İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy101 Physics I
Midterm Examination
December 01, 2023 14:30 – 16:00
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

INSTRUCTOR:

DURATION: 90 minutes

◇ **Answer all the questions.**
◇ **Write the solutions explicitly and clearly. Use the physical terminology.**
◇ **You are allowed to use Formulae Sheet.**
◇ **Calculator is allowed.**
◇ **You are not allowed to use any other electronic equipment in the exam.**

Question	Grade	Out of
1A		15
1B		10
1C		10
2		20
3		20
4		20
5		20
TOTAL		115

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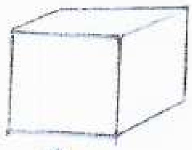
1. A) The side of a cube of metal is measured to be (1.60±0.05) cm and its mass is measured to be (30.1±0.4) g

   i Find the perimeter of one face of the cube with the uncertainty.

   ii Find the volume and uncertainty in the volume.

   iii Determine the density of the solid in kilograms per cubic meter and the uncertainty in the density.

   You should be using the correct number of significant figures in your result.



a

$a = (1.60 \pm 0.05)\,cm = (1.60 \pm 0.05)\times 10^{-2}\,m$

$m = (30.1 \pm 0.4)\,g = (30.1 \pm 0.4)\times 10^{-3}\,kg$

$\rightsquigarrow$ 3 sig figs //

i) Perimeter: $4a = 4(1.60 \pm 0.05)\times 10^{-2}\,m = \underline{(6.40 \pm 0.20)\times 10^{-2}\,m}$

ii) volume: $V = a^3 \rightsquigarrow C = A^n,\ \Delta C = C|n|\frac{\Delta A}{A}$

$\Rightarrow V = a^3 = (1.60\times 10^{-2}\,m)^3 = 4.10\times 10^{-6}\,m^3.$

$\Delta V = a^3|3|\frac{\Delta a}{a} = 4.10\times 10^{-6}|3|\frac{0.05}{1.60} = 0.38\times 10^{-6}\,m^3$

$\Rightarrow$ Volume: $\underline{(4.10 \pm 0.38)\times 10^{-6}\,m^3}$

iii) density: $\rho = m/V \rightsquigarrow C = \frac{A}{B},\ \Delta C = |C|\sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$

$\rho = \frac{m}{V} = \frac{30.1\times 10^{-3}\,kg}{4.10\times 10^{-6}\,m^3} = 7341\,kg/m^3 \sim 7.34\times 10^{3}\,kg/m^3$

$\Delta\rho = \left|\frac{30.1\times 10^{-3}\,kg}{4.10\times 10^{-6}\,m^3}\right|\sqrt{\left(\frac{0.4}{30.1}\right)^2 + \left(\frac{0.38}{4.10}\right)^2} = 687\,kg/m^3$

$\Rightarrow$ density: $\rho = \underline{(7.34 \pm 0.69)\times 10^{3}\,kg/m^3}$ Results in 3 sig figs

B) A rock is thrown vertically upward from ground level at time $t = 0\ s$. At $t = 1.5\ s$ it passes the top of a tall tower, and $1.0\ s$ later it reaches its maximum height. What is the height of the tower?

y_{max} - - - - - - - $t = 2.5\,s$

$\uparrow$

$y_0 = 0$ ———— $t = 0$

at $y_{max} \Rightarrow v_y = 0$ (2)

$$y = y_0 + v_{0y} t - \frac{1}{2} g t^2 \quad (1)$$

$$v_y = v_{0y} - g t \quad (1)$$

$$\rightarrow 0 = v_{0y} - 9.8\,m/s^2 (2.5\,s) \quad (1)$$

$$v_{0y} = 24.5\,m/s \quad (0.5)\ (0.5)$$

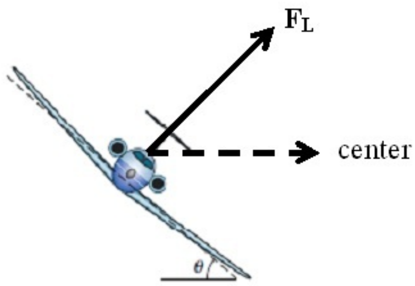
$\Rightarrow y(t = 1.5\,s)$: height of the tower

$$y(t=1.5\,s) = y_0 + v_{0y}(1.5\,s) - \frac{1}{2} g (1.5\,s)^2 \quad (3)$$

$$= 0 + (24.5\,m/s)(1.5\,s) - (4.9\,m/s^2)(1.5\,s)^2$$

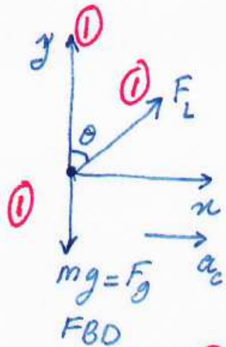
$$= 25.725\,m \quad (0.5)\ (0.5) \rightsquigarrow \boxed{\text{height} \cong 26\,m}$$

C) An airplane is flying in a horizontal circle at a speed of 480 km/h as given in the figure below. Its wings are tilted at angle $\theta = 40°$ to the horizontal. Assume that the required force is provided entirely by an "aerodynamic lift" ($\mathbf{F_L}$) that is perpendicular to the wing surface.



i What is the radius of the circle in which the plane is flying ?

ii What is the magnitude of $\mathbf{F_L}$ if the air plane has a mass of 240 $\times 10^3$ kg ?



x: $F_L \sin\theta = m a_x = m\frac{v^2}{R}$ (1)

y: $F_L \cos\theta - mg = m a_y = 0$ (2)

$v = 480\frac{km}{h}\cdot\frac{1000m}{1km}\cdot\frac{1h}{3600s} = 133\,m/s$ (1)

$\theta = 40°$

(2) $\rightarrow F_L = \frac{mg}{\cos\theta}$

$\rightarrow$ (1) $\frac{mg}{\cos\theta}\sin\theta = m\frac{v^2}{R}$

i) $R = \frac{v^2}{(\tan\theta)g} = \frac{(133\,m/s)^2}{(9.8\,m/s^2)\tan 40} = \boxed{2151\,m}$ (1)

ii) $F_L = \frac{mg}{\cos\theta} = \frac{(240\times10^3 kg)(9.8\,m/s^2)}{\cos 40} = \boxed{3.07\times10^6 N}$ (1)

2. Three vectors are given by $\vec{a} = 3.0\hat{i} + 3.0\hat{j} - 2.0\hat{k}$, $\vec{b} = -1.0\hat{i} - 4.0\hat{j} + 2.0\hat{k}$, and $\vec{c} = 2.0\hat{i} + 2.0\hat{j} + 1.0\hat{k}$. Find (a) $\vec{a}.(\vec{b} \times \vec{c})$, (b) $\vec{a}.(\vec{b} + \vec{c})$, and (c) $\vec{a} \times (\vec{b} + \vec{c})$.

Three vectors:

$\vec{a} = 3.0\hat{i} + 3.0\hat{j} - 2.0\hat{k}$

$\vec{b} = -1.0\hat{i} - 4.0\hat{j} + 2.0\hat{k}$

$c = 2.0\hat{i} + 2.0\hat{j} + 1.0\hat{k}$

i) $\vec{a}.(\vec{b} \times \vec{c}) = ?$ (scalar) 4

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -4 & 2 \\ 2 & 2 & 1 \end{vmatrix} \Rightarrow \vec{b} \times \vec{c} = (b_y c_z - b_z c_y)\hat{i} + (b_z c_x - b_x c_z)\hat{j} + (b_x c_y - b_y c_x)\hat{k}$$

$$\vec{b} \times \vec{c} = ((-4)(1) - (2)(2))\hat{i} + ((2)(2) - (-1)(1))\hat{j} + ((-1)(2) - (-4)(2))\hat{k} = -8\hat{i} + 5\hat{j} + 6\hat{k} \quad 2$$

$$\vec{a}.(\vec{b} \times \vec{c}) = (3\hat{i} + 3\hat{j} - 2\hat{k}).(-8\hat{i} + 5\hat{j} + 6\hat{k}) = -24 + 15 - 12 = \boxed{-21} \quad 4$$

ii) $\vec{a}.(\vec{b} + \vec{c}) = ?$ (scalar)

$\vec{b} + \vec{c} = 1\hat{i} - 2\hat{j} + 3\hat{k}$ 2

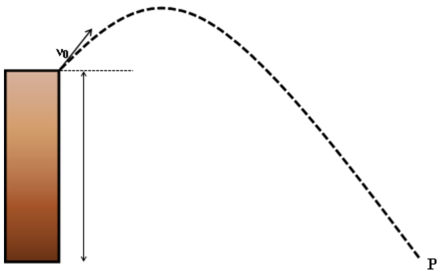
$$\vec{a}.(\vec{b} + \vec{c}) = (3\hat{i} + 3\hat{j} - 2\hat{k}).(1\hat{i} - 2\hat{j} + 3\hat{k}) = 3 - 6 - 6 = \boxed{-9} \quad 4$$

iii) $\vec{a} \times (\vec{b} + \vec{c}) = ?$ (vector)

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & -2 \\ 1 & -2 & 3 \end{vmatrix} \rightsquigarrow ((3)(3) - (-2)(-2))\hat{i} + ((-2)(1) - (3)(3))\hat{j} + ((3)(-2) - (3)(1))\hat{k}$$

$$= \boxed{5\hat{i} - 11\hat{j} - 9\hat{k}} \quad 4$$

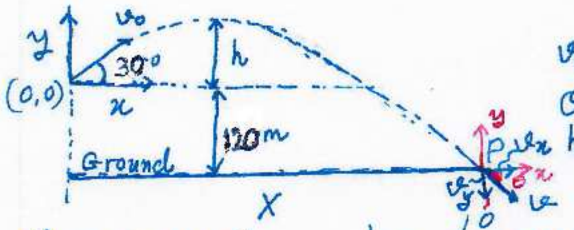
3. A projectile is shot from the edge cliff 120 m above ground level with an initial speed of 60 m/s at an angle of 30° with the horizontal.



i Determine the distance X of point P from the base of the vertical cliff.

ii What is the velocity v at point P in magnitude-angle notation and in unit-vector notation?

iii Find the maximum height reached by projectile **above ground**.



$v_0 = 60\ m/s$

$\theta = 30°$

$h_0 = 120\ m$

$v_{0x} = v_0 \cos\theta$

$v_{0y} = v_0 \sin\theta$

$\sin 30 = 0.5$

$\cos 30 = 0.87$

i) $x = x_0 + v_{0x} t$

$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$

$\Rightarrow x - x_0 = v_0 \cos\theta\, t$

$X = (60\ m/s) \cos 30\ (8.88\ s)$ (1)

$\boxed{\cong 461\ m}$ (1) (1)

$y - y_0 = v_0 \sin\theta\, t - \frac{1}{2} g t^2$ (1)

$-120\ m = (60\ m/s) \sin 30°\, t - \frac{1}{2}\, 9.8\ m/s^2\, t^2$ (1)

$\Rightarrow 4.9 t^2 - 30\ t - 120 = 0$

$t_{1,2} = \frac{-(-30) \pm \sqrt{(-30)^2 - 4(4.9)(-120)}}{2(4.9)}$ (2)

$t_1 = -2.76\ s \rightsquigarrow$ not physical $\leftarrow$ minus sign

$t_2 = 8.88\ s = t$ (1)

ii) At point P

$v_x = v_{0x} = v_0 \cos\theta$

$v_y = v_{0y} - g t = v_0 \sin\theta - g t$

$v_x = (60\ m/s) \cos 30°$ (1)

$v_y = (60\ m/s) \sin 30° - (9.8\ m/s^2)(8.88\ s)$ (1)

$v_x \cong 52\ m/s$ & $v_y = -57\ m/s$

$\vec{v} = v_x \hat{i} + v_y \hat{j} = \boxed{(52\, \hat{i} - 57\, \hat{j})\ m/s}$ (1) (1)

$\rightarrow |\vec{v}| = \sqrt{(52\ m/s)^2 + (-57\ m/s)^2}$

(1) $\boxed{= 77.16\ m/s}$ $\theta = \tan^{-1} \frac{-60.7}{53.2}$

$\boxed{\cong -48°}$ (1)

iii) $h_{max} = h_0 + h \rightsquigarrow$ What is h?

$v_y = v_{0y} - g t = 0$ at max height (2)

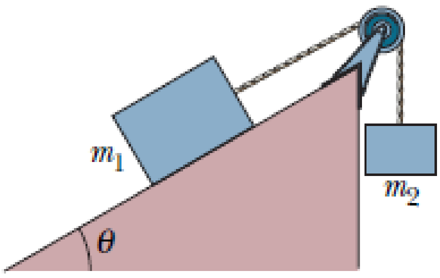
$t_f = \frac{v_{0y}}{g} \rightarrow h = v_{0y} \frac{v_{0y}}{g} - \frac{1}{2} g \left(\frac{v_{0y}}{g}\right)^2$

$h = \frac{1}{2} \frac{v_0^2 \sin^2\theta}{g}$ (2) $= \frac{1}{2} (60\ m/s)^2 \frac{\sin^2 30°}{9.8\ m/s^2}$

$\boxed{h \cong 46\ m}$ (1) $\Rightarrow h_{max} = 120\ m + 46\ m = \boxed{166\ m}$ (1)

3

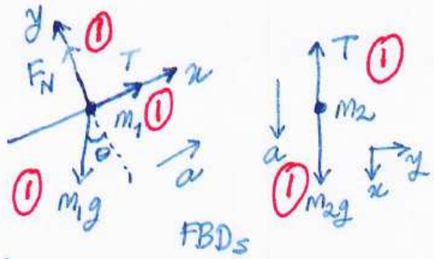
4. A block of mass $m_1 = 6\ kg$ on a frictionless plane inclined at angle $\theta = 60°$ is connected by a cord over a massless, frictionless pulley to a second block of mass $m_2 = 8\ kg$ as given in figure below.



What are

i the magnitude of the acceleration of each block?

ii the tension in the cord?



(2) m_1: x: $T - m_1 g \sin\theta = m_1 a_{x,1}$ (1) $\quad$ m_2: x: $m_2 g - T = m_2 a_{x,2}$ (3)

y: $F_N - m_1 g \cos\theta = m_1 a_{y,1}$ (2) $\quad$ y: ~ (2)

where $a_{x,1} = a_{x,2} = a$ system acceleration

$a_{y,1} = 0$

i) (2) ⇝ $F_N = m_1 g \cos\theta$

(1) ⇝ $T = m_1 a + m_1 g \sin\theta$ ⇝ (3) $m_2 g - (m_1 a + m_1 g \sin\theta) = m_2 a$

$$\Rightarrow a = \frac{m_2 g - m_1 g \sin\theta}{m_1 + m_2} \quad (2)$$

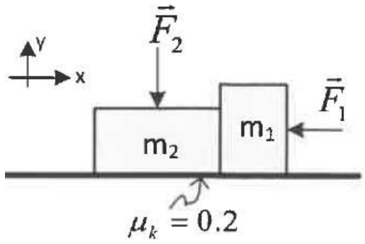
$$= \frac{(8kg)(9.8 m/s^2) - (6kg)(9.8 m/s^2)\sin 60}{6kg + 8kg} = \boxed{1.96\ m/s^2} \quad (1)\ (1)$$

ii) (3) ⇝ $T = m_2(g - a)$ (2) $= 8kg(9.8 m/s^2 - 1.96 m/s^2) = \boxed{62.7\ N}$ (2) (1)

or (1) ⇝ $T = m_1(a + g \sin\theta)$

5.

Two blocks ($m_1 = 4\ kg$ and $m_2 = 1\ kg$) on a rough horizontal surface ($\mu_k = 0.2$ for both blocks) are pushed to the left by a horizontal force $F_1 = 80\ N$. Another force $F_2 = 20\ N$ is vertically pressing the block m_2 to the surface. There is no friction between the blocks. Use the coordinate system as depicted in the figure. Take $g = 10\ m/s^2$.

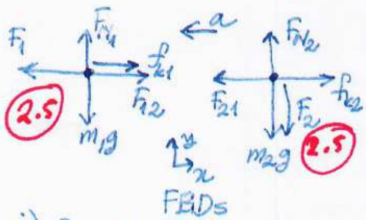


i Find the normal force **vectors** (use unit vector notation) exerted by the surface on each block

ii Find the frictional force **vectors** on each block

iii Determine the acceleration **vector** of each block.

iv Find the action-reaction force **vectors** exerted by each block on the other.



m_1

x: (1) $f_{k1} + F_{12} - F_1 = m_1 a_{x,1}$

y: (2) $F_{N_1} - m_1 g = m_1 a_{y,1}$ (3)

m_2

x: (3) $f_{k2} - F_{21} = m_2 a_{x2}$

y: (4) $F_{N_2} - F_2 - m_2 g = m_2 a_{y2}$ (3)

where $a_{x,1} = a_{x2} = a$ & $a_{y,1} = a_{y2} = \phi$

i) (2) $\rightsquigarrow F_{N_1} = m_1 g = (4\ kg)(10\ m/s^2) = 40\ N \rightsquigarrow \boxed{\vec{F}_{N_1} = 40\ N(\hat{j})}$ (1)

(4) $\rightsquigarrow F_{N_2} = F_2 + m_2 g = 20\ N + (1\ kg)(10\ m/s^2) = 30\ N \rightsquigarrow \boxed{\vec{F}_{N_2} = 30\ N(\hat{j})}$ (1)

ii) $f_{k1} = \mu_k F_{N_1} = 0.2 * 40\ N = 8\ N \rightsquigarrow \boxed{\vec{f}_{k1} = 8\ N(\hat{i})}$ (1)

$f_{k2} = \mu_k F_{N_2} = 0.2 * 30\ N = 6\ N \rightsquigarrow \boxed{\vec{f}_{k2} = 6\ N(\hat{i})}$ (1)

iii) (1) & (3) $\rightsquigarrow f_{k1} + F_{12} - F_1 = -m_1 a$ & $F_{21} = f_{k2} + m_2 a$ & $|F_{21}| = |F_{12}|$

$\rightsquigarrow f_{k1} + f_{k2} + m_2 a - F_1 = -m_1 a \Rightarrow a = -\dfrac{f_{k1} + f_{k2} - F_1}{(m_1 + m_2)} = \dfrac{8\ N + 6\ N - 80\ N}{(1\ kg + 4\ kg)} = 13.2\ m/s^2$ (2)

$\Rightarrow \boxed{\vec{a} = 13.2\ m/s^2(-\hat{i})}$ (1)

iv) Action-Reaction pair $\vec{F}_{12} = -\vec{F}_{21}$

(3) $\rightsquigarrow f_{k2} + m_2 a = F_{21} \rightsquigarrow F_{21} = 6\ N + (1\ kg)(13.2\ m/s^2) = 19.2\ N \rightsquigarrow \boxed{\begin{array}{l} F_{21} = 19.2\ N(-\hat{i}) \\ F_{12} = 19.2\ N(+\hat{i}) \end{array}}$ (1) (1)