rightarrow m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v$
 $\Rightarrow v = \frac{m_1 v_{1i}}{m_1 + m_2} = \frac{3.2 \text{ kg}}{(3.2 \text{ kg} + 2.0 \text{ kg})} 3.0 \text{ m/s} = 1.8 \text{ m/s } +x\text{-direction}$

Now, we have projectile motion
 $K_2 + U_2 = K_1 + U_1$
 $? + 0 = \frac{1}{2} (3.2 \text{ kg} + 2.0 \text{ kg}) (1.8 \text{ m/s})^2 + (5.2 \text{ kg}) (9.8 \frac{\text{m}}{\text{s}^2}) (0.40 \text{ m})$
 $\Rightarrow K_2 = 29 \text{ J}$

13. (Center of Mass and Linear Momentum) The mass $m_1 = 1.0 \text{ kg}$ moving with a speed $v_1 = 5.0 \text{ m/s}$ on a horizontal frictionless floor collides with a mass $m_2 = 2.0 \text{ kg}$ initially at rest. After the collision, m_1 moves at a speed $v_{1f} = 3.0 \text{ m/s}$ and m_2 moves at a speed v_{2f} along the directions shown in the figure.



- i Find the speed v_{2f} of m_2 after the collision.
- ii Is this collision elastic or not? Prove your answer.
- iii Find the velocity of the center of mass of the system after collision. Give your answer in terms of unit vector notation.
- iv Find the impulse delivered to m_1 during the collision. Give your answer in terms of unit vector notation.

Answer: i) $v_{2f} = 2.0 \text{ m/s}$ ii) **Inelastic** iii) $\vec{v}_{com,f} = 5/3 \text{ m/s } \hat{i}$ iv) $\vec{J}_1 = (-3.2\hat{i} + 2.4\hat{j}) \text{ kgm/s}$

5 a) Find the speed v_{2f} of m_2 after the collision.

$$\vec{P}_i = \vec{P}_f$$

$$m_1 \vec{u}_{1i} + m_2 \vec{u}_{2i} = m_1 \vec{u}_{1f} + m_2 \vec{u}_{2f}$$

y-components

$$m_1 u_{1iy} = m_1 u_{1fy} + m_2 u_{2fy} \Rightarrow (1.0) u_1 \sin 53 - m_2 u_2 \sin 37 \Rightarrow$$

$$\Rightarrow (1.0)(3.0) \frac{4}{5} = (2.0) u_{2f} \frac{3}{4}$$

$$u_{2f} = 2.0 \text{ m/s}$$

5 b) Is this collision elastic or not? Prove your answer.

In elastic collision ΔE is conserved
check?

$K_i > K_f$ so the collision is not elastic. It is inelastic collision

$$K_i = \frac{1}{2} m_1 u_{1i}^2 + \frac{1}{2} m_2 u_{2i}^2 = \frac{1}{2} (1.0) (3.0)^2 = \frac{2.25}{2} \text{ J} = 12.5 \text{ J}$$

$$K_f = \frac{1}{2} m_1 u_{1f}^2 + \frac{1}{2} m_2 u_{2f}^2 = \frac{1}{2} (1.0) (3.0)^2 + \frac{1}{2} (2.0) (2.0)^2 = 8.5 \text{ J}$$

5 c) Find the velocity of the center of mass of the system after collision. Give your answer in terms of unit vector notation.

$$\vec{v}_{\text{com},f} = \frac{m_1 \vec{u}_{1f} + m_2 \vec{u}_{2f}}{m_1 + m_2}$$

$$v_{\text{com},x} = \frac{(1.0)(3.0) \cos 53 + (2.0)(2.0) \cos 37}{3}$$

$$v_{\text{com},x} = \frac{3 \cdot \frac{3}{5} + 4 \cdot \frac{4}{5}}{3} = \frac{5}{3} \text{ m/s}$$

$$v_{\text{com},y} = \frac{(1.0)(3.0) \sin 53 - (2.0)(2.0) \sin 37}{3}$$

$$= 0 \quad v_{\text{com},y} = \frac{5}{3} \hat{j} \text{ m/s}$$

$$\vec{v}_{\text{com},i} = \frac{m_1 \vec{u}_{1i} + m_2 \vec{u}_{2i}}{m_1 + m_2} = \frac{(1.0)(3.0) \hat{i}}{3}$$

$$\vec{v}_{\text{com},i} = \frac{5}{3} \hat{i} \text{ m/s}$$

SAME \vec{p} is conserved!

5 d) Find the impulse delivered to m_1 during the collision. Give your answer in terms of unit vector notation.

$$\vec{J}_1 = \vec{P}_{1f} - \vec{P}_{1i}$$

$$P_{1fx} = m_1 u_{1fx} = (1.0)(3.0) \cos 53 = 1.8 \text{ kgm/s}$$

$$P_{1fy} = m_1 u_{1fy} = (1.0)(3.0) \sin 53 = 2.4 \text{ kgm/s}$$

$$P_{1ix} = m_1 u_{1ix} = (1.0)(3.0) = 3.0 \text{ kgm/s}$$

$$P_{1iy} = 0$$

$$\vec{J}_1 = 2.4 \hat{j} + 1.8 \hat{i} - 3.0 \hat{i} \text{ kgm/s}$$

$$\vec{J}_1 = -1.2 \hat{i} + 2.4 \hat{j} \text{ kgm/s}$$

14. (Rotation) The angular position of a point on the rim of a rotating wheel is given by $\theta = 4.0t + 3.0t^2 + t^3$, where θ is in radians and t is in seconds. What are the angular velocities at

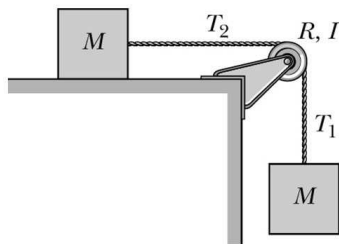
i $t = 2.0$ s and $t = 4.0$ s,

ii What is the average and instantaneous angular acceleration for that time interval?

Answer: i) $w(t = 2.0 \text{ s}) = 28 \text{ rad/s}$ $w(t = 4.0 \text{ s}) = 76 \text{ rad/s}$ ii) 24 rad/s^2

$\theta = 4.0t + 3.0t^2 + t^3$
 i) $w = \frac{d\theta}{dt} = 4.0 + 6.0t + 3t^2$
 $\left\{ \begin{array}{l} t=2.0 \text{ s} \rightarrow w(t=2) = 28 \text{ rad/s} \\ t=4.0 \text{ s} \rightarrow w(t=4) = 76 \text{ rad/s} \end{array} \right.$
 ii) $\alpha_{\text{avg}} = \frac{w(t=4) - w(t=2)}{4-2} = \frac{76 - 28}{2} = 24 \text{ rad/s}^2$
 $\alpha = \frac{dw}{dt} = 6.0 + 6.0t$
 $\left\{ \begin{array}{l} \alpha(t=4) = 30 \text{ rad/s}^2 \\ \alpha(t=2) = 18 \text{ rad/s}^2 \end{array} \right.$
 $\frac{30+18}{2} = 24$

15. (Rotation) In figure, two 6.2 kg blocks are connected by a massless string over a pulley of radius 2.40 cm and rotational inertia $7.40 \times 10^{-4} \text{ kg}\cdot\text{m}^2$. The string does not slip on the pulley; it is not known whether there is friction between the table and the sliding block; the pulley axis is frictionless. When this system is released from rest, the pulley turns through 0.650 rad in 91.0 ms and the acceleration of the block is constant. What are



- the magnitude of the pulley's angular acceleration,
- the magnitude of either block's acceleration,
- string tension T_1 ,
- string tension T_2 ?

Answer: i) 157.0 rad/s^2 ii) 3.8 m/s^2 iii) $T_1 = 37.4 \text{ N}$ iv) $T_2 = 32.6 \text{ N}$

What are

- the magnitude of the pulley's angular acceleration, $\alpha = ?$
- the magnitude of either block's acceleration, $a = ?$
- string tension T_1 , $T_1 = ?$
- string tension T_2 , $T_2 = ?$

From Given figure:

$$a_1 = a_2 = R\alpha \quad (1)$$

• apply Newton's 2nd Law to m_1 & m_2 & disk

For $m_1 \Rightarrow m_1 g - T_1 = m_1 a_1 \quad (2)$

For $m_2 \Rightarrow T_2 - m_2 g = m_2 a_2 \quad (3)$

For disk $\Rightarrow T_1 R - T_2 R = I \alpha \quad (4)$

ii) $a = R \alpha \Rightarrow a = 3.8 \text{ m/s}^2 \quad (5)$

iii) From (2) $\Rightarrow T_1 = m_1 g - m_1 a_1$
 $T_1 = (6.2 \text{ kg}) \times (9.8 \text{ m/s}^2) - (6.2 \text{ kg}) \times (3.8 \text{ m/s}^2)$
 $T_1 = 60.76 - 23.56$
 $\Rightarrow T_1 = 37.2 \text{ N} \quad (5)$

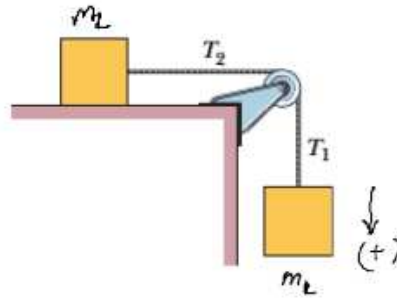
iv) From (4) $\Rightarrow T_1 R - T_2 R = I \alpha$
 $37.2 \times 0.024 - T_2 \times 0.024 = (7.4 \times 10^{-4} \text{ kg}\cdot\text{m}^2) \times (15.9)$
 $\Rightarrow 0.89 - 0.024 T_2 = 117.7 \times 10^{-3}$
 $\Rightarrow 0.024 T_2 = 0.773 \Rightarrow T_2 = 32.2 \text{ N} \quad (5)$

Additional calculation for angular acceleration:

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2 \quad (5)$$

$0.65 \text{ rad} = 0 + \frac{1}{2} \alpha (0.0910 \text{ s})^2$
 $\Rightarrow \alpha = 159 \text{ rad/s}^2 \quad (5)$

tension T_2 ?



$$T_2 - f = m_2 a_2$$

$$m_1 g - T_1 = m_1 a$$

$$(T_1 - T_2) R = I \alpha$$

$$\Delta \theta = 0.650 \text{ rad}$$

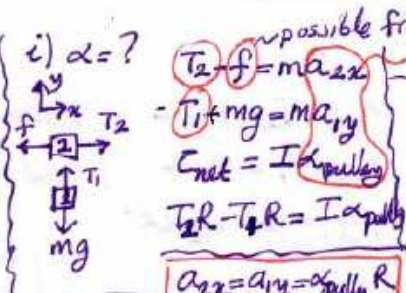
$$\Delta t = t = 91.0 \text{ ms}$$

10 Solved Problems



9 (7)

$m_1 = m_2 = 6.2 \text{ kg} = m$
 pulley radius = 2.40 cm
 $I = 7.40 \times 10^{-4} \text{ kg m}^2$
 Friction? not known if exists
 Released from rest
 $\Delta \theta = 0.650 \text{ rad}$
 $\Delta t = 91.0 \text{ ms}$
 acceleration: constant



i) $\alpha = ?$

$$T_2 - f = m a_{2x}$$

$$T_1 + mg = m a_{1y}$$

$$C_{net} = I \alpha_{pulley}$$

$$T_2 R - T_1 R = I \alpha_{pulley}$$

$$a_{2x} = a_{1y} = \alpha_{pulley} R$$

unknowns } constant acceleration

$$\theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$0 - 0 = 0 + \frac{1}{2} \alpha (91.0 \times 10^{-3})^2$$

$$\Delta \theta = 0.650 \text{ rad} = \frac{1}{2} \alpha (91.0 \text{ ms})^2$$

$$\alpha = 157.1 \text{ rad/s}^2$$

ii) $a = \alpha R = (0.024 \text{ m})(157 \text{ rad/s}^2) = 3.77 \text{ m/s}^2$

iii) $T_1 = ?$ $-T_1 + mg = ma$

$$T_1 = m(a + g) = m(9.8 \text{ m/s}^2 - 3.77 \text{ m/s}^2)$$

$$T_1 = 37.4 \text{ N}$$

iv) $T_2 R - T_1 R = I \alpha$

$$T_2 = \frac{T_1 R + I \alpha}{R} = T_1 + \frac{I \alpha}{R} = 37.4 \text{ N} + \frac{7.40 \times 10^{-4} \text{ kg m}^2 (157 \text{ rad/s}^2)}{(0.024 \text{ m})} = 32.56 \text{ N}$$

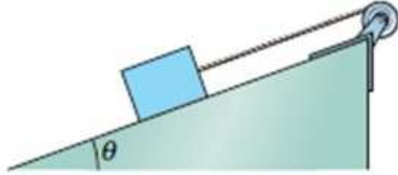
in (iii)

$$(T_1 - T_2) = \frac{I \alpha}{R}$$

$$T_2 = T_1 - \frac{I \alpha}{R}$$

$T_2 < T_1$

16. (Rotation - 40%) In figure, a wheel of radius 0.20 m is mounted on a frictionless horizontal axle. A massless cord is wrapped around the wheel and attached to a 2.0 kg box that slides on a frictionless surface inclined at angle $\theta = 20^\circ$ with the horizontal. The box accelerates down the surface at 2.0 m/s^2 .



What is the rotational inertia of the wheel about the axle?

What is the rotational inertia of the wheel about the axle?

① $mg \sin \theta - T = m \cdot a$

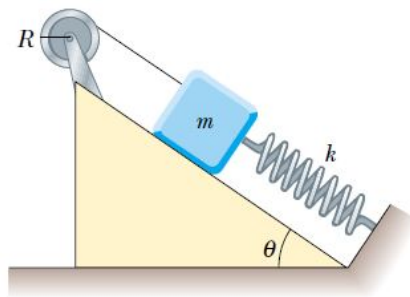
② $T \cdot R = I \cdot \alpha$

③ $I = \frac{(mg \sin \theta - ma) R}{\alpha}$

④ $\alpha = \frac{a}{R}$

⑤ $I = \frac{(2\text{ kg}) \cdot (9.8\text{ m/s}^2) \sin 20 - (2\text{ kg}) \cdot (2\text{ m/s}^2)}{(0.2\text{ m})^2} = 0.054\text{ kg m}^2$

17. (Rotation - 40%) The reel shown in Figure has radius $R = 0.300 \text{ m}$ and moment of inertia $I = 1.00 \text{ kg}\cdot\text{m}^2$. One end of the block of mass $m = 0.500 \text{ kg}$ is connected to a spring of force constant $k = 50.0 \text{ N/m}$, and the other end is fastened to a cord wrapped around the reel. The reel axle and the plane having an inclination of $\theta = 37^\circ$ are frictionless. The reel is wound counterclockwise so that the spring stretches a distance $d = 0.200 \text{ m}$ from its unstretched position and is then released from rest.



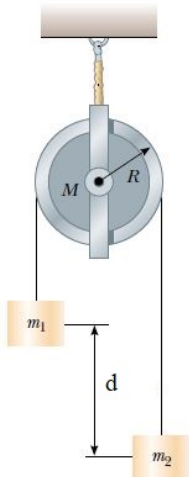
Find the angular speed of the reel when the spring is again unstretched.

Answer: 1.74 rad/s

$R = 0.300 \text{ m}$
 $I = 1.00 \text{ kg}\cdot\text{m}^2$
 $m = 0.500 \text{ kg}$
 $k = 50.0 \text{ N/m}$
 $d = 0.200 \text{ m}$
 $\theta = 37^\circ$

$E_i = E_f$
 $K_i + U_i = K_f + U_f$
 $mg(ds\sin\theta) = \frac{1}{2}m v^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}kx^2, \quad v = R\omega, \quad x = d$
 $mgd\sin\theta = \frac{1}{2}mR^2\omega^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}kd^2$
 $\omega = \left(\frac{2mgd\sin\theta + kd^2}{I + mR^2} \right)^{1/2}$
 $\omega = \left(\frac{2 \cdot (0.500 \text{ kg}) \cdot (9.80 \text{ m/s}^2) \cdot (0.200 \text{ m}) \cdot \sin 37^\circ + (50.0 \text{ N/m}) \cdot (0.200 \text{ m})^2}{(1.00 \text{ kg}\cdot\text{m}^2) + (0.500 \text{ kg}) \cdot (0.300 \text{ m})^2} \right)^{1/2}$
 $\omega = 1.74 \text{ rad/s}$

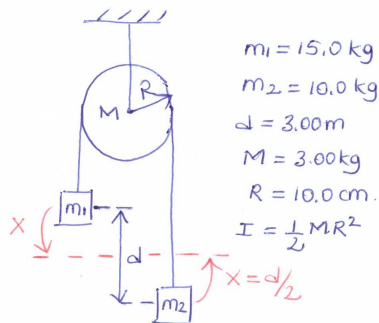
18. (Rotation - 40%) A $m_1 = 15.0 \text{ kg}$ object and a $m_2 = 10.0 \text{ kg}$ object are suspended, joined by a cord that passes over a pulley with a radius of $R = 10.0 \text{ cm}$ and a mass of $M = 3.00 \text{ kg}$ as seen in the Figure.



The cord has a negligible mass and does not slip on the pulley. The pulley rotates on its axis without friction. The objects start from rest $d = 3.00 \text{ m}$ apart. Treat the pulley as a uniform disk, and determine the speeds of the two objects as they pass each other.

Hint: $I_{CM} = \frac{1}{2}MR^2$

Answer: 2.36 m/s



$$\begin{aligned} m_1 &= 15.0 \text{ kg} \\ m_2 &= 10.0 \text{ kg} \\ d &= 3.00 \text{ m} \\ M &= 3.00 \text{ kg} \\ R &= 10.0 \text{ cm} \\ I &= \frac{1}{2}MR^2 \end{aligned}$$

$$\Delta K + \Delta U = 0$$

$$\left(\underbrace{\frac{1}{2}(m_1+m_2)v^2}_{K_f} + \underbrace{\frac{1}{2}I\omega^2}_{K_i} - 0 \right) + \left(\underbrace{-m_1g\frac{d}{2}}_{\Delta U_{\text{grav},1}} + \underbrace{m_2g\frac{d}{2}}_{\Delta U_{\text{grav},2}} \right) = 0$$

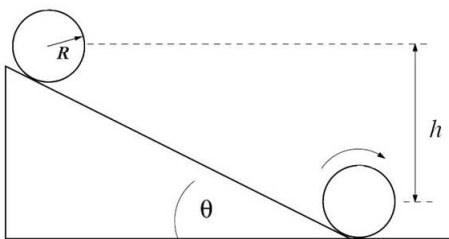
$$\omega = \frac{v}{R}$$

$$\frac{1}{2}(m_1+m_2)v^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{v^2}{R^2} = \frac{g \cdot d}{2}(m_1 - m_2)$$

$$v = \left(\frac{g \cdot d (m_1 - m_2)}{(m_1 + m_2) + \frac{M}{2}} \right)^{1/2}$$

$$= \left(\frac{(9.80 \text{ m/s}^2)(3.00 \text{ m})(15.0 \text{ kg} - 10.0 \text{ kg})}{(15.0 \text{ kg} + 10.0 \text{ kg}) + \frac{(3.00 \text{ kg})}{2}} \right)^{1/2} = \boxed{2.36 \text{ m/s}}$$

19. (Rolling, Torque, and Angular Momentum) A hollow cylinder of outer radius $R = 5 \text{ cm}$ and mass $M = 50 \text{ g}$ with moment of inertia about the center of mass $I_{com} = MR^2$ starts from rest and moves down an incline tilted at an angle $\theta = 20^\circ$ from the horizontal. The center of mass of the cylinder has dropped a vertical distance $h = 30 \text{ cm}$ when it reaches the bottom of the incline. The coefficient of static friction between the cylinder and the surface is μ_s . The cylinder rolls without slipping down the incline. **Answer: i) $\mu_s > 0.18$ ii) $v = 1.7 \text{ m/s}$**



- i) What is the minimum value for the coefficient of static friction μ_s such that the cylinder rolls without slipping down the incline plane?
- ii) What is the magnitude of the velocity of the center of mass of the cylinder when it reaches the bottom of the incline?

$m g \sin \theta - f = m a$
 $N - m g \cos \theta = 0$
 $\tau = I_{com} \cdot \alpha$
 $f R = m R^2 \left(\frac{a}{R} \right) \Rightarrow f = m a$ (for no slipping)

$\Rightarrow m g \sin \theta - m a = m a$
 $\Rightarrow m g \sin \theta = 2 m a \Rightarrow a = \frac{1}{2} g \sin \theta = 1.6 \text{ m/s}^2$ (3)

$f = m \cdot \frac{1}{2} g \sin \theta = \frac{0.05 \cdot 9.8 \sin 20}{2} \Rightarrow f = 0.08 \text{ N}$ (3)

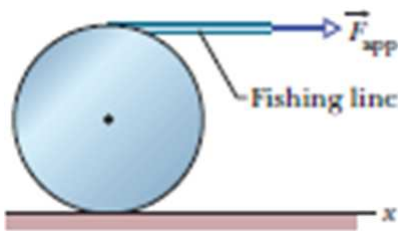
For rolling without slipping $f < \mu_s \cdot N$ (6)
 $\frac{1}{2} m g \sin \theta < \mu_s \cdot m g \cos \theta$
 $\Rightarrow \mu_s > \frac{1}{2} \tan \theta \Rightarrow \mu_s > 0.18$

ii) $U_i + K_i = U_f + K_f$
 $m g h = \frac{1}{2} m v_f^2 + \frac{1}{2} I_{com} \omega_f^2$
 $m g h = \frac{1}{2} m v_f^2 + \frac{1}{2} m R^2 \frac{v_f^2}{R^2}$ (3)
 $\Rightarrow m g h = \frac{1}{2} m v_f^2 + \frac{1}{2} m v_f^2$
 $\Rightarrow v_f = \sqrt{g h}$

$v_f = \sqrt{g h}$
 $v_f = \sqrt{9.8 \times 0.3}$
 $\Rightarrow v_f = 1.7 \text{ m/s}$ (8)

$\left\{ \omega = \frac{v_f}{R} = \frac{1.7}{0.05} = 34 \right\}$

20. (Rolling, Torque, and Angular Momentum - 40%) In figure given below, a constant horizontal force \vec{F}_{app} of magnitude 12 N is applied to a uniform solid cylinder by fishing line wrapped around the cylinder. The mass of the cylinder is 10 kg , its radius is 0.10 m , and the cylinder rolls smoothly on the horizontal surface. (**Hint:** $I_{COM} = \frac{1}{2}MR^2$)



- What is the magnitude of the acceleration of the center of mass of the cylinder?
- What is the magnitude of the angular acceleration of the cylinder about the center of mass?
- In unit-vector notation, what is the frictional force acting on the cylinder?

$g(71)$ $\vec{F}_{app} = 12\text{ N}$
 $m = 10\text{ kg}$
 $R = 0.10\text{ m}$
 rolls smoothly

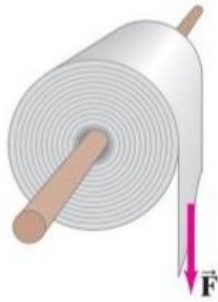
$\text{cw} \rightarrow \text{as positive (as our choice!)}$
 $I_p = I_{com} + MR^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$
 $F_{app} \cdot 2R = \tau = I_p \alpha \rightarrow 2R F_{app} = (\frac{3}{2}MR^2) \alpha$
 $\alpha = \frac{4}{3} \frac{F_{app}}{MR} = \frac{4}{3} \frac{(12\text{ N})}{(10\text{ kg})(0.10\text{ m})} = 16\text{ rad/s}^2 \Rightarrow a = \alpha R$
 $a = 1.6\text{ m/s}^2$

$\text{ii) } \alpha = 16\text{ rad/s}^2$
 $\text{iii) } f = ?$

$F_{app} + f = ma \quad (1)$
 $F_{app} - f = ma \quad (2)$
 $12 - f = 10 \cdot 1.6 = 16 = f$
 $\Rightarrow f = -4\text{ N} \Rightarrow \vec{f} = +4\text{ N} \hat{i}$

$\vec{f} \leftarrow \vec{a} \rightarrow \vec{F}_{app}$
 $12 - f = ma$
 $\vec{a} \rightarrow \text{concordant}$
 $12\text{ N} - (10\text{ kg})(1.6\text{ m/s}^2) = -f$
 $\Rightarrow \vec{f} = +4\text{ N} \hat{i}$

21. (Rolling, Torque, and Angular Momentum - 40%) The radius of the roll of paper shown in Figure is 7.6 cm and its moment of inertia is $I = 3.3 \times 10^{-3} \text{ kg}\cdot\text{m}^2$. A force of 2.5 N is exerted on the end of the roll for 1.3 s , but the paper does not tear so it begins to unroll. A constant friction torque of $0.11 \text{ m}\cdot\text{N}$ is exerted on the roll which gradually brings it to a stop.



Assuming that the paper's thickness is negligible, calculate

- the length of paper that unrolls during the time that the force is applied (1.3 s) and
- the length of paper that unrolls from the time the force ends to the time when the roll has stopped moving.

Answer: i) 1.56 m ii) 1.13 m

i. $\Delta S_1 = R \Delta \theta_1$

the angular acceleration is constant, and so constant acceleration relationship can be used.

$$\Delta \theta_1 = \omega_0 t_1 + \frac{1}{2} \alpha_1 t_1^2$$

Take clockwise torques as positive. Write Newton's second law for the rotational motion:

$$\sum \tau = I \alpha_1$$

$$FR - \tau_{fr} = I \alpha_1 \rightarrow \alpha_1 = \frac{FR - \tau_{fr}}{I}$$

$$\Delta S_1 = R \left(\frac{1}{2} \left(\frac{FR - \tau_{fr}}{I} \right) t_1^2 \right) = (0.076 \text{ m}) \frac{1}{2} \left(\frac{(2.5 \text{ N})(0.076 \text{ m}) - (0.11 \text{ N}\cdot\text{m})}{3.3 \times 10^{-3} \text{ kg}\cdot\text{m}^2} \right) (1.3 \text{ s})^2$$

$\Delta S_1 = 1.56 \text{ m}$

ii. Now the external force removed, but the frictional torque is still present.

$$\omega_1 = \omega_0 + \alpha_1 t_1 = \left(\frac{FR - \tau_{fr}}{I} \right) t_1 = \frac{[(2.5 \text{ N})(0.076 \text{ m}) - (0.11 \text{ N}\cdot\text{m})]}{3.3 \times 10^{-3} \text{ kg}\cdot\text{m}^2} (1.3 \text{ s}) = 31.515 \text{ rad/s}$$

the initial angular velocity is the final angular velocity the motion in part (i)

$$\sum \tau = I \alpha_2$$

$$-\tau_{fr} = I \alpha_2$$

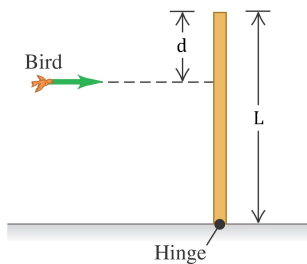
$$\alpha_2 = -\frac{\tau_{fr}}{I}$$

$$\omega_2^2 = \omega_1^2 + 2 \Delta \theta_2 \alpha$$

$$0 = (31.515 \text{ rad/s})^2 + 2 \Delta \theta_2 \left(-\frac{0.11 \text{ N}\cdot\text{m}}{3.3 \times 10^{-3} \text{ kg}\cdot\text{m}^2} \right) \rightarrow \Delta \theta_2 = 14.90 \text{ rad}$$

$$\Delta S_2 = R \Delta \theta_2 = (0.076 \text{ m})(14.90 \text{ rad}) = 1.13 \text{ m}$$

22. (Rolling, Torque, and Angular Momentum - 40%) A 500 g bird is flying horizontally at 2.24 m/s, not paying much attention, when it suddenly flies into a stationary vertical bar, hitting it $d = 25.0$ cm below the top as seen in the Figure. The bar is uniform, $l = 0.750$ m long, has a mass of 1.50 kg, and is hinged at its base. The collision stuns the bird so that it just drops to the ground afterward (but soon recovers to fly happily away).



What is the angular velocity of the bar

- i just after it is hit by the bird, and
- ii just as it reaches the ground?

Hint: For long thin rod the moment of inertia about an axis through end is $I_{CM} = \frac{1}{3}MR^2$

Answer: i) 2.00 rad/s ii) 6.58 rad/s

- i) Angular momentum is conserved in the collision between the bird and the bar:

$$L_i = L_f$$

$$mv(L-d) = \left(\frac{1}{3}ML^2\right)\omega \quad (\text{for the axis at hinge})$$

$$\omega = \frac{3mv(L-d)}{ML^2} = \frac{3(0.5\text{ kg})(2.24\text{ m/s})(0.750\text{ m} - 0.250\text{ m})}{(1.5\text{ kg})(0.750\text{ m})^2}$$

$$\boxed{\omega = 2.00\text{ rad/s}}$$

- ii) Energy is conserved in the motion of the bar after the collision:

$$E_1 = E_2$$

$$\frac{1}{2}I\omega_1^2 + Mg\frac{L}{2} = \frac{1}{2}I\omega_2^2, \quad I = \frac{1}{3}ML^2$$

$$\frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega_1^2 + Mg\frac{L}{2} = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega_2^2$$

$$\omega_2 = \sqrt{\omega_1^2 + \frac{3g}{L}} = \sqrt{(2.00\text{ rad/s})^2 + \frac{3(9.80\text{ m/s}^2)}{(0.750\text{ m})}} = 6.58\text{ rad/s}.$$



İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy101 Physics I
Final Examination
May 22, 2019 10:30 – 12:30
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

INSTRUCTOR:

DURATION: 120 minutes

- ◇ Answer all the questions.
- ◇ Write the solutions explicitly and clearly.
Use the physical terminology.
- ◇ You are allowed to use Formulae Sheet.
- ◇ Calculator is allowed.
- ◇ You are not allowed to use any other electronic equipment in the exam.
- ◇ I declare hereby that I fulfilled the requirements for the attendance according to the University regulations and I accept that my examination will not be valid otherwise.

Question	Grade	Out of
1A		15
1B		15
2		20
3		20
4		20
5		20
TOTAL		110

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1. A) A force $\vec{F} = (cx - 3x^2)\hat{x}$ acts on a particle as the particle moves along an x axis, with \vec{F} in newtons, x in meters, and c a constant. At $x = 0$, the particle's kinetic energy is 20.0 J; at $x = 3.00$ m, it is 11.0 J. Find c .

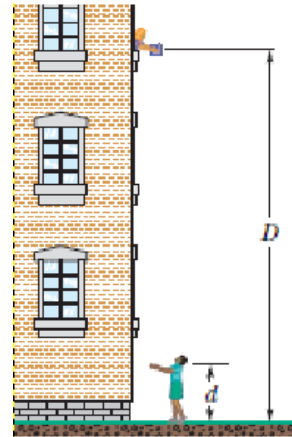
$$\begin{aligned} W &= \int_{x_i}^{x_f} \vec{F} \cdot d\vec{x} = \int_{x_i}^{x_f} F_x dx = \int_{x_i}^{x_f} (cx - 3x^2) dx \\ &= \left. \frac{c}{2}x^2 - x^3 \right|_0^3 = 4.5c - 27 \\ W &= \Delta K = K_f - K_i = 11 - 20 = -9 \text{ J} \\ \Rightarrow 4.5c - 27 &= -9 \rightarrow \boxed{c = 4 \text{ N/m}} \end{aligned}$$

B) You drop a 2.00 kg physics book to a friend who stands on the ground at distance $D=10.0$ m below. If your friend's outstretched hands are at distance $d=1.50$ m above the ground (see Figure),

- i How much work W_g does the gravitational force do on the book as it drops to her hands?
- ii What is the change ΔU in the gravitational potential energy of the book-Earth system during the drop?

If the gravitational potential energy U of that system is taken to be zero at ground level,

- iii what is U when the book is released?
- iv what is U when it reaches her hands?
- v Now take U to be 100 J at ground level and again find W_g , ΔU , U at the release point, and U at her hands.



$$\begin{aligned}
 \text{i) } W_g &= \vec{F}_g \cdot \vec{d} = mgd \cos \theta \quad \left\{ \begin{array}{l} d = 10\text{m} - 1.5\text{m} = 8.5\text{m} \\ \theta = 0^\circ \end{array} \right. \\
 &= (2\text{kg})(9.8\text{m/s}^2)8.5\text{m} \cos 0^\circ \\
 &= \underline{167\text{J}} \\
 \text{ii) } \Delta U &= U_f - U_i = mg(y_f - y_i) = (2\text{kg})(9.8\text{m/s}^2)(1.5\text{m} - 10\text{m}) \\
 &= \underline{-167\text{J}} \\
 \text{iii) } U_i &= mgy_i = (2\text{kg})(9.8\text{m/s}^2)10\text{m} = \underline{196\text{J}} \quad \text{iv) } \underline{29\text{J}} \\
 \text{v) } W_g &= \underline{167\text{J}}, \quad \Delta U = \underline{-167\text{J}}, \quad U_i = mgy_i + 100\text{J} \\
 &= \underline{296\text{J}} \\
 U_f &= mgy_f + 100\text{J} \\
 &= \underline{129\text{J}}
 \end{aligned}$$

2. A rod of length 30.0 cm has linear density (mass per length) given by $\lambda = 50.0 \frac{g}{m} + 20.0x \frac{g}{m}$ where x is the distance from one end, measured in meters.

i What is the mass of the rod?

ii How far from the $x = 0$ end is its center of mass?

2) a) $M = \int_0^{0.3} \lambda dx = \int_0^{0.3} (50 + 20x) dx$ (5)

$$M = (50x + 10x^2)_0^{0.3} = 15.9 \text{ g}$$
 (5)

b) $x_{cm} = \frac{\int x dm}{M} = \frac{1}{M} \int_0^{0.3} x dx =$

$$= \frac{1}{M} \int_0^{0.3} (50x + 20x^2) dx = \frac{1}{15.9} \left(25x^2 + \frac{20x^3}{3} \right)_0^{0.3}$$
 (5)
$$\approx 0.153 \text{ m}$$
 (5)

3. A soccer player kicks a soccer ball of mass 0.40 kg that is initially at rest. The foot of the player is in contact with the ball for 2.0×10^{-3} s, and the force of the kick is given by

$$F(t) = [(12.0 \times 10^9)t^2 - (4.0 \times 10^{12})t^3] \text{ N}$$

for $0 \leq t \leq 2.0 \times 10^{-3}$ s, where t is in seconds. Find the magnitudes of

- the impulse on the ball due to the kick,
- the average force on the ball from the player's foot during the period of contact,
- the maximum force on the ball from the player's foot during the period of contact,
- the ball's velocity immediately after it loses contact with the player's foot.

$m = 0.40 \text{ kg}$
 initially at rest
 Contact time = $2.0 \times 10^{-3} \text{ s}$

i) $J = ?$ impulse $\vec{F}_N = \frac{\vec{J}}{\Delta t}$ OR $J = \int F(t) dt$

$$J = \int_0^{2 \times 10^{-3}} [(12 \times 10^9)t^2 - (4 \times 10^{12})t^3] dt$$

$$\Rightarrow J = 4 \times 10^9 t^3 - 1 \times 10^{12} t^4 \Big|_0^{2 \times 10^{-3}}$$

$$= (4 \times 10^9)(8 \times 10^{-9}) - (1 \times 10^{12})(16 \times 10^{-12}) = \underline{\underline{16 \text{ N s}}}$$

ii) $F_{av} = \frac{J}{\Delta t} = \frac{16 \text{ N s}}{2.0 \times 10^{-3} \text{ s}} = \underline{\underline{8 \times 10^3 \text{ N}}}$

iii) $F_{max} = ?$ during the period of contact $\left\{ \begin{array}{l} \text{at maximum;} \\ \frac{dF(t)}{dt} = 0 \end{array} \right.$

$$\rightarrow 24 \times 10^9 t - 12 \times 10^{12} t^2 = 0 \rightarrow t = 2 \times 10^{-3} \text{ s}$$

$$\Rightarrow F(t = 2 \times 10^{-3} \text{ s}) = F_{max} = (12 \times 10^9)(2 \times 10^{-3})^2 - (4 \times 10^{12})(2 \times 10^{-3})^3 = \underline{\underline{16 \times 10^3 \text{ N}}}$$

iv) $v = ?$ when contact is lost

$$\Delta \vec{p} = \vec{p}_f - \vec{p}_i = m\vec{v}_f - m\vec{v}_i \rightarrow \Delta p = m v = J \rightarrow v = \frac{J}{m} = \frac{16 \text{ N s}}{0.40 \text{ kg}} = \underline{\underline{40 \text{ m/s}}}$$

4. The angular position of a point on a rotating wheel is given by $\theta(t) = 2.0 + 4.0t^2 + 2.0t^3$, where θ is in radians and t is in seconds. At $t = 0$,
- what is the point's angular position?
 - what is its angular velocity?
 - what is its angular velocity at $t = 4.0$ s?
 - Calculate its angular acceleration at $t = 2.0$ s.
 - Is its angular acceleration constant?

3) a) at $t=0$, $\theta_0 = 2.0$ rad. (2)

b) $\omega = \frac{d\theta}{dt} = 8t + 6t^2$ (4)
at $t=0$ to obtain $\omega_0 = 0$

c) For $t=4$ s, $\omega_4 = (8)(4) + (6)(4)^2 = 128$ rad/s (3)

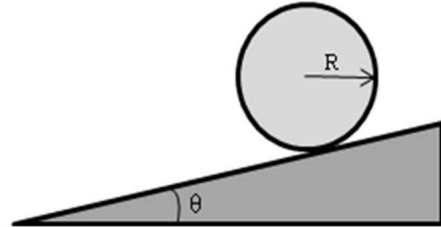
d) $\alpha = \frac{d\omega}{dt} = 8 + 12t$ (3)
 $\alpha_2 = 8 + (12)(2) = 32$ rad/s² at $t=2$ s. (1)

e) Angular acceleration depends on time;
it is not constant. (2)

5. A uniform ball, of mass $M = 6.0 \text{ kg}$ and radius R , rolls smoothly from rest down a ramp at angle $\theta = 30.0^\circ$ (see Figure, $I = \frac{2}{5}MR^2$)

i) The ball descends a vertical height $h = 1.20 \text{ m}$ to reach the bottom of the ramp. What is its speed at the bottom?

ii) What are the magnitude and direction of the frictional force (f_s) on the ball as it rolls down the ramp?



$M = 6 \text{ kg}$
 $\theta = 30^\circ$
 $I = \frac{2}{5}MR^2$
 $h = 1.2 \text{ m}$

i) Mechanical Energy is conserved for the ball-earth system
 $\rightarrow F_N$ & f_s does not work

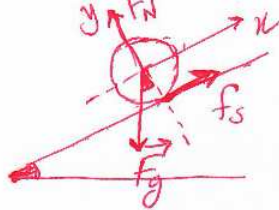
$$K_f + U_f = K_i + U_i \rightarrow K_f = U_i \rightarrow \frac{1}{2}I_{\text{com}}\omega^2 + \frac{1}{2}Mv_{\text{com}}^2 = Mgh$$

$$\rightarrow \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v_{\text{com}}}{R}\right)^2 + \frac{1}{2}Mv_{\text{com}}^2 = Mgh$$

$$\frac{7}{10}Mv_{\text{com}}^2 = Mgh \rightarrow v = \sqrt{\frac{10}{7}gh}$$

$$v = \sqrt{\frac{10}{7}(9.8 \text{ m/s}^2)(1.2 \text{ m})} = \underline{4.1 \text{ m/s}}$$

ii)



Newton's 2nd law in x -direction
 $-Mg \sin 30^\circ + f_s = Ma_{\text{com},x}$

Newton's 2nd law in angular form
 $\tau_{\text{net}} = I_{\text{com}}\alpha \rightarrow f_s R = \frac{2}{5}MR^2\alpha$

$a_{\text{com},x} = \alpha R \rightarrow \frac{5}{2}f_s = -Ma_{\text{com},x}$

$$\Rightarrow -Mg \sin 30^\circ + f_s = -\frac{5}{2}f_s \rightarrow f_s = \frac{2}{7}Mg \sin 30^\circ$$

$$= \frac{2}{7}(6 \text{ kg})(9.8 \text{ m/s}^2) \frac{1}{2} = \underline{8.4 \text{ N}}$$