İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy101 Physics I
Final Examination
January 19, 2024 08:30 – 10:00
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

INSTRUCTOR:

DURATION: 90 minutes

◇ **Answer all the questions.**
◇ **Write the solutions explicitly and clearly. Use the physical terminology.**
◇ **You are allowed to use Formulae Sheet.**
◇ **Calculator is allowed.**
◇ **You are not allowed to use any other electronic equipment in the exam.**
◇ **I declare hereby that I fulfilled the requirements for the attendance according to the University regulations and I accept that my examination will not be valid otherwise.**

Question	Grade	Out of
1A		15
1B		20
2		15
3		15
4		15
5		20
TOTAL		100

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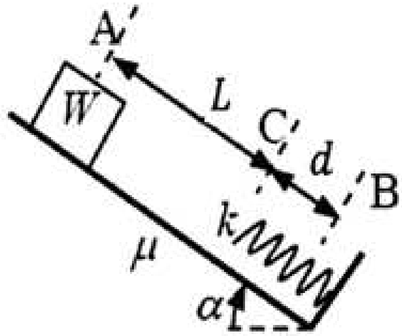
1. A) A block, initially at rest, of weight $W = 10.0\ N$ is released from the point A moving down an inclined plane with angle $\alpha = 53^o$. It meets a spring with a spring constant $k = 500.0\ N/m$ at the point C and compresses the spring a distance $d = 10.0\ cm$ stopping at the point B momentarily. The surface has a coefficient of friction, μ between points A and C. There is no friction between C and B. The distance between A and C, is $L = 50.0\ cm$. Take $g = 9.8\ m/s^2$.



a Calculate μ by using the work-kinetic energy theorem.

b After stopping at B momentarily, the block bounces back and goes up on the incline some distance and stops. What is this stopping distance measured from the point C?

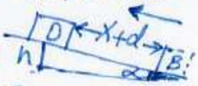
$W = \Delta K$: Work – Kinetic Energy Theorem

i) $\Delta K = 0$ since $v_A = v_B = \phi$ (1) & $\sin\alpha = \dfrac{H}{L+d}$ (1)

$\Rightarrow W = 0 = mgH - f_k L - \frac{1}{2}kd^2$ (3) $= mgH - \mu mg\cos\alpha L - \frac{1}{2}kd^2$

$\Rightarrow \mu_k = \dfrac{mg(L+d)\sin\alpha - \frac{1}{2}kd^2}{mgL\cos\alpha} = \dfrac{(10N)(0.5m+0.1m)\sin 53 - \frac{1}{2}500\frac{N}{m}(0.1m)^2}{(10N)(0.5m)\cos 53} = \boxed{0.76}$ (1) (1)

ii) $\Delta K = 0$ since $v_B = v_D = \phi$ (1) & $\sin\alpha = \dfrac{h}{X+d}$ (1)

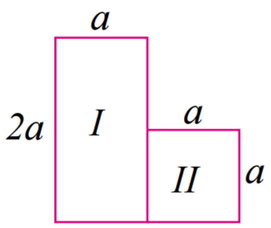


$\Rightarrow W = 0 = -mgh - f_k X + \frac{1}{2}kd^2 = -mgh - \mu mg\cos\alpha X + \frac{1}{2}kd^2$ (3)

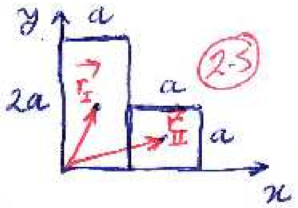
$\Rightarrow -mg(X+d)\sin\alpha - \mu mg X\cos\alpha + \frac{1}{2}kd^2 = 0 \Rightarrow X = \dfrac{\frac{1}{2}kd^2 - mgd\sin\alpha}{\mu_k mg\cos\alpha + mg\sin\alpha}$

$\Rightarrow X = \dfrac{\frac{1}{2}500\frac{N}{m}(0.1m)^2 - (10N)(0.1m)\sin 53}{(0.76)(10N)\cos 53 + (10N)\sin 53}$ (2) $= \boxed{0.135m}$ (1)

B) Consider 'L' shaped plate with uniform mass per unit area σ.



Find the center of mass, $\vec{r}_{COM}$ in unit vector notation.



I: $A_I = 2a^2$ & $\sigma = m_I/A_I \rightsquigarrow m_I = \sigma 2a^2$ 2.5

$r_I = \frac{a}{2}\hat{i} + a\hat{j}$ 2.5

II: $A_{II} = a^2$ & $\sigma = m_{II}/A_{II} \rightsquigarrow m_{II} = \sigma a^2$ 2.5

$r_{II} = \frac{3}{2}a\hat{i} + \frac{a}{2}\hat{j}$ 2.5

$$\vec{r}_{com} = \frac{\sum \vec{r}_i m_i}{\sum m_i} = \frac{(a/2\,\hat{i} + a\hat{j})\cancel{\sigma} 2\cancel{a^2} + (3/2\,a\hat{i} + a/2\,\hat{j})\cancel{\sigma}\cancel{a^2}}{\cancel{\sigma} 2\cancel{a^2} + \cancel{\sigma}\cancel{a^2}}$$ 2.5 2.5

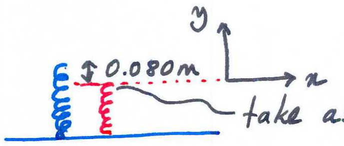
$$= \frac{5}{6}a(\hat{i} + \hat{j})$$ 2.5

2. A 5.0 g marble is fired vertically upward using a spring gun. The spring must be compressed 8.0 cm, if the marble is to just reach a target 20 m above the marble's position on the compressed spring.

   i What is the change (ΔU_g) in the gravitational potential energy of the marble-Earth system during the 20 m ascent?

   ii What is the change (ΔU_s) in the elastic potential energy of the spring during its launch of the marble?

   iii What is the spring constant of the spring?



take as reference point

i) $\Delta U_g = U_{g,f} - \cancel{U_{g,i}}^{\,0} = mgh = (5.0\times10^{-3}\,kg)(9.8\,m/s^2)(20\,m)\ \boxed{=0.98\,J}$

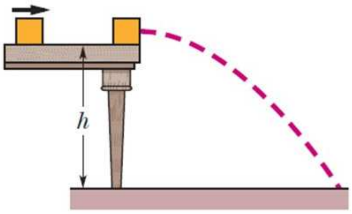
ii) Conservation of mechanical energy, $\Delta E_{mech} = 0 = \Delta U + \Delta K$

$\Delta U_g + \underbrace{\Delta U_s}_{\text{change in spring's elastic potential}} + \underbrace{\cancel{\Delta K}^{\,0}}_{\text{Kinetic Energy}} = 0 \quad \Big\} \quad \Delta U_s = -\Delta U_g \boxed{=-0.98\,J}$

iii) $\Delta U_s = -0.98\,J = \cancel{U_{s,i}}^{\,0} - U_{s,f} \Rightarrow U_{s,f} = 0.98\,J = \frac{1}{2}kx^2$

$\rightarrow k = \frac{2U_{s,f}}{x^2} = \frac{2(0.98\,J)}{(0.080\,m)^2} \boxed{=3.1\times10^{2}\,\frac{N}{m}} = 3.1\,N/cm$

3. A 3.2 kg box slides on a horizontal frictionless table and collides with a 2.0 kg box initially at rest on the edge of the table, at height $h = 0.40\,m$. The speed of the 3.2 kg box is 3.0 m/s just before the collision.



If the two boxes stick together because of packing tape on their sides, what is their kinetic energy just before they strike the floor?

Stick Together $\rightsquigarrow$ completely inelastic collision

$\Rightarrow$ conservation of momentum

$\rightsquigarrow m_1 v_{1i} + m_2 \overset{0}{\cancel{v_{2i}}} = (m_1+m_2)\,v$ 3 $\vec{P_i} = \vec{P_f}$ in x-direction

$\rightsquigarrow v = \dfrac{m_1}{m_1+m_2} v_{1i} = \dfrac{3.2\,kg}{3.2\,kg+2\,kg}\, 3\,m/s = 1.8\,m/s$ 1 +x-direction 3

That is the velocity after impact. Next, we have projectile motion. Now, Conservation of Mechanical Energy

$K_i + U_i = K_f + \underbrace{\cancel{U_f}}_{0 \text{ at ground}}$ 3 $\Big\}\; m = m_1 + m_2$

$\frac{1}{2} m v^2 + mgh = K_f$ 3 $\rightsquigarrow KE = \frac{1}{2}(5.2\,kg)(1.8\,m/s)^2 + (5.2\,kg)(9.8\,m/s^2)\,0.40\,m$

$= \underline{28.8\ J}$ 1 1

4. The angular position of a point on a rotating wheel is given by $\theta(t) = 2.0 + 4.0t^2 + 2.0t^3$, where θ is in radians and t is in seconds. At t = 0,

    i what is the point's angular position?

    ii what is its angular velocity?

    iii what is its angular velocity at t = 4.0 s?

    iv Calculate its angular acceleration at t = 2.0 s.

    v Is its angular acceleration constant? Why?

$\Theta(t) = 2t^3 + 4t^2 + 2$

i) at $t=0$, $\Theta(\phi) = 2$ rad 2

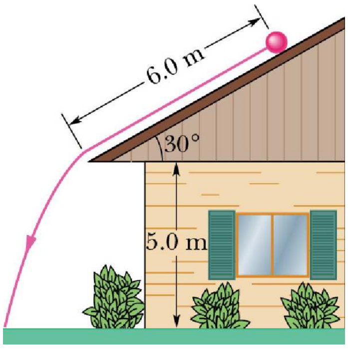
ii) $\omega(t) = \frac{d\Theta(t)}{dt} = 6t^2 + 8t$ 2, at $t=0$, $\omega(\phi) = \phi$ 2

iii) $t=4 \rightsquigarrow \omega(t=4) = 6*4^2 + 8*4 = 128$ rad/s 3

iv) $\alpha(t) = \frac{d\omega(t)}{dt} = 12t + 8$ 2, at $t=2$, $\alpha(t=2) = 32$ rad/s^2 2

v) α has time dependency $\Rightarrow$ NOT constant 2

5. In Figure, a solid cylinder of radius 10 cm and mass 12 kg starts from rest and rolls without slipping a distance L = 6.0 m down a roof that is inclined at the angle $\theta = 30°$. ($I = 1/2MR^2$)



i What is the angular speed of the cylinder about its center as it leaves the roof?

ii The roof's edge is at height H=5.0 m. How far horizontally from the roof's edge does the cylinder hit the level ground?

Rolling Motion. Conservation of Emech

i) $\cancel{K_i} + U_i = K_f + \cancel{U_f}$ 3 $\quad$ $h = L\sin 30$ $\quad$ $v = R\omega$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ 3

$mgL\sin 30° = \frac{1}{2}mR^2\omega^2 + \frac{1}{2}\,\frac{1}{2}mR^2\omega^2 \sim \omega^2 = \frac{4}{3}\frac{L\sin 30°\, g}{R^2}$

$\sim \omega = \frac{2}{R}\sqrt{\frac{L\sin 30°}{3}}$ 3 $= \frac{2}{0.1m}\sqrt{\frac{6 \times \sin 30° \times 9.8\,m/s^2}{3}} \simeq 63\,rad/s$ 1

ii) $v = R\omega = (0.1m)(63\,rad/s) = 6.3\,m/s = v_0$ 2

now, we have projectile motion

1 $y - y_0 = v_{0y}t + \frac{1}{2}gt^2$ $\quad$ $v_{0x} = v_0\cos 30°$, $v_{0y} = v_0\sin 30°$

$5m - \phi = v_0\sin 30\, t + \frac{1}{2}gt^2$

$\rightarrow 4.9t^2 + 3.15t - 5 = 0$ Quadratic equation 3 $\quad t_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t_{12} = \frac{-3.15 \pm \sqrt{3.15^2 - 4 \cdot 4.9 \cdot (-5)}}{2 \cdot 4.9}$

1 $t_1 = 0.74s$ $\quad$ $\cancel{t_2 = -1.38s}$

1 $\Rightarrow x - x_0 = v_0\cos 30\, t$

$= 63\,m/s\,\cos 30\; 0.74s$

$= 4.03m$ 2