İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy101 Physics I
Final Examination
January 19, 2024 08:30 – 10:00
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

INSTRUCTOR:

DURATION: 90 minutes

◇ **Answer all the questions.**
◇ **Write the solutions explicitly and clearly. Use the physical terminology.**
◇ **You are allowed to use Formulae Sheet.**
◇ **Calculator is allowed.**
◇ **You are not allowed to use any other electronic equipment in the exam.**
◇ **I declare hereby that I fulfilled the requirements for the attendance according to the University regulations and I accept that my examination will not be valid otherwise.**

Question	Grade	Out of
1A		15
1B		20
2		15
3		15
4		15
5		20
TOTAL		100

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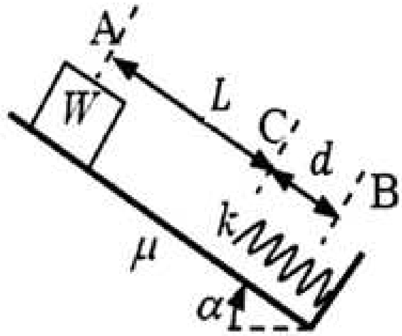
1. A) A block, initially at rest, of weight $W = 10.0\ N$ is released from the point A moving down an inclined plane with angle $\alpha = 53^o$. It meets a spring with a spring constant $k = 500.0\ N/m$ at the point C and compresses the spring a distance $d = 10.0\ cm$ stopping at the point B momentarily. The surface has a coefficient of friction, μ between points A and C. There is no friction between C and B. The distance between A and C, is $L = 50.0\ cm$. Take $g = 9.8\ m/s^2$.

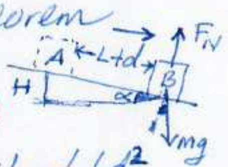


a Calculate μ by using the work-kinetic energy theorem.

b After stopping at B momentarily, the block bounces back and goes up on the incline some distance and stops. What is this stopping distance measured from the point C?

$W = \Delta K$: Work – Kinetic Energy Theorem

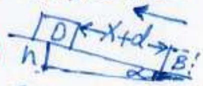
i) $\Delta K = 0$ since $v_A = v_B = \phi$ (1) & $\sin\alpha = \dfrac{H}{L+d}$ (1)



$\Rightarrow W = 0 = mgH - f_k L - \frac{1}{2}kd^2$ (3) $= mgH - \mu mg\cos\alpha L - \frac{1}{2}kd^2$

$\Rightarrow \mu_k = \dfrac{mg(L+d)\sin\alpha - \frac{1}{2}kd^2}{mgL\cos\alpha} = \dfrac{(10N)(0.5m+0.1m)\sin 53 - \frac{1}{2}500\frac{N}{m}(0.1m)^2}{(10N)(0.5m)\cos 53} = \boxed{0.76}$ (1) (1)

ii) $\Delta K = 0$ since $v_B = v_D = \phi$ (1) & $\sin\alpha = \dfrac{h}{X+d}$ (1)

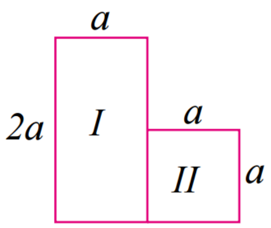


$\Rightarrow W = 0 = -mgh - f_k X + \frac{1}{2}kd^2 = -mgh - \mu mg\cos\alpha X + \frac{1}{2}kd^2$ (3)

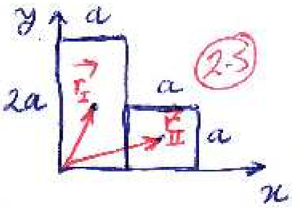
$\Rightarrow -mg(X+d)\sin\alpha - \mu mgX\cos\alpha + \frac{1}{2}kd^2 = 0 \Rightarrow X = \dfrac{\frac{1}{2}kd^2 - mgd\sin\alpha}{\mu_k mg\cos\alpha + mg\sin\alpha}$

$\Rightarrow X = \dfrac{\frac{1}{2}500\frac{N}{m}(0.1m)^2 - (10N)(0.1m)\sin 53}{(0.76)(10N)\cos 53 + (10N)\sin 53}$ (2) $= \boxed{0.135m}$ (1)

B) Consider 'L' shaped plate with uniform mass per unit area σ.



Find the center of mass, $\vec{r}_{COM}$ in unit vector notation.



2.5

I: $A_I = 2a^2$ & $\sigma = m_I/A_I \rightarrow m_I = \sigma 2a^2$ 2.5

$r_I = \frac{a}{2}\hat{i} + a\hat{j}$ 2.5

II: $A_{II} = a^2$ & $\sigma = m_{II}/A_{II} \rightsquigarrow m_{II} = \sigma a^2$ 2.5

$r_{II} = \frac{3}{2}a\hat{i} + \frac{a}{2}\hat{j}$ 2.5

$$\vec{r}_{com} = \frac{\sum \vec{r}_i m_i}{\sum m_i} = \frac{(a/2\,\hat{i} + a\hat{j})\cancel{\sigma} 2\cancel{a^2} + (3/2\,a\hat{i} + a/2\,\hat{j})\cancel{\sigma}\cancel{a^2}}{\cancel{\sigma}2\cancel{a^2} + \cancel{\sigma}\cancel{a^2}}$$ 2.5 2.5

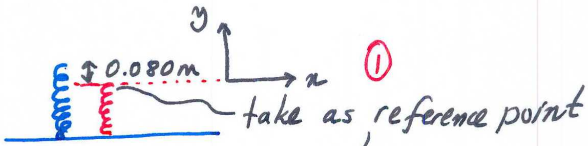
$$= \frac{5}{6}a(\hat{i} + \hat{j})$$ 2.5

2. A 5.0 g marble is fired vertically upward using a spring gun. The spring must be compressed 8.0 cm, if the marble is to just reach a target 20 m above the marble's position on the compressed spring.

   i What is the change (ΔU_g) in the gravitational potential energy of the marble-Earth system during the 20 m ascent?

   ii What is the change (ΔU_s) in the elastic potential energy of the spring during its launch of the marble?

   iii What is the spring constant of the spring?



(1)

i) $\Delta U_g = U_{g,f} - \cancel{U_{g,i}}^{\,0} = mgh = (5.0\times10^{-3}\,kg)(9.8\,m/s^2)(20\,m)$ (1) (1) (1) $\boxed{=0.98\,J}$ (1)

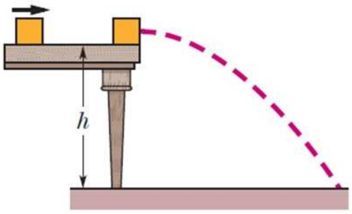
ii) Conservation of mechanical energy, $\Delta E_{mech} = 0 = \Delta U + \Delta K$ (1)

$\Delta U_g + \underbrace{\Delta U_s}_{\text{change in spring's elastic potential}} + \underbrace{\cancel{\Delta K}}_{0\ \text{Kinetic Energy}} = 0$ (1) (1) $\Bigg\}\ \Delta U_s = -\Delta U_g \boxed{=-0.98\,J}$ (1) (1)

iii) $\Delta U_s = -0.98\,J = \cancel{U_{s,i}}^{\,0} - U_{s,f} \Rightarrow U_{s,f} = 0.98\,J = \frac{1}{2}kx^2$ (1) (1)

$\rightarrow k = \frac{2U_{s,f}}{x^2} = \frac{2(0.98\,J)}{(0.080\,m)^2} \boxed{= 3.1\times10^2\,\frac{N}{m}} = 3.1\,N/cm$ (1) (1) (1)

3. A 3.2 kg box slides on a horizontal frictionless table and collides with a 2.0 kg box initially at rest on the edge of the table, at height $h = 0.40\,m$. The speed of the 3.2 kg box is 3.0 m/s just before the collision.



If the two boxes stick together because of packing tape on their sides, what is their kinetic energy just before they strike the floor?

Stick Together ⇝ completely inelastic collision

⇒ conservation of momentum

$$\rightsquigarrow m_1 v_{1i} + m_2 \underbrace{\cancel{v_{2i}}}_{0} = (m_1+m_2)\, v \quad 3 \quad \vec{P_i} = \vec{P_f} \text{ in } x\text{-direction}$$

$$\rightsquigarrow v = \frac{m_1}{m_1+m_2} v_{1i} = \frac{3.2\,kg}{3.2\,kg + 2\,kg}\, 3\,m/s = 1.8\,m/s \quad 1 \quad +x\text{-direction}$$

3

That is the velocity after impact. Next, we have projectile motion. Now, Conservation of Mechanical Energy

$$K_i + U_i = K_f + \underbrace{\cancel{U_f}}_{0 \text{ at ground}} \quad 3 \qquad \Big\}\; m = m_1 + m_2$$

$$\frac{1}{2} m v^2 + mgh = K_f \quad 3 \quad \rightsquigarrow KE = \frac{1}{2}(5.2\,kg)(1.8\,m/s)^2 + (5.2\,kg)(9.8\,m/s^2)\,0.40\,m$$

$$= \underline{28.8\ J} \quad 1 \quad 1$$

4. The angular position of a point on a rotating wheel is given by $\theta(t) = 2.0 + 4.0t^2 + 2.0t^3$, where θ is in radians and t is in seconds. At t = 0,

   i what is the point's angular position?

   ii what is its angular velocity?

   iii what is its angular velocity at t = 4.0 s?

   iv Calculate its angular acceleration at t = 2.0 s.

   v Is its angular acceleration constant? Why?

$\theta(t) = 2t^3 + 4t^2 + 2$

i) at $t=0$, $\theta(\phi) = 2$ rad 2

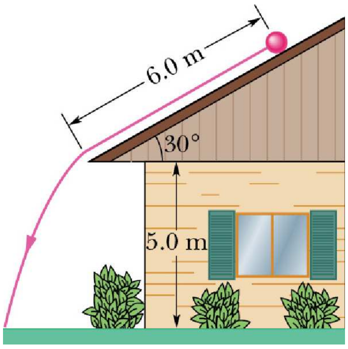
ii) $\omega(t) = \frac{d\theta(t)}{dt} = 6t^2 + 8t$ 2, at $t=0$, $\omega(\phi) = \phi$ 2

iii) $t=4 \rightsquigarrow \omega(t=4) = 6 * 4^2 + 8 * 4 = 128$ rad/s 3

iv) $\alpha(t) = \frac{d\omega(t)}{dt} = 12t + 8$ 2, at $t=2$, $\alpha(t=2) = 32$ rad/s^2 2

v) α has time dependency $\Rightarrow$ NOT constant 2

5. In Figure, a solid cylinder of radius 10 cm and mass 12 kg starts from rest and rolls without slipping a distance L = 6.0 m down a roof that is inclined at the angle $\theta = 30°$. ($I = 1/2MR^2$)



i What is the angular speed of the cylinder about its center as it leaves the roof?

ii The roof's edge is at height H=5.0 m. How far horizontally from the roof's edge does the cylinder hit the level ground?

Rolling Motion. Conservation of Emech

i) $\cancel{K_i} + U_i = K_f + \cancel{U_f}$ 3

$h = L \sin 30$

$v = R\omega$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ 3

$mgL \sin 30° = \frac{1}{2}mR^2\omega^2 + \frac{1}{2}\cdot\frac{1}{2}mR^2\omega^2 \sim \omega^2 = \frac{4}{3}\frac{L \sin 30° g}{R^2}$

$\sim \omega = \frac{2}{R}\sqrt{\frac{L \sin 30°}{3}}$ 3 $= \frac{2}{0.1m}\sqrt{\frac{6 * \sin 30° * 9.8 m/s^2}{3}} \cong 63 rad/s$ 1

ii)

$v = R\omega = (0.1m)(63 rad/s) = 6.3 m/s = v_0$ 2

now, we have projectile motion

1 $y - y_0 = v_{0y}t + \frac{1}{2}gt^2$ $\left\{ \begin{array}{l} v_{0x} = v_0 \cos 30° \\ v_{0y} = v_0 \sin 30° \end{array} \right.$

$5m - 0 = v_0 \sin 30 t + \frac{1}{2}gt^2$

$\rightsquigarrow 4.9t^2 + 3.15t - 5 = 0$ { Quadratic equation 3 $t_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t_{12} = \frac{-3.15 \pm \sqrt{3.15^2 - 4 \cdot 4.9 \cdot (-5)}}{2 \cdot 4.9}$

1 $t_1 = 0.74s$ $\cancel{t_2 = -1.38s}$

1 $\Rightarrow x - x_0 = v_0 \cos 30 t$

$= 63 m/s \cos 30 \; 0.74s$

$= 4.03m$ 2

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1C		15
2		15
3		20
4		20
5		20
TOTAL		115

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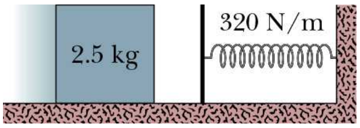
1. A) A 2.5 kg block slides into a spring with a spring constant of 320 N/m. When block stops, it has compressed the spring by 7.5 cm. The coefficient friction between the block and the horizontal surface is 0.25.



What is the block's speed just as the block reaches the spring?

Work-Energy Theorem $\underbrace{\Delta K + \Delta U}_{\Delta E_{mec}} + \Delta E_{th} = W = 0$ (no external force) 3

$\Delta K = K_f - K_i$ 2 $\left\{ K_f = 0 \text{ since } v_f = 0 \text{ \& } m = 2.5\,kg \right.$

$\Delta U = -W_s = \frac{1}{2}kx^2$ 3 $\left\{ \begin{array}{l} k = 320\,N/m \\ x = 7.5\times10^{-2}\,m \end{array} \right.$

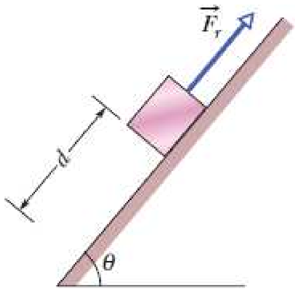
$\Delta E_{th} = f_k x = (\mu_k mg)x$ 3 $\left\{ \mu_k = 0.25 \right.$

$\rightsquigarrow -\frac{1}{2}mv_i^2 + \frac{1}{2}kx^2 + \mu_k mgx = 0$ 2

$$v_i = \frac{2}{m}\sqrt{\frac{1}{2}kx^2 + \mu_k mgx} = \frac{2}{2.5kg}\sqrt{\frac{1}{2}320\frac{N}{m}(7.5\times10^{-2}m)^2 + (0.25\cdot 2.5kg\cdot 9.8m/s^2)(7.5\times10^{-2}m)}$$ 1

$= 0.93\,m/s$ 1

B) In Figure, a block of ice slides down a frictionless ramp at angle $\theta = 50^o$ while an ice worker pulls on the block (via a rope) with a force $\vec{F}_r$ that has a magnitude of 50 N and is directed up the ramp. As the block slides through distance $d = 0.50\ m$ along the ramp, its kinetic energy increases by 80 J. How much greater would its kinetic energy have been if the rope had not been attached to the block?



$\Delta K + \Delta U + W = 0$

5 $\quad \vec{F}_r \cdot \vec{d}$

$F_r d \cos 180^\circ$

$|W| = (50N)(0.5m) = 25\ J$

$(\Delta K + 25J) + \Delta U = 0$

increase $\quad$ does not change

$\boxed{25\ J}$ 5

$h = d \sin 50$

$\Delta U = mgh$

C) A force $\vec{F} = (cx - 3x^2)\hat{x}$ acts on a particle as the particle moves along an x axis, with $\vec{F}$ in newtons, x in meters, and c a constant. At $x = 0$, the particle's kinetic energy is 20.0 J; at $x = 3.00\ m$, it is 11.0 J. Find c.

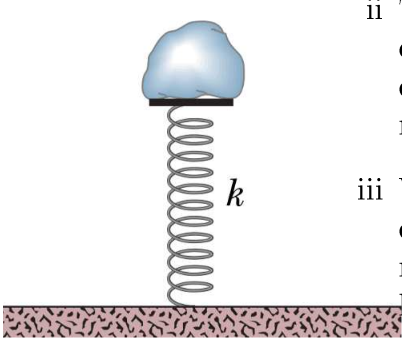
$$W = \int \vec{F} \cdot d\vec{x} = \int_{x_i}^{x_f} F_x\, dx = \int_{x_i}^{x_f} (cx - 3x^2)\, dx$$ 2 3

$$= \frac{c}{2}x^2 - x^3 \Big|_0^3 = 4.5c - 27$$ 3

$$W = \Delta K = K_f - K_i = 11 - 20 = -9J$$ 5

$$\Rightarrow 4.5c - 27 = -9 \rightsquigarrow \boxed{c = 4\ N/m}$$ 1 1

2. Figure below shows an 8.00 kg stone at rest on a spring. The spring is compressed 10.0 cm by the stone.

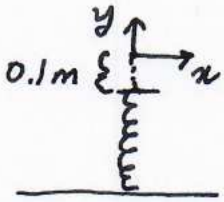


i What is the spring constant?

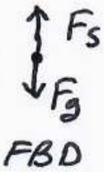
ii The stone is pushed down an additional 30.0 cm and released. What is the elastic potential energy of the compressed spring just before that release?

iii What is the change in the gravitational potential energy of the stone-Earth system when the stone moves from the release point to its maximum height?

iv What is that maximum height, measured from the release point?



$m = 8\ kg$
$y = 0.1\ m$
at rest



i) $F_s - F_g = ma = 0$ (3) at rest

$-ky - mg = 0$

$-k(-0.1m) = (8kg)(9.8 m/s^2)$

$\Rightarrow \underline{\underline{k = 784\ N/m}}$ (1) (1)

ii) pushed down 0.3 m further

$\Delta K + \Delta U = 0 \rightarrow \underbrace{\cancel{K_f}}_{0} + U_f = \underbrace{\cancel{K_i}}_{0} + U_i$ (3)

$\rightarrow U_f = \frac{1}{2}kx^2 = \frac{1}{2}(784 N/m)(-0.4m)^2 = \underline{\underline{62.7 J}}$ (1) (1) just before release

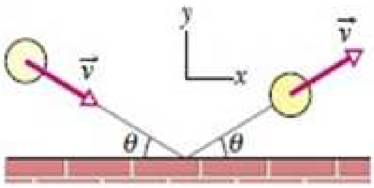
iii) at maximum height $\underbrace{\cancel{K_f}}_{0} + U_f = \underbrace{\cancel{K_i}}_{0} + U_i$

(2) $\underline{\underline{U_f = 62.7 J}}$ change in gravitational potential

iv) $U_f = mgh = 62.7 J$ (1)

$h = \frac{62.7 J}{mg} = \frac{62.7 J}{(8kg)(9.8 m/s^2)} = \underline{\underline{0.8 m}}$ (1) (1)

3. In Figure, a 300 g ball with a speed v of 6.0 m/s strikes a wall at an angle θ of 30° and then rebounds with the same speed and angle. It is in contact with the wall for 10 ms. In unit vector notation, what are



i the impulse on the ball from the wall,

ii the average force on the wall from the ball?

$\vec{v}_i = v\cos\theta\,\hat{i} - v\sin\theta\,\hat{j} = 5.2\,\hat{i} - 3.0\,\hat{j}$ 3

rebounds with same speed & angle $|\vec{v}_i| = |\vec{v}_f|$

$\vec{v}_f = v\cos\theta\,\hat{i} + v\sin\theta\,\hat{j} = 5.2\,\hat{i} + 3.0\,\hat{j}$ 3

i) $\vec{J} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i = 2\,(0.30\,kg)\,(3.0\,m/s)\,\hat{j}$

3 $= \underline{(1.8\ Ns)\,\hat{j}}$ $\underline{\text{upward}}$

2 1

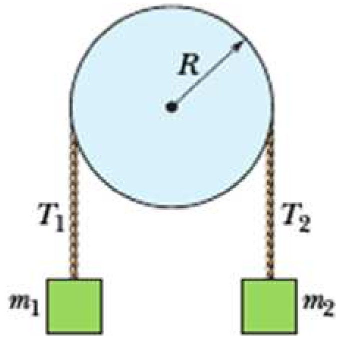
ii) $\dfrac{\vec{J}}{\Delta t} = \vec{F} = \dfrac{1.8}{0.010}\,\hat{j} = (180\,N)\,\hat{j}$ average force on the ball from the wall

3 2 1

2

Newton's third law: $\underline{(-180\,N)\,\hat{j}}$: average force on the wall from the ball

4. In Figure, block 1 has mass $m_1 = 460\ g$, block 2 has mass $m_2 = 500\ g$, and the pulley, which is mounted on a horizontal axle with negligible friction, has radius R=5.00 cm. When released from rest, block 2 falls 75.0 cm in 5.00 s without the cord slipping on the pulley.



In unit-vector notation, what are

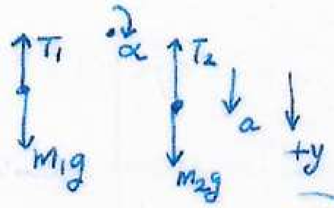
i What is the magnitude of the acceleration of the blocks?

ii What are tension T_2 and tension T_1?

iii What is the magnitude of the pulley's angular acceleration?

iv What is its rotational inertia?

$M_1 = 460 g$
$m_2 = 500 g$
pully $R = 5.00 cm$
relaset from rest
block 2 $\rightarrow$ 75.0 cm falls in 5.00 s



$a_t = \alpha R$
without the cord slipping $\rightarrow a_t = a$

i)

③ $M_2 g - T_2 = m_2 a$
$m_1 g - T_1 = -m_1 a$

Unknowns T_1, T_2, a ?
need one more equation

$y - y_0 = v_0 t + \frac{1}{2} a t^2$

$0.75 m - 0 m = \frac{1}{2} a (5 s)^2$

① $\rightarrow a = 6 \times 10^{-2} m/s^2$ ①

of the system

ii)

$T_2 = m_2 (g - a)$
$= 0.5 kg (9.8 - 6 \times 10^{-2}) m/s^2$
$= 4.87 N$ ②

$T_1 = m_1 (g + a)$
$= 0.46 kg (9.8 + 6 \times 10^{-2}) m/s^2$
$= 4.54 N$ ②

iii) $\alpha = ?$ $\alpha = \frac{a_t}{R} = \frac{a}{R}$ ② ①

$\alpha = \frac{6 \times 10^{-2} m/s^2}{5 \times 10^{-2} m} = 1.20 rad/s^2$ ① ①

iv) $I = ?$ $-\tau = I\alpha \rightarrow -(T_1 - T_2) R = I\alpha$ ②

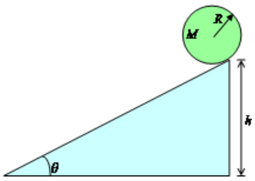
cw ②

$\boxed{T_2 - T_1 = 4.87 N - 4.54 N}$

$I = \frac{(4.87 - 4.54) N \; 0.05 m}{1.20 rad/s^2} = \boxed{1.38 \times 10^{-2} kg m^2}$

① ①

5. A solid ball of radius $R = 0.2\ m$ and mass $M = 3\ kg$ is placed at the top of a ramp of height $h = 1.2\ m$ and $\theta = 37°$. (Hint: $I = \frac{2}{5}mR^2$)



i If the ramp surface is frictionless, calculate the velocity of the ball's center of mass (v_{com}) and its angular velocity (ω) at the bottom of the ramp.

ii Calculate the minimum value of the coefficient of static friction (μ_s) that would cause smooth rolling (no slipping) of the ball down the ramp. Calculate v_{com} and ω at the bottom of the ramp for this case.

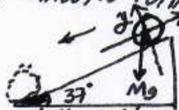
iii If the coefficient of kinetic friction (μ_k) between the ball and the ramp surface is 0.1 and it is known that the ball does not roll smoothly down the ramp (there is sliding), calculate v_{com} and ω at the bottom of the ramp.

i) frictionless → no rolling ⇒ $v_{com} = v$ & $\boxed{\omega = 0}$

$\cancel{U_f} + K_f = U_i + \cancel{K_i}$ (1) → $\frac{1}{2}mv^2 = mgh$ → $v = \sqrt{2gh}$ (1)

at the bottom ⇒ $v = \sqrt{2(9.8 m/s^2)(1.2 m)} = \boxed{4.85 m/s}$ (1)

ii) smooth rolling, $a_{com,x} = \dfrac{g\sin\theta}{1 + I_{com}/MR^2}$ & $f_s = -I_{com}\dfrac{a_{com,x}}{R^2}$ (2)



at the bottom

→ $a_{com,x} = \dfrac{(9.8 m/s^2)\sin 37}{1 + \frac{2}{5}MR^2/MR^2} = -4.21\ m/s^2$ (1)

→ $f_s = -\frac{2}{5}MR^2\dfrac{(-4.21 m/s^2)}{R^2} = 5.05 N$ (1)

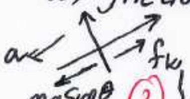
⇒ $f_s = \mu_s F_N$ (1) → $\mu_s = \dfrac{f_s}{F_N} = \dfrac{5.05 N}{mg\cos 37} = \dfrac{5.05 N}{(3 kg)(9.8 m/s^2)\cos 37} = \boxed{0.22}$ (1)

$\cancel{U_f} + K_f = U_i + \cancel{K_i}$ → $\frac{1}{2}Mv_{com}^2 + \frac{1}{2}I_{com}\omega^2 = mgh$ | $v_{com} = \omega R$

$\frac{1}{2}Mv_{com}^2 + \frac{1}{2}\frac{2}{5}MR^2\left(\frac{v_{com}}{R}\right)^2 = mgh$ (2) → $v_{com} = \sqrt{\frac{10}{7}gh} = \boxed{4.1 m/s}$ (1)

$\omega = v_{com}/R = \dfrac{4.1 m/s}{0.2 m} = \boxed{20.5 \frac{rad}{s}}$ (1)

iii) friction & motion & sliding



(2)

$f_k - mg\sin\theta = ma_{com}$

$\mu_k \cancel{m}g\cos\theta - \cancel{m}g\sin\theta = \cancel{m}a_{com}$

$a_{com} = g(\mu_k\cos\theta - \sin\theta)$

$= 9.8 m/s^2(0.1\cos 37 - \sin 37)$

(1) $= -5.12 m/s^2$ } $v_{com} = a_{com}t$

$L = \frac{1}{2}a_{com}t^2$

$-\dfrac{1.2 m}{\sin 37} = \frac{1}{2}(-5.12)t^2$

$t = 0.88 s$

⇒ $\boxed{v_{com} = 4.52 m/s}$ (1)

$f_k R = \tau = I\alpha$ ⇒ $\alpha = \dfrac{f_k R}{I}$ (1)

$\alpha = \dfrac{(0.1)\cancel{M}g\cos 37\ \cancel{R}}{2/5\ \cancel{M}R^{\cancel{2}}} = 9.78 \frac{rad}{s^2}$ (1)

$\omega = \alpha t$

$= (9.78 rad/s^2)(0.88 s)$

$= \boxed{8.6 rad/s}$ (1)

İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy101 Physics I
Final Examination Take Home Assignment
Due date: June 05, 2020 10:30
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

◇ **Answer all the questions.**
◇ **Write the solutions explicitly and clearly.**
Use the physical terminology.
◇ **I declare hereby that I fulfilled the requirements for the attendance according to the University regulations and I accept that my examination will not be valid otherwise.**

Question	Grade	Out of
TOTAL		100

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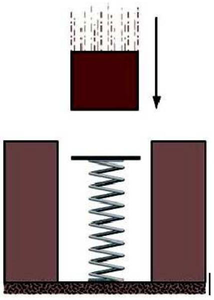
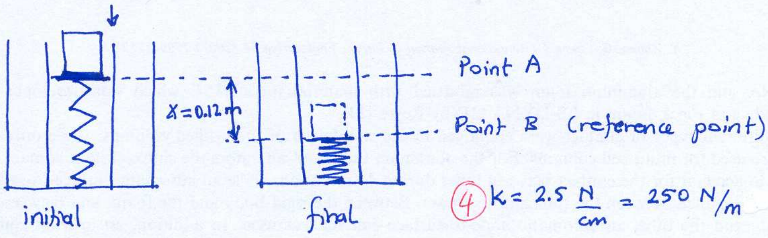
1. (Kinetic Energy and Work) A 250 g block is dropped onto a relaxed vertical spring that has a spring constant of $k = 2.5\ N/cm$. The block becomes attached to the spring and compresses the spring 12 cm before momentarily stopping.



What is the speed of the block just before it hits the spring (assume that friction is negligible)?

Answer: $v = 3.5\ m/s$



4 $k = 2.5\ \frac{N}{cm} = 250\ N/m$

Conservation of mechanical energy:

4 $U_A + K_A = U_B + K_B$ $0\ (h_B = 0)$ $0\ (v_B = 0)$

4 $mgh_A + \frac{1}{2} m v_A^2 = \cancelto{0}{mgh_B} + \frac{1}{2} kx^2 + \cancelto{0}{\frac{1}{2} m v_B^2}$

6 $(0.25)(9.8)(0.12) + \frac{1}{2}(0.25)\, v_A^2 = \frac{1}{2}(250)(0.12)^2 \Rightarrow \boxed{v_A = 3.5\ m/s}$ 2

= OR =

Kinetic energy work theorem

$\Delta K = W_{gravitational\ force} + W_{spring\ force}$ 5

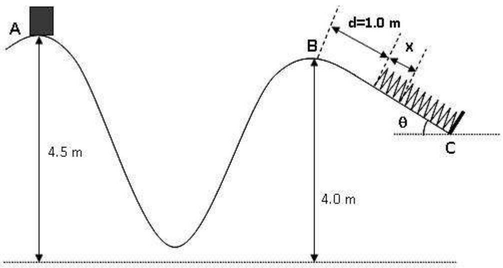
$W_g = -\Delta U = -(\cancelto{0}{U_A} - U_B) = U_B = mgh_B = (0.25)(9.8)(0.12) = 0.29\ J$ 5

$W_s = -\frac{1}{2} kx^2 = -\frac{1}{2}(250)(0.12)^2 = -1.8\ J$ 5

$\Delta K = K_B - K_A = \cancelto{0}{\frac{1}{2} m v_B^2} - \frac{1}{2} m v_A^2 = W_g + W_s$ 2

3 $-\frac{1}{2}(0.25)\, v_A^2 = (0.29 - 1.8) \Rightarrow \boxed{v_A = 3.5\ m/s}$

2. (Kinetic Energy and Work) An object having a mass of 1 kg, which is initially at rest at point **A**, starts its motion along frictionless path of **A-C** and finally, stops momentarily after it compresses the spring ($k = 100\ N/m$) by x m. Take $g = 10\ m/s^2$.

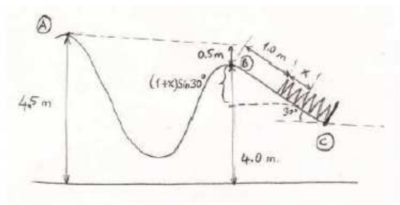


i Find the compression distance, x, if angle θ of the inclined plane is 30^o and the distance travelled on inclined plane, d, just before hitting the spring is 1.0 m.

ii The object starts moving in opposite direction after it stops momentarily. What is the speed of the object just after it starts its motion in opposite direction?

iii Determine the final position of the object at the end of motion in opposite direction along path $C - A$.

Answer: i) $x = 0.5\ m$ **ii)** $v = 5\ m/s$ **iii)** A



a) $mg\,(0.5 + (1+x)\sin 30) = \frac{1}{2} kx^2$

$0.5\,mg + 0.5\,mg\,(1+x) = \frac{1}{2}(100)x^2 \Rightarrow 100x^2 = 2mg + mgx$

$100x^2 = 20 + 10x \Rightarrow x^2 - 0.1x - 0.2 = 0$

$(x+0.4)(x-0.5) = 0$

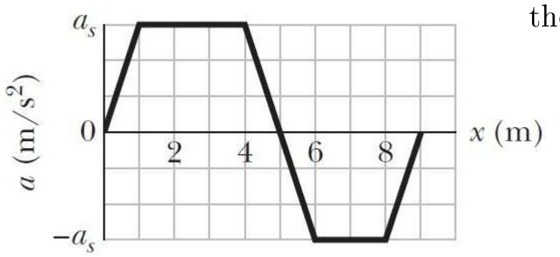
$x = 0.5\,m$

b) $\frac{1}{2} kx^2 = \frac{1}{2} m v_o^2 \Rightarrow 100\left(\frac{1}{2}\right)^2 = (1)\, v_o^2$

$\boxed{v_o = 5\ m/s}$

c) Since there is no friction, no energy is lost so that all the elastic potential energy stored in the spring will transform into potential and kinetic energies and finally, the object will be at its initial point of A.

3. (Kinetic Energy and Work) Figure gives the acceleration of a 2.00 kg particle as an applied force F_a moves it from rest along an $x-$axis from $x = 0$ to $x = 9.0\ m$. The scale of the figure's vertical axis is set by $a_s = 6.0\ m/s^2$.



How much work has the force done on the particle when the particle reaches

i x= 4.0 m,

ii x= 7.0 m,

iii What is the particle's speed and direction of travel when it reaches $x=$ 4.0 m.

Answer: i) 42 J **ii)** 30 J **iii)** 6.5 m/s $+x$ *direction*

ii What is the average and instantaneous angular acceleration for that time interval?

1.5

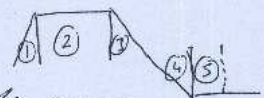
1) a) $W = Fx = (ma_s)x$, area under the curve

$0 \le x \le 4$ 1.5 $6 + 36$

~~2×6~~ $\cancel{\frac{2kg \times 6 m/s^2 (1m - 0m)}{2}} + 2kg \times 6 m/s^2 (4m - 1m)$

i) 42J

ii) $0 \le x \le 7$



6 $6 + 36 + 6 - 6 - 12 = 30J$

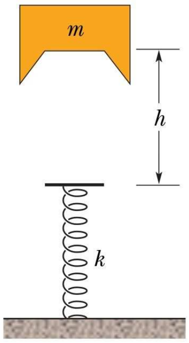
0.6 0.6 0.6 0.6 0.6

iii) $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \Delta K = W$

1 $v = 6.48 \rightarrow v = 6.5\ m/s$ $+x$ direction

$= \sqrt{42.3}$ 1 1

4. (Kinetic Energy and Work - 40%) A block of mass $m = 2.0$ kg is dropped from height $h = 40$ cm onto a spring of spring constant $k = 1960$ N/m (See the figure below).



Find the maximum distance the spring is compressed.

System: Block + spring + Earth

No external forces ⇒ Mechanical energy is conserved

$$K_i + U_i = K_f + U_f$$

$K_i = 0$, block is dropped.

(+1)

$K_f = 0$ to have maximum distance of compression.

(+5) (+7) (+5)

$$mgh = mgy + \frac{1}{2} k y^2$$

$$(2.0\,\text{kg})(9.8\,\text{m/s}^2)(0.40\,\text{m}) = (2.0\,\text{kg})(9.8\,\text{m/s}^2)\, y + \frac{1}{2}(1960\,\text{N/m})\, y^2$$

$$0.40\,\text{m} = y + \left(50\,\frac{1}{\text{m}}\right) y^2$$

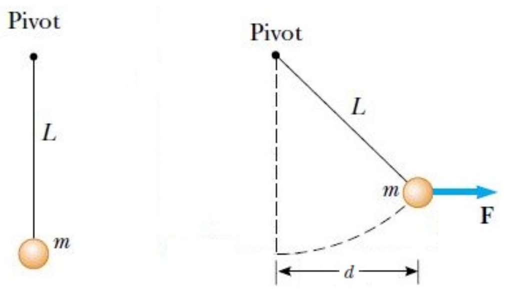
(+2)

$$0 = 500 y^2 + 10 y - 4$$

Roots: $y = -0.1$ m, $y = 0.08$ m

Since y should be negative, $\boxed{y_{max} = -0.10\,\text{m}}$

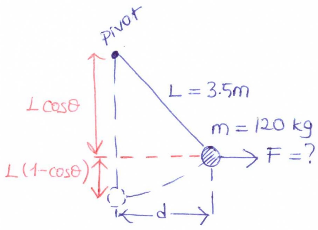
5. (Kinetic Energy and Work- 40%) A 120 kg mass hangs by vertical rope $L = 3.5\ m$ long as in Figure a. A person then displaces the mass to a position $d = 2.0\ m$ sideways from its original position, always keeping the rope taut, and holds it in this new position as in Figure b.



(a) (b)

As the mass is moved to this position, how much work is done by the person?

Answer: 738 J



$d = 2.0\,m.$

$W_{tot} = \overset{0}{\cancel{K_2}} - \overset{0}{\cancel{K_1}}$ (Since the mass starts and ends at rest)

$W_{grav.} + \underset{0}{\cancel{W_T}} + W_F = 0$

(The tension in the rope is radial and displacement is tangential so there is no component of T in the direction in the displacement during the motion and the tension in the rope does not work.)

$W_F = -W_{grav}$, $W_{grav} = -\Delta U_{grav.} = U_1 - U_2 = mg(y_1 - y_2)$

$= mg\,[0 - L(1-\cos\theta)]$, $\theta = \sin^{-1}\left(\frac{d}{L}\right) = \sin^{-1}\left(\frac{2\,m}{3.5\,m}\right)$

$\theta = 34.85^\circ$

$= -(120\,kg)(9.80\,m/s^2)\,[(3.5m)(1-\cos 34.85^\circ)]$

$= -738\,J$

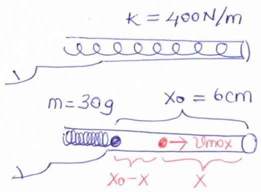
$\boxed{W_F = +738\,J}$

6. (Kinetic Energy and Work- 40%) The spring of a spring gun has force constant $k = 400\ N/55$ and negligible mass. The spring is compressed 6.00 cm, and a ball with mass 30.0 g is placed in the horizontal barrel against the compressed spring. The spring is then released, and the ball is propelled out the barrel of the gun. The barrel is 6.00 cm long, so the ball leaves the barrel at the same point that it loses contact with the spring. The gun is held so he barrel is horizontal.

   i Calculate the speed of the ball as it leaves the barrel if a constant resisting force of 6.00 N acts on the ball as it moves along the barrel.

   ii What is the greatest speed the ball have along the barrel if a constant resisting force of 6.00 N acts on the ball as it moves along the barrel?

Answer: **i)** 4.90 m/s **ii)**5.20 m/s



i) $f \leftarrow\bullet\rightarrow F_{spring}$, $f = 6.00\,N$

$W_{tot} = K_f - \cancel{K_i}^{\,0}$

$W_f + W_{Fspring} = \frac{1}{2} m v^2$, $W_f = \vec{f}\cdot\vec{x_0} = -f x_0$

$W_{Fspring} = \frac{1}{2} k \underbrace{x_i}_{x_0}{}^2 - \frac{1}{2} k \cancel{x_f}^2_{\,0}$

$-f x_0 + \frac{1}{2} k x_0^2 = \frac{1}{2} m v^2$

$-(6.00\,N)(0.06\,cm) + \frac{1}{2}(400\,N/m)(0.06\,m)^2 = \frac{1}{2}(30\times10^{-3}\,kg)\,v^2$

$\boxed{v = 4.90\ m/s}$

ii) The greatest speed occurs when the acceleration (and the net force) are zero.

$f \leftarrow\bullet\rightarrow F_{spring}$ $\quad \Sigma\vec{F} = 0 \rightarrow kx = f$

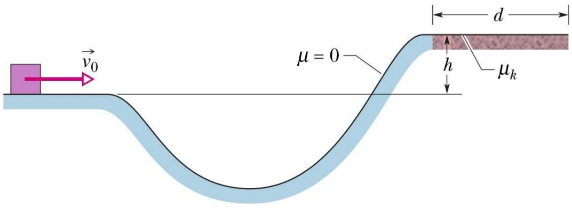
$x = f/k$

$W_f + W_{spring} = K_f - \cancel{K_i}^{\,0}$

$-f(x - x_0) + \frac{1}{2} k x_0^2 - \frac{1}{2} k x^2 = \frac{1}{2} m v_{max}^2$

$-(6.00N)\left(\frac{6.00N}{400\,N/m} - 0.06m\right) + \frac{1}{2}(400\,N/m)\left(\frac{6.00\,N}{400\,N/m}\right)^2 = \frac{1}{2}(30\times10^{-3}\,kg)\,v_{max}^2 \rightarrow \boxed{v_{max} = 5.20\ m/s}$

7. (Potential Energy and Conservation of Energy) In the figure below, a block slides along a track from one level to a higher level after passing through an intermediate valley. The track is frictionless until the block reaches the higher level. There a frictional force stops the block in a distance d.



The block's initial speed v_0 is 6.0 m/s, the height difference h is 1.1 m, and μ_k is 0.60. Find d.

Answer: $d = 1.2\ m$

System: Block + Earth + track

No external force, then

$$0 = \Delta E_{mech} + \Delta E_{th}$$

$$0 = \left(\frac{1}{2} m \cancel{v_f^2} + mgh\right) - \frac{1}{2} m v_i^2 + f_k d$$

0

(+1)

$$0 = mgh - \frac{1}{2} m v_i^2 + \mu_k (mg) d \qquad (F_N = mg \text{ at the higher level})$$

(+6) (+6) (+6)

(OR you can reach same equation by taking block + Earth as a system and having f_k as the external force;

$$W_{fric} = \Delta E_{mech}$$)

(+2)

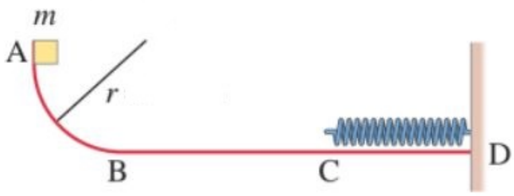
Then, solving for d;

$$d = \frac{1}{\mu_k}\left(\frac{v_i^2}{2g} - h\right) = \frac{1}{0.60}\left[\frac{(6.0\ m/s)^2}{2(9.8\ m/s^2)} - (1.1\ m)\right]$$

$$\Rightarrow \boxed{d = 1.2\ m}$$

3

8. (Potential Energy and Conservation of Energy - 40%) Consider the track shown in Figure below. The section AB is one quadrant of a circle of radius 2.0 m and is frictionless. B to C is a horizontal span 3.0 m long with a coefficient of kinetic friction $\mu_k = 0.25$. The section CD under the spring is frictionless. A block of mass 1.0 kg is released from rest at A. After sliding on the track, it compresses the spring by 0.20 m.



Determine the stiffness constant k for the spring.

Answer: 612 $N/55$

$\Delta U_{int} = -W_{other}$ ΔU_{int} : Change in internal energy.

W_{other} : The work done by forces other than conservative forces (gravitational force, elastic force, ...)

$\Delta K + \Delta U + \Delta U_{int} = 0$ (law of conservation of energy)

$(\underbrace{\cancel{K_f}}_{0} - \underbrace{\cancel{K_i}}_{0}) + (U_f - U_i) + (-W_{F_{fric.}}) = 0$, $U_f = \frac{1}{2} k x_{max}^2$

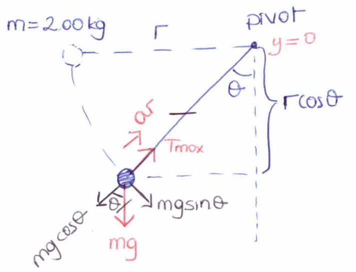
$U_i = mgr$

$W_{F_{fric.}} = \vec{F}_{fric}\,\vec{\Delta x} = \mu_k mg \underbrace{\Delta x}_{|BC|} \underbrace{\cos 180^\circ}_{-1}$

$\frac{1}{2} k x_{max}^2 - mgr + \mu_k mg|BC| = 0$

$\frac{1}{2} k (0.20m)^2 - (1.0kg)(9.80 m/s^2)(2.0m) + (0.25)(1.0kg)(9.80 m/s^2)(3.0m) \rightsquigarrow \boxed{k = 612.5\ N/m}$

9. (Potential Energy and Conservation of Energy - 40%) A 2.00 kg ball is attached to the bottom end of a length of rope with a breaking strength of 44.5 N The top end of the rope is held stationary. The ball is released from rest with the line taut and horizontal ($\theta = 90.0°$). At what angle (measured from the vertical) will the rope break?
Answer: 40.8°



$\Sigma F_r = m a_r$

$T_{max} - mg\cos\theta = m\frac{v^2}{r} \Rightarrow v^2 = \frac{r}{m}(T_{max} - mg\cos\theta)$ (at the breaking point)

$E_i = E_f$

$\underbrace{\cancel{K_i}}_{0} + \underbrace{\cancel{U_i}}_{0} = K_f + U_f$

$= \frac{1}{2} m v^2 + mgy$, $y = -r\cos\theta$

$0 = \frac{1}{2}\cancel{m}\left(\frac{\cancel{r}}{\cancel{m}}(T_{max} - mg\cos\theta)\right) - mg\cancel{r}\cos\theta$

$0 = \underbrace{T_{max}/2}_{\frac{T_{max}}{2}} - \frac{3}{2} mg\cos\theta \Rightarrow \theta = \cos^{-1}\left(\frac{T_{max}}{3mg}\right)$

$\theta = \cos^{-1}\left(\frac{44.5\ N}{3 \cdot 2.00\ kg \cdot 9.80\ m/s^2}\right)$

$\boxed{\theta = 40.8°}$

10. (Center of Mass and Linear Momentum) A system containing three objects, which is initially at rest, accelerates by the application of a net force. Masses and time dependent position vectors of objects are given as;

Find;

$m_1 = 1\ kg,\ m_2 = 3\ kg,\ m_3 = 8\ kg$

$$\vec{r_1} = (t^2 - 4)\hat{\imath} + 2\hat{\jmath}$$
$$\vec{r_2} = (4 - t)\hat{\imath} + 2\hat{\jmath}$$
$$\vec{r_3} = 2\hat{\imath} + (t - 3)\hat{\jmath}$$

i Position vector of system's center of mass, r_{com}, as a function of time,

ii Acceleration of the center of mass of the system, a_{com},

iii Net force acting on the system, F_{net}.

Answer: i) $\vec{r}_{com} = \left[(t^2 - 3t + 24)\hat{\imath} + (8t - 16)\hat{\jmath}\right]/12$ **ii)** $\vec{a}_{com} = \hat{\imath}/6\ m/s^2$ **iii)** $\vec{F}_{net} = 2\hat{\imath}\ N$

4) b) i) $\vec{r}_{com} = \dfrac{m_1\vec{r_1} + m_2\vec{r_2} + m_3\vec{r_3}}{m_1 + m_2 + m_3}$

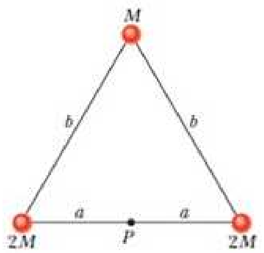
$$\vec{r}_{com} = \frac{(1)[(t^2-4)\hat{\imath} + 2\hat{\jmath}] + (3)[(4-t)\hat{\imath} + 2\hat{\jmath}] + (8)[2\hat{\imath} + (t-3)\hat{\jmath}]}{1+3+8}$$

$$\vec{r}_{com} = \frac{(t^2 - 3t + 24)\hat{\imath} + (8t - 16)\hat{\jmath}}{12} \;//$$

ii) $\vec{a}_{com} = \dfrac{d^2\vec{r}_{com}}{dt^2} = \dfrac{2\hat{\imath}}{12} = \dfrac{\hat{\imath}}{6}\ m/s^2 \;//$

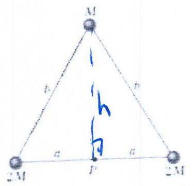
iii) $\vec{F}_{net} = M.\vec{a}_{com} \Rightarrow \vec{F}_{net} = (1+3+8).\dfrac{\hat{\imath}}{6} \Rightarrow \boxed{\vec{F}_{net} = 2\hat{\imath}\ N} \;//$

11. (Center of Mass and Linear Momentum) The rigid body shown in figure given below consists of three particles connected by **massless** rods. It is to be rotated about an axis perpendicular to its plane through point P.



If $M = 0.40\ kg$, $a = 30\ cm$, and $b = 50\ cm$, What is the rotational inertia about point P?

Answer: $I = 0.208\ kgm^2$



If $M = 0.40\ kg$, $a = 30\ cm$, and $b = 50\ cm$, What is the rotational inertia about point P?

(2) (2) (2)

$b^2 - a^2$

$$I = \sum_{i=1}^{3} m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

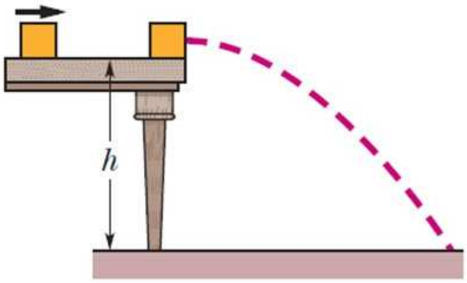
$$= 2M(a^2) + (2M)a^2 + M(h^2)$$

$$= 4Ma^2 + Mb^2 - Ma^2 = \boxed{3Ma^2 + Mb^2} \quad (2)$$

$$= M(3a^2 + b^2) = (0.4kg)\left(3.(0.3m)^2 + (0.5m)^2\right]$$

$$\boxed{I = 0.208\ kg\ m^2} \quad (2)$$

12. (Center of Mass and Linear Momentum) A 3.2 kg box slides on a horizontal frictionless table and collides with a 2.0 kg box initially at rest on the edge of the table, at height $h = 0.40\ m$. The speed of the 3.2 kg box is 3.0 m/s just before the collision.



If the two boxes stick together because of packing tape on their sides, what is their kinetic energy just before they strike the floor?

Answer: $KE = 29\ J$

Stick Together → completely inelastic collision

⇒ conservation of momentum

$$\Rightarrow m_1 v_{1i} + m_2 \overset{0}{\cancel{v_{2i}}} = (m_1 + m_2)\, v \quad 3 \quad \vec{P}_i = \vec{P}_f \text{ in } x\text{-direction}$$

$$\rightarrow v = \frac{m_1}{m_1 + m_2} v_{1i} = \frac{3.2\,kg}{3.2\,kg + 2\,kg}\, 3\,m/s = 1.8\,m/s \quad 1 \quad +x\text{-direction}$$

3

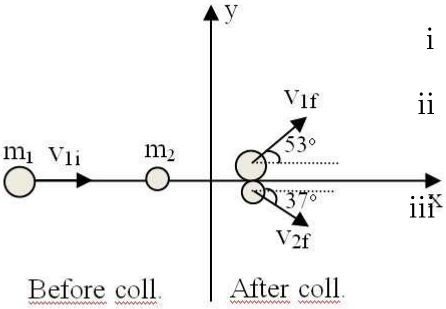
That is the velocity after impact. Next, we have projectile motion. Now, Conservation of Mechanical Energy

$$K_i + U_i = K_f + \underbrace{U_f}_{0 \text{ at ground}} \quad 3 \qquad \Big\}\ m = m_1 + m_2$$

$$\frac{1}{2} m v^2 + mgh = K_f \quad 3 \quad \rightarrow KE = \frac{1}{2}(5.2\,kg)(1.8\,m/s)^2 + (5.2\,kg)(9.8\,m/s^2)\,0.40\,m$$

$$= \underline{28.8\ J} \quad 1 \quad 1$$

13. (Center of Mass and Linear Momentum) The mass $m_1 = 1.0\ kg$ moving with a speed $v_1 = 5.0\ m/s$ on a horizontal frictionless floor collides with a mass $m_2 = 2.0\ kg$ initially at rest. After the collision, m_1 moves at a speed $v_{1f} = 3.0\ m/s$ and m_2 moves at a speed v_{2f} along the directions shown in the figure.



i Find the speed v_{2f} of m_2 after the collision.

ii Is this collision elastic or not? Prove your answer.

iii Find the velocity of the center of mass of the system after collision. Give your answer in terms of unit vector notation.

iv Find the impulse delivered to m_1 during the collision. Give your answer in terms of unit vector notation.

Answer: i) $v_{2f} = 2.0\ m/s$ **ii) Inelastic iii)** $\vec{v}_{com_f} = 5/3\ m/s\ \hat{i}$ **iv)** $\vec{J}_1 = (-3.2\hat{i} + 2.4\hat{j})\ kgm/s$

5 a) Find the speed v_{2f} of m_2 after the collision.

$\vec{P}_i = \vec{P}_f$

$m_1\vec{v}_{1i} + m_2\cancel{\vec{v}_{2i}}^{0} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

y-components

$\cancel{m_1 v_{1iy}} = m_1 v_{1fy} + m_2 v_{2fy} \Rightarrow (m_1) v_1 \sin 53 - m_2 v_2 \sin 37 \Rightarrow$

$\Rightarrow (1.0)(3.0)\frac{4}{\cancel{5}} = (2.0) v_{2f} \frac{3}{\cancel{5}}$

$v_{2f} = 2.0\ m/s$

0

5 b) Is this collision elastic or not? Prove your answer.

In elastic collision K.E is conserved

check ?

$K_i = \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 \cancel{v_{2i}^2} = \frac{1}{2}(1.0)(5.0)^2 = \frac{25}{2} J = 12.5 J$

$K_f = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 = \frac{1}{2}(1.0)(3.0)^2 + \frac{1}{2}(2.0)(2.0)^2 = 8.5 J$

$K_i > K_f$ so the collision is not elastic. It is inelastic collision

5 c) Find the velocity of the center of mass of the system after collision. Give your answer in terms of unit vector notation.

$\vec{v}_{com,f} = \frac{m_1\vec{v}_{1f} + m_2\vec{v}_{2f}}{m_1+m_2}$

$v_{comfx} = \frac{(1.0)(3.0)\cos 53 + (2.0)(2.0)\cos 37}{3}$

$v_{comfx} = \frac{3\cdot\frac{3}{5} + 4\cdot\frac{4}{5}}{3} = \frac{5}{3} m/s$

$v_{comfy} = \frac{(1.0)(3.0)\sin 53 - (2.0)(2.0)\sin 37}{3}$

$= 0$

$\vec{v}_{comf} = \frac{5}{3}\hat{i}\ m/s$

$\vec{v}_{comi} = \frac{m_1\vec{v}_{1i} + \cancel{m_2\vec{v}_{2i}}^{0}}{m_1+m_2} = \frac{(1.0)(5.0)}{3}$

$\vec{v}_{comi} = \underbrace{\frac{5}{3}\hat{i}}_{4}\ \underbrace{m/s}_{1}$ SAME since $\vec{p}$ is conserved!

5 d) Find the impulse delivered to m_1 during the collision. Give your answer in terms of unit vector notation.

$\vec{J}_1 = \vec{P}_{1f} - \vec{P}_{1i}$

$P_{1fx} = m_1 v_{1fx} = (1.0)(3.0)\underbrace{\cos 53}_{3/5} = 1.8\ kgm/s$

$P_{1fy} = m_1 v_{1fy} = (1.0)(3.0)\underbrace{\sin 53}_{4/5} = 2.4\ kgm/s$

$P_{1ix} = m_1 v_{1ix} = (1.0)(5.0) = 5.0\ kgm/s$

$P_{1iy} = 0$

$\vec{J}_1 = 2.4\hat{j} + 1.8\hat{i} - 5.0\hat{i}\ kgm/s$

$\vec{J}_1 = \underbrace{-3.2\hat{i} + 2.4\hat{j}}_{4}\ \underbrace{kgm/s}_{1}$

14. (Rotation) The angular position of a point on the rim of a rotating wheel is given by $\theta = 4.0t + 3.0t^2 + t^3$, where θ is in radians and t is in seconds. What are the angular velocities at

i t= *2.0 s* and t = *4.0s*,

ii What is the average and instantaneous angular acceleration for that time interval?

Answer: i) $w(t = 2.0\ s) = 28\ rad/s$ $w(t = 4.0\ s) = 76\ rad/s$ **ii)** $24\ rad/s^2$

$\theta = 4.0t + 3.0t^2 + t^3$

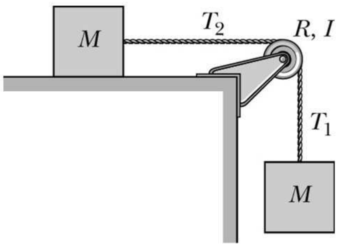
i) $\omega = \frac{d\theta}{dt} = 4.0 + 6.0t + 3t^2$ (1) $\begin{cases} t = 2.0s \rightarrow \omega(t=2s) = 28\ rad/s & (1) \\ t = 4.0s \rightarrow \omega(t=4s) = 76\ rad/s & (1) \end{cases}$

ii) $\alpha_{avg} = \frac{\omega(t=4s) - \omega(t=2s)}{4-2} = \frac{48}{2} = 24\ rad/s^2$ (1) (0.5)

$\alpha = \frac{d\omega}{dt} = 6.0 + 6.0t$ (1) $\begin{cases} \alpha(t=4s) = 30\ rad/s^2 \\ \alpha(t=2s) = 18\ rad/s^2 \end{cases}$ (0.5)

$\frac{30+18}{2} = 24$

15. (Rotation) In figure, two 6.2 kg blocks are connected by a massless string over a pulley of radius 2.40 cm and rotational inertia 7.40×10^{-4} $kg.m^2$. The string does not slip on the pulley; it is not known whether there is friction between the table and the sliding block; the pulley axis is frictionless. When this system is released from rest, the pulley turns through 0.650 rad in 91.0 ms and the acceleration of the block is constant. What are



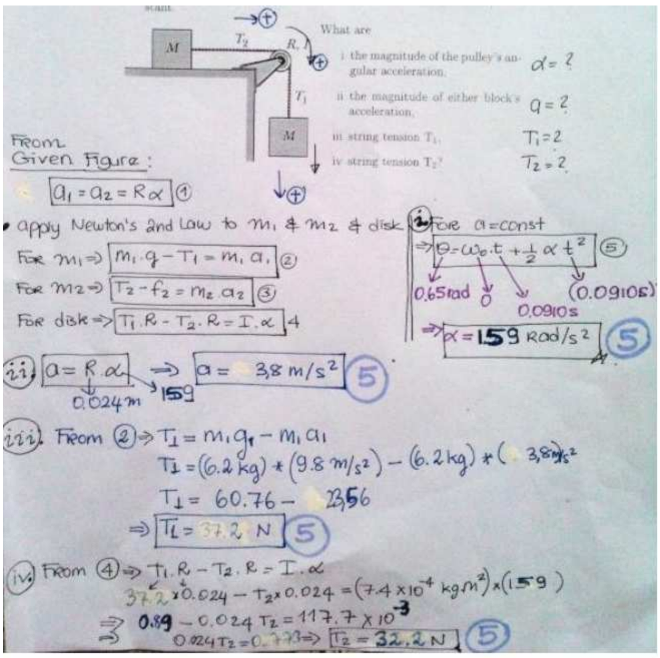
i the magnitude of the pulley's angular acceleration,

ii the magnitude of either block's acceleration,

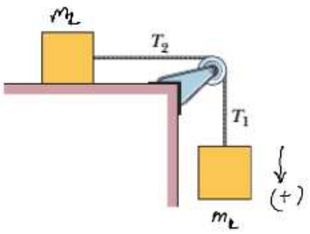
iii string tension T_1,

iv string tension T_2?

Answer: i) 157.0 rad/s **ii)** 3.8 m/s^2 **iii)** $T_1 = 37.4\ N$ **iv)** $T_2 = 32.6\ N$



tension T_2?



$T_2 - f = m_2 a_2$

$m_1 g - T_1 = m_1 a$

$(T_1 - T_2) R = I\alpha$

$\Delta\theta = 0.650$ rad

$\Delta t = t = 91.$ ms

10 Solved Problems

9 (71)

$m_1 = m_2 = 6.2$ kg $= m$

pulley radius $= 2.40$ cm

$I = 7.40 \times 10^{-4}$ kg m^2

Friction? not known if exists

Released from rest

$\Delta\theta = 0.650$ rad

$\Delta t = 91.0$ ms

acceleration: constant

i) $\alpha = ?$

$T_2 - f = m a_{2x}$ possible friction

$-T_1 + mg = m a_{1y}$ unknowns

$\tau_{net} = I \alpha_{pulley}$

$T_1 R - T_2 R = I \alpha_{pully}$

$\boxed{a_{2x} = a_{1y} = \alpha_{pully} R}$

constant acceleration

$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$

$\Delta\theta = 0.650$ rad $= \frac{1}{2}\alpha(91$ ms$)^2$

$\boxed{\alpha = 157.\ \text{rad/s}^2}$

ii) $a = \alpha R = (0.024\text{m})(157\ \text{rad/s}^2) = \boxed{3.77\ \text{m/s}^2}$

iii) $T_1 = ?$ $-T_1 + mg = ma$

$T_1 = m(a + g) = m(9.8\ \text{m/s}^2 - 3.77\ \text{m/s}^2)$

$\boxed{T_1 = 37.4\text{N}}$

iv) $T_2 R - T_1 R = I\alpha$

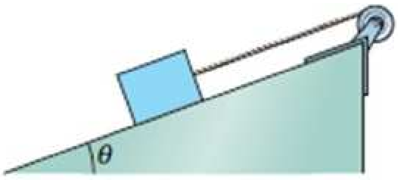
$T_2 = \frac{T_1 R + I\alpha}{R} = T_1 + \frac{I\alpha}{R} = 37.4\text{N} + \frac{7.40 \times 10^{-4}\ \text{kg m}^2 (-157\ \text{rad/s}^2)}{(0.024\text{m})} = 32.56\text{N}$?

$\boxed{T_2 < T_1}$

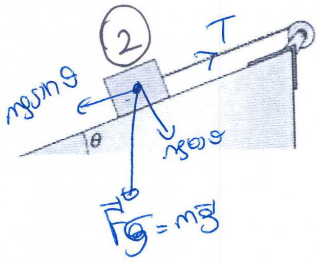
$(T_1 - T_2) = \frac{I\alpha}{R}$

$T_2 = T_1 - \frac{I\alpha}{R}$

16. (Rotation - 40%) In figure, a wheel of radius 0.20 m is mounted on a frictionless horizontal axle. A massless cord is wrapped around the wheel and attached to a 2.0 kg box that slides on a frictionless surface inclined at angle $\theta = 20°$ with the horizontal. The box accelerates down the surface at 2.0 m/s^2.



What is the rotational inertia of the wheel about the axle?



What is the rotational inertia of the wheel about the axle?

(3) $mg\sin\theta - T = m \cdot a$ $\Rightarrow$ $mg\sin\theta - \frac{I\alpha}{R} = m \cdot a$

(2) $T \cdot R = I \cdot \alpha$

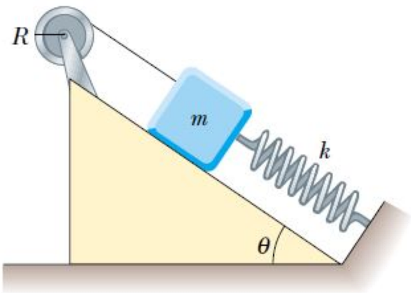
$mg\sin\theta - ma = \frac{I\alpha}{R}$

$I = \frac{(mg\sin\theta - ma)R}{\alpha}$ (2) $\boxed{a = \alpha \cdot R}$ $\Rightarrow$ $\alpha = a/R$

(2) $I = \frac{(mg\sin\theta - ma)R}{a/R}$

$I = \frac{(2kg)\cdot(9.8m/s^2)\sin 20 - (2kg)\cdot(2m/s^2)}{2m/s^2 / (0.2m)^2}$ (2) $= 0.054 kg\,m^2$ (1)

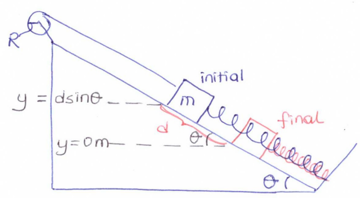
17. (Rotation - 40%) The reel shown in Figure has radius $R = 0.300\ m$ and moment of inertia $I = 1.00\ kg.m^2$. One end of the block of mass $m = 0.500\ kg$ is connected to a spring of force constant $k = 50.0\ N/55$, and the other end is fastened to a cord wrapped around the reel. The reel axle and the plane having an inclination of $\theta = 37°$ are frictionless. The reel is wound counterclockwise so that the spring stretches a distance $d = 0.200\ m$ from its unstretched position and is then released from rest.



Find the angular speed of the reel when the spring is again unstretched.

Answer: $1.74\ rad/s$

$R = 0.300m$
$I = 1.00\ kgm^2$
$m = 0.500\ kg$
$k = 50.0\ N/m$
$d = 0.200m$
$\theta = 37°$



$$E_i = E_f$$

$$\underset{0}{\cancel{K_i}} + U_i = K_f + U_f$$

$$mg(d\sin\theta) = \frac{1}{2}mv^2 + \cancel{\frac{1}{2}kd^2} + \frac{1}{2}I\omega^2 + \frac{1}{2}kx^2, \quad v = R\omega,\ x = d$$

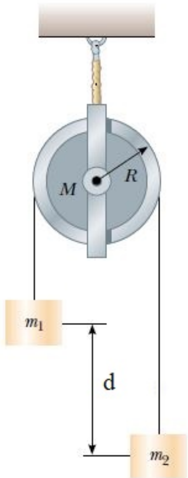
$$mg\,d\sin\theta = \frac{1}{2}mR^2\omega^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}kd^2$$

$$\omega = \left(\frac{2mgd\sin\theta + kd^2}{I + mR^2}\right)^{1/2}$$

$$\omega = \left(\frac{2\cdot(0.500kg)(9.80m/s^2)(0.200m)\sin 37° + (50.0N/m)(0.200m)^2}{(1.00kg\cdot m^2) + (0.500kg)\cdot(0.300m)^2}\right)^{1/2}$$

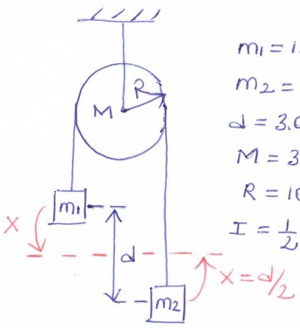
$$\boxed{\omega = 1.74\ rad/s}$$

18. (Rotation - 40%) A $m_1 = 15.0\ kg$ object and a $m_2 = 10.0\ kg$ object are suspended, joined by a cord that passes over a pulley with a radius of $R = 10.0\ cm$ and a mass of $M = 3.00\ kg$ as seen in the Figure.



The cord has a negligible mass and does not slip on the pulley. The pulley rotates on its axis without friction. The objects start from rest $d = 3.00\ m$ apart. Treat the pulley as a uniform disk, and determine the speeds of the two objects as they pass each other.
Hint: $I_{CM} = \frac{1}{2}MR^2$

Answer: $2.36\ m/s$



$m_1 = 15.0\ kg$
$m_2 = 10.0\ kg$
$d = 3.00\ m$
$M = 3.00\ kg$
$R = 10.0\ cm$
$I = \frac{1}{2}MR^2$

$$\Delta K + \Delta U = 0$$

$$\Big(\underbrace{\tfrac{1}{2}(m_1+m_2)v^2 + \tfrac{1}{2}I\omega^2}_{K_f} - \underbrace{0}_{K_i}\Big) + \Big(\underbrace{-m_1 g\, d/2}_{\Delta U_{grav.1}} + \underbrace{m_2 g\, d/2}_{\Delta U_{grav.2}}\Big) = 0$$

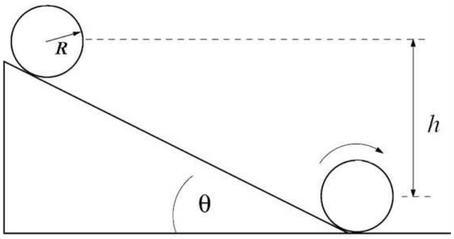
$$\omega = v/R$$

$$\frac{1}{\cancel{2}}(m_1+m_2)v^2 + \frac{1}{\cancel{2}}\left(\frac{1}{2}M\cancel{R^2}\right)\frac{v^2}{\cancel{R^2}} = g\frac{d}{\cancel{2}}(m_1-m_2)$$

$$v = \left(\frac{g\,d\,(m_1-m_2)}{(m_1+m_2) + M/2}\right)^{1/2}$$

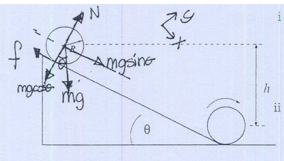
$$= \left(\frac{(9.80\ m/s^2)(3.00\ m)(15.0\ kg - 10.0\ kg)}{(15.0\ kg + 10.0\ kg) + \frac{(3.00\ kg)}{2}}\right)^{1/2} = \boxed{2.36\ m/s}$$

19. (Rolling, Torque, and Angular Momentum) A hollow cylinder of outer radius $R = 5\ cm$ and mass $M = 50\ g$ with moment of inertia about the center of mass $I_{com} = MR^2$ starts from rest and moves down an incline tilted at an angle $\theta = 20^o$ from the horizontal. The center of mass of the cylinder has dropped a vertical distance $h = 30\ cm$ when it reaches the bottom of the incline. The coefficient of static friction between the cylinder and the surface is μ_s. The cylinder rolls without slipping down the incline. **Answer: i)** $\mu_s > 0.18$ **ii)** $v = 1.7\ m/s$



i What is the minimum value for the coefficient of static friction μ_s such that the cylinder rolls without slipping down the incline plane?

ii What is the magnitude of the velocity of the center of mass of the cylinder when it reaches the bottom of the incline?



i What is the minimum value for the coefficient of static friction μ_s such that the cylinder rolls without slipping down the incline plane?

ii What is the magnitude of the velocity of the center of mass of the cylinder when it reaches the bottom of the incline?

$\boxed{mg\sin\theta - f = ma}$ $\Rightarrow mg\sin\theta - ma = ma.$

$\boxed{N - mg\cos\theta = 0}$ $\Rightarrow \cancel{m}g\sin\theta = 2\cancel{m}a \Rightarrow \boxed{a = \frac{1}{2}g\sin\theta} = 1.6\ m/s^2$ (3)

$\boxed{\tau = I_{com}\cdot\alpha}$

$f.\cancel{R} = m\cancel{R^2}.\left(\frac{a}{\cancel{R}}\right) \Rightarrow \boxed{f = ma}$ $\Rightarrow f = m.\frac{1}{2}g\sin\theta = \frac{0.05}{2}.9.8\sin 20$

FOR no slipping

$\Rightarrow \boxed{f = 0.08N}$ (3)

• FOR Rolling without slipping $\boxed{f < \mu_s.N}$ ★ (6)

$\frac{1}{2}\cancel{mg}\sin\theta < \mu_s.\cancel{mg}\cos\theta$

$\Rightarrow \mu_s > \frac{1}{2}\tan\theta \Rightarrow \boxed{\mu_s > 0.18}$

i) $U_i + \cancel{K_i}^{0} = \cancel{U_f}^{0} + K_f$

$\boxed{mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}I_{com}\omega_f^2}$

$mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}m\underbrace{R^2\omega_f^2}_{v_f^2}$

3

$\Rightarrow \cancel{m}gh = \frac{1}{2}\cancel{m}v_f^2 + \frac{1}{2}\cancel{m}v_f^2$

$\Rightarrow \boxed{v_f = \sqrt{gh}}$

$v_f = \sqrt{gh}$

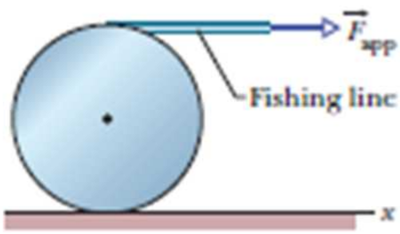
$v_f = \sqrt{9.8 * 0.3}$

$\Rightarrow \boxed{v_f = 1.7\ m/s}$

(8)

$\left\{\omega = \frac{v_f}{R} = \frac{1.7}{0.05} = 34\right\}$

20. (Rolling, Torque, and Angular Momentum - 40%) In figure given below, a constant horizontal force $\vec{F}_{app}$ of magnitude 12 N is applied to a uniform solid cylinder by fishing line wrapped around the cylinder. The mass of the cylinder is 10 kg, its radius is 0.10 m, and the cylinder <u>*rolls smoothly*</u> on the horizontal surface. (**Hint**: $I_{COM} = \frac{1}{2}MR^2$)



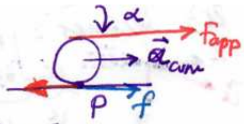
i What is the magnitude of the acceleration of the center of mass of the cylinder?

ii What is the magnitude of the angular acceleration of the cylinder about the center of mass?

iii In unit-vector notation, what is the frictional force acting on the cylinder?

9(71) $\vec{F}_{app} = 12N$
$M = 10\ kg$
$R = 0.10m$
rolls smoothly

cw → as positive (as our choice!)



i) $I_p = I_{com} + MR^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$

$F_{app} 2R = \tau = I_p \alpha \rightarrow 2R F_{app} = (\frac{3}{2}MR^2)\alpha$

$\alpha = \frac{4}{3}\frac{F_{app}}{MR} = \frac{4}{3}\frac{(12N)}{(10\ kg)(0.10m)} \boxed{= 16\ rad/s^2} \Rightarrow a = \alpha R$ $\boxed{a = 1.6\ m/s^2}$

ii) $\alpha = 16\ rad/s^2$

iii) $f = ?$

$F_{app} + f = ma$ ①
$F_{app} - f = ma_x$ ②

$12N - 10kg\ 1.6\ m/s^2 = f$ ②

$\Rightarrow \boxed{f = -4N} \xrightarrow{?} \boxed{\vec{f} = +4N\hat{i}}$

$\vec{f}$ $\vec{a}$ $\vec{F}_{app}$ ②

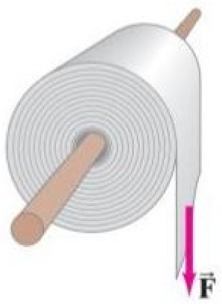
$12 - f = Ma_x$

$\vec{a}$ $\vec{F}_{app}$ $\vec{f}$ Contradiction

$12N - (10\ kg)1.6\ m/s^2 = -f$

① ✓ $\Rightarrow \boxed{\vec{f} = +4N\hat{i}}$

21. (Rolling, Torque, and Angular Momentum - 40%) The radius of the roll of paper shown in Figure is 7.6 cm and its moment of inertia is $I = 3.3 \times 10^{-3}\ kg.m^2$. A force of 2.5 N is exerted on the end of the roll for 1.3 s, but the paper does not tear so it begins to unroll. A constant friction torque of 0.11 $m.N$ is exerted on the roll which gradually brings it to a stop.

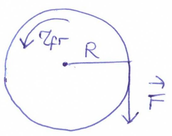


Assuming that the paper's thickness is negligible, calculate

i the length of paper that unrolls during the time that the force is applied (1.3 s) and

ii the length of paper that unrolls from the time the force ends to the time when the roll has stopped moving.

Answer: i) 1.56 m **ii)**1.13 m



i. $\Delta S_1 = R\,\Delta\theta_1$

the angular acceleration is constant, and so constant acceleration relationship can be used.

$\Delta\theta_1 = \underbrace{\cancel{\omega_i}}_{0} t_1 + \frac{1}{2}\alpha_1 t_1^2$

Take clockwise torques as positive. Write Newton's second law for the rotational motion:

$\Sigma\tau = I\alpha_1$

$FR - \tau_{fr} = I\alpha_1 \rightarrow \alpha_1 = \dfrac{FR - \tau_{fr}}{I}$

$$\Delta S_1 = R\left(\frac{1}{2}\left(\frac{FR-\tau_{fr}}{I}\right)t_1^2\right) = (0.076m)\,\frac{1}{2}\left(\frac{(2.5N)(0.076m)-(0.11\,N.m)}{3.3\times10^{-3}\,kg.m^2}\right).(1.3\,s)^2$$

$\boxed{\Delta S_1 = 1.56m}$

ii. Now the external force removed, but the frictional torque is still present.

$$\omega_1 = \underbrace{\cancel{\omega_i}}_{0} + \alpha_1 t_1 = \left(\frac{FR-\tau_{fr}}{I}\right)t_1 = \frac{[(2.5N)(0.076m)-(0.11N.m)]}{3.3\times10^{-3}\,kg.m^2}(1.3s) = 31.515\ rad/s.$$

the initial angular velocity is the final angular velocity the motion in part (i)

$\Sigma\tau = I\alpha_2$

$-\tau_{fr} = I\alpha_2$

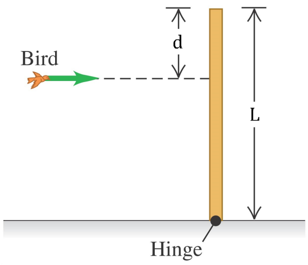
$\alpha_2 = -\dfrac{\tau_{fr}}{I}$

$\underbrace{\cancel{\omega_2^2}}_{0} = \omega_1^2 + 2\Delta\theta_2\alpha$

$0 = (31.515\,rad/s)^2 + 2.\Delta\theta_2.\left(-\dfrac{0.11\,N.m}{3.3\times10^{-3}\,kg.m^2}\right) \rightarrow \Delta\theta_2 = 14.90\ rad.$

$\Delta S_2 = R\,\Delta\theta_2 = (0.076m)(14.90rad) = \boxed{1.13m}$

22. (Rolling, Torque, and Angular Momentum - 40%) A 500 g bird is flying horizontally at 2.24 m/s, not paying much attention, when it suddenly flies into a stationary vertical bar, hitting it $d = 25.0\ cm$ below the top as seen in the Figure. The bar is uniform, $l = 0.750\ m$ long, has a mass of 1.50 kg, and is hinged at its base. The collision stuns the bird do that it just drops to the ground afterward (but soon recovers to fly happily away).



What is the angular velocity of the bar

i just after it is hit by the bird, and

ii just as it reaches the ground?

Hint: For long thin rod the moment of inertia about an axis through end is $I_{CM} = \frac{1}{3}MR^2$

Answer: i) 2.00 rad/s **ii)** 6.58 rad/s

i) Angular momentum is conserved in the collision between the bird and the bar :

$$L_i = L_f$$

$$mv(L-d) = \left(\frac{1}{3}ML^2\right)\omega \quad \text{(for the axis at hinge)}$$

$$\omega = \frac{3mv}{ML^2}(L-d) = \frac{3(0.5\,kg)(2.24\,m/s)(0.750m - 0.250m)}{(1.5\,kg)(0.750m)^2}$$

$$\boxed{\omega = 2.00\ rad/s}$$

ii) Energy is conserved in the motion of the bar after the collision ;

$$E_1 = E_2$$

$$\frac{1}{2}I\omega_1^2 + Mg\frac{L}{2} = \frac{1}{2}I\omega_2^2 \ , \quad I = \frac{1}{3}ML^2$$

$$\cancel{\frac{1}{2}}\left(\frac{1}{3}\cancel{M}L^{\cancel{2}}\right)\omega_1^2 + \cancel{M}g\frac{\cancel{L}}{\cancel{2}} = \frac{1}{\cancel{2}}\left(\frac{1}{3}\cancel{M}L^{\cancel{2}}\right)\omega_2^2$$

$$\omega_2 = \sqrt{\omega_1^2 + \frac{3g}{L}} = \sqrt{(2.00\,rad/s)^2 + \frac{3(9.80\,m/s^2)}{(0.750m)}} = 6.58\ rad/s.$$

İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy101 Physics I
Final Examination
May 22, 2019 10:30 – 12:30
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

INSTRUCTOR:

DURATION: 120 minutes

◇ **Answer all the questions.**
◇ **Write the solutions explicitly and clearly. Use the physical terminology.**
◇ **You are allowed to use Formulae Sheet.**
◇ **Calculator is allowed.**
◇ **You are not allowed to use any other electronic equipment in the exam.**
◇ **I declare hereby that I fulfilled the requirements for the attendance according to the University regulations and I accept that my examination will not be valid otherwise.**

Question	Grade	Out of
1A		15
1B		15
2		20
3		20
4		20
5		20
TOTAL		110

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1. A) A force $\vec{F} = (cx - 3x^2)\hat{x}$ acts on a particle as the particle moves along an x axis, with $\vec{F}$ in newtons, x in meters, and c a constant. At $x = 0$, the particle's kinetic energy is 20.0 J; at $x = 3.00$ m, it is 11.0 J. Find c.

$$W = \int \vec{F} \cdot d\vec{x} = \int_{x_i}^{x_f} F_x \, dx = \int_{x_i}^{x_f} (cx - 3x^2) \, dx$$

2 3

$$= \frac{c}{2}x^2 - x^3 \Big|_0^3 = 4.5c - 27$$ 3

$$W = \Delta K = K_f - K_i = 11 - 20 = -9 \text{ J}$$ 5

$$\Rightarrow 4.5c - 27 = -9 \rightsquigarrow \boxed{c = 4 \text{ N/m}}$$

1 1

B) You drop a 2.00 kg physics book to a friend who stands on the ground at distance D=10.0 m below. If your friend's outstretched hands are at distance d=1.50 m above the ground (see Figure),

i How much work Wg does the gravitational force do on the book as it drops to her hands?

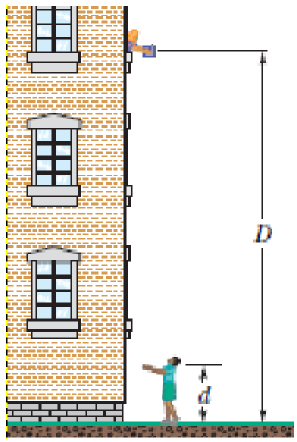
ii What is the change ΔU in the gravitational potential energy of the book–Earth system during the drop?

If the gravitational potential energy U of that system is taken to be zero at ground level,

iii what is U when the book is released?

iv what is U when it reaches her hands?

v Now take U to be 100 J at ground level and again find Wg, ΔU, U at the release point, and U at her hands.



i) $W_g = \vec{F}_g \cdot \vec{d} = mgd\cos\theta$ | $d = 10m - 1.5m = 8.5m$, $\theta = 0°$

$= (2kg)(9.8m/s^2)\,8.5m\cos 0°$

$= \underline{\underline{167J}}$

ii) $\Delta U = U_f - U_i = mg(y_f - y_i) = (2kg)(9.8m/s^2)(1.5m - 10m)$

$= \underline{\underline{-167J}}$

iii) $U_i = mgy_i = (2kg)(9.8m/s^2)10m = \underline{\underline{196J}}$ iv) $\underline{\underline{29J}}$

v) $W_g = \underline{\underline{167J}}$, $\Delta U = \underline{\underline{-167J}}$, $U_i = mgy_i + 100J$

$= \underline{\underline{296J}}$

$U_f = mgy_f + 100J$

$= \underline{\underline{129J}}$

2. A rod of length 30.0 cm has linear density (mass per length) given by $\lambda = 50.0\frac{g}{m} + 20.0x\frac{g}{m}$ where x is the distance from one end, measured in meters.

   i What is the mass of the rod?

   ii How far from the $x = 0$ end is its center of mass?

2)

a) $M = \int_0^{0.3} \lambda\, dx = \int_0^{0.3} (50 + 20x)\, dx$ 5

$M = (50x + 10x^2)\Big|_0^{0.3} = 15.9\,g$ 5

b) $X_{CM} = \frac{\int x\, dm}{M} = \frac{1}{M}\int_0^{0.3} \lambda x\, dx =$

$= \frac{1}{M} \int_0^{0.3} (50x + 20x^2)\, dx = \frac{1}{15.9}\left(25x^2 + \frac{20x^3}{3}\right)\Big|_0^{0.3}$ 5

$\simeq 0.153\,m$

5

3. A soccer player kicks a soccer ball of mass 0.40 kg that is initially at rest. The foot of the player is in contact with the ball for 2.0×10^{-3} s, and the force of the kick is given by

$$F(t) = [(12.0 \times 10^9)t^2 - (4.0 \times 10^{12})t^3]\ N$$

for $0 \le t \le 2.0 \times 10^{-3}$ s, where t is in seconds. Find the magnitudes of

i the impulse on the ball due to the kick,

ii the average force on the ball from the player's foot during the period of contact,

iii the maximum force on the ball from the player's foot during the period of contact,

iv the ball's velocity immediately after it loses contact with the player's foot.

$m = 0.40$ kg
initially at rest
Contact time $= 2.0 \times 10^{-3}$ s

i) $J = ?$ impulse $\vec{F}_N = \frac{\vec{J}}{\Delta t}$ OR $J = \int F(t)\,dt$

$$= \int_0^{2\times10^{-3}} \left((12\times10^9)t^2 - (4\times10^{12})t^3\right) dt$$

$$\Rightarrow J = 4\times10^9 t^3 - 1\times10^{12} t^4 \Big|_0^{2\times10^{-3}}$$

$$= (4\times10^9)(8\times10^{-9}) - (1\times10^{12})(16\times10^{-12}) = \underline{\underline{16\,Ns}}$$

ii) $F_{av} = \frac{J}{\Delta t} = \frac{16\,Ns}{2.0\times10^{-3}\,s} = \underline{\underline{8\times10^3\,N}}$

iii) $F_{max} = ?$ during the period of contact $\left\{ \text{at maximum; } \frac{dF(t)}{dt} = 0 \right.$

$$\rightarrow 24\times10^9 t - 12\times10^{12} t^2 = 0 \rightsquigarrow t = 2\times10^{-3}\,s$$

$$\Rightarrow F(t = 2\times10^{-3}\,s) = F_{max} = (12\times10^9)(2\times10^{-3}\,s)^2 - (4\times10^{12})(2\times10^{-3}\,s)^3 = \underline{\underline{16\times10^3\,N}}$$

iv) $v = ?$ when contact is lost

$$\Delta\vec{p} = \vec{P}_f - \vec{P}_i = m\vec{v}_f - \underbrace{m\vec{v}_i}_{0} \rightsquigarrow \Delta p = mv = J \rightsquigarrow v = \frac{J}{m} = \frac{16\,Ns}{0.40\,kg} = \underline{\underline{40\,m/s}}$$

4. The angular position of a point on a rotating wheel is given by $\theta(t) = 2.0 + 4.0t^2 + 2.0t^3$, where θ is in radians and t is in seconds. At t = 0,

   i what is the point's angular position?

   ii what is its angular velocity?

   iii what is its angular velocity at t = 4.0 s?

   iv Calculate its angular acceleration at t = 2.0 s.

   v Is its angular acceleration constant?

$\Theta(t) = 2t^3 + 4t^2 + 2$

i) at $t=0$, $\underline{\Theta(\phi) = 2 \text{ rad}}$ 2

ii) $\omega(t) = \frac{d\Theta(t)}{dt} = 6t^2 + 8t$ 2, at $t=0$, $\underline{\omega(\phi) = \phi}$ 2

iii) $t = 4 \rightsquigarrow \omega(t=4) = 6 * 4^2 + 8 * 4 = \underline{128 \text{ rad/s}}$ 3

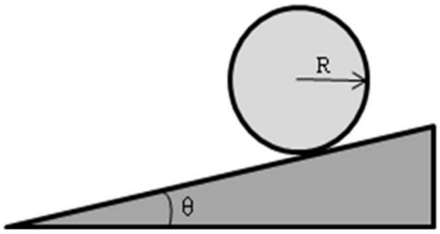
iv) $\alpha(t) = \frac{d\omega(t)}{dt} = 12t + 8$ 2, at $t=2$, $\underline{\alpha(t=2) = 32}$ rad/s^2 2

v) α has time dependency $\Rightarrow$ NOT constant 2

5. A uniform ball, of mass $M = 6.0$ kg and radius R, rolls smoothly from rest down a ramp at angle $\theta = 30.0°$ (see Figure, $I = \frac{2}{5}MR^2$)

i The ball descends a vertical height $h = 1.20$ m to reach the bottom of the ramp. What is its speed at the bottom?

ii What are the magnitude and direction of the frictional force (f_s) on the ball as it rolls down the ramp?



$M = 6\,kg$
$\theta = 30°$
$I_{com} = \frac{2}{5}MR^2$
$h = 1.2\,m$

i) Mechanical Energy is conserved for the ball-earth system
$\rightsquigarrow F_N$ & f_s does not work

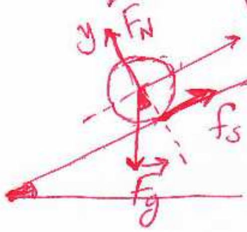
$$K_f + \underbrace{\cancel{U_f}}_{0} = \underbrace{\cancel{K_i}}_{0} + U_i \rightsquigarrow K_f = U_i \rightsquigarrow \frac{1}{2} I_{com}\omega^2 + \frac{1}{2} M v_{com}^2 = Mgh$$

$$\rightsquigarrow \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v_{com}}{R}\right)^2 + \frac{1}{2} M v_{com}^2 = Mgh$$

$$\frac{7}{10}\cancel{M} v_{com}^2 = \cancel{M}gh \rightsquigarrow v = \sqrt{\frac{10}{7}gh}$$

$$\rightsquigarrow v = \sqrt{\frac{10}{7}(9.8\,m/s^2)(1.2\,m)} = \underline{4.1\,m/s}$$

ii)



Newton's 2nd law in x-direction
$$-Mg\,Sin30° + f_s = M a_{com,x}$$
Newton's 2nd law in angular form
$$\tau_{net} = I_{com}\alpha \rightsquigarrow f_s R = \frac{2}{5}MR^2\alpha$$
$$a_{com,x} = -\alpha R \rightsquigarrow \frac{5}{2} f_s = -M a_{com,x}$$

$$\Rightarrow -Mg\,Sin30° + f_s = -\frac{5}{2} f_s \rightsquigarrow f_s = \frac{2}{7} Mg\,Sin30°$$
$$= \frac{2}{7}(6\,kg)(9.8\,m/s^2)\,\frac{1}{2} = \underline{\underline{8.4\,N}}$$