Chapter 2

Motion Along A Straight Line

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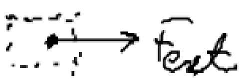
2-2 Motion

- We find moving objects all around us.
- One purpose of physics is to study the motion of objects
  - how fast they move (velocity)
  - how far they move in a given amount of time... (displacement)
- The classification and comparison of motions: **kinematics.**

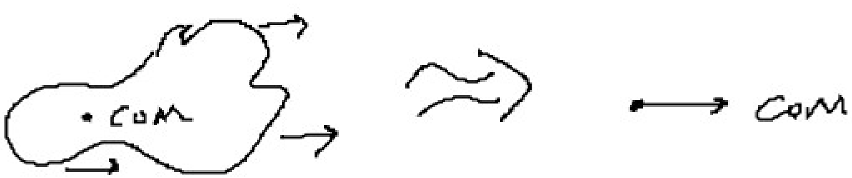
1. We study the basic physics of motion where the object moves along a single axis (1D).
2. Forces cause motion.

   - We will find out, as a result of application of force, *if the objects speed up, slow down, or maintain the same rate.*

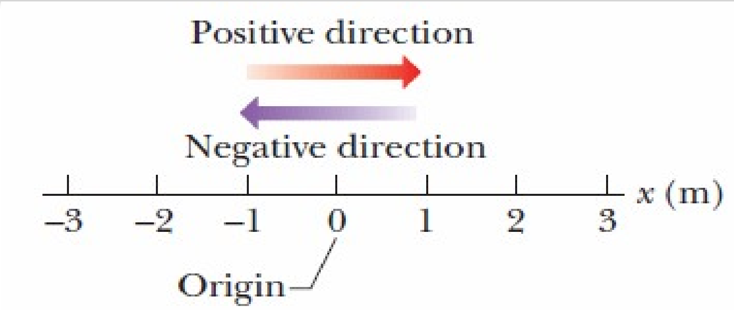
3. The moving object here will be considered **as a particle**. (COM: center of mass)
   - If we deal with a stiff, extended object, we will assume that all particles on the body move in the same fashion.



   - We will study the <u>motion of a particle</u>, which will represent the entire body.



- To locate an object = to find its position *relative to some reference* point (**origin**).



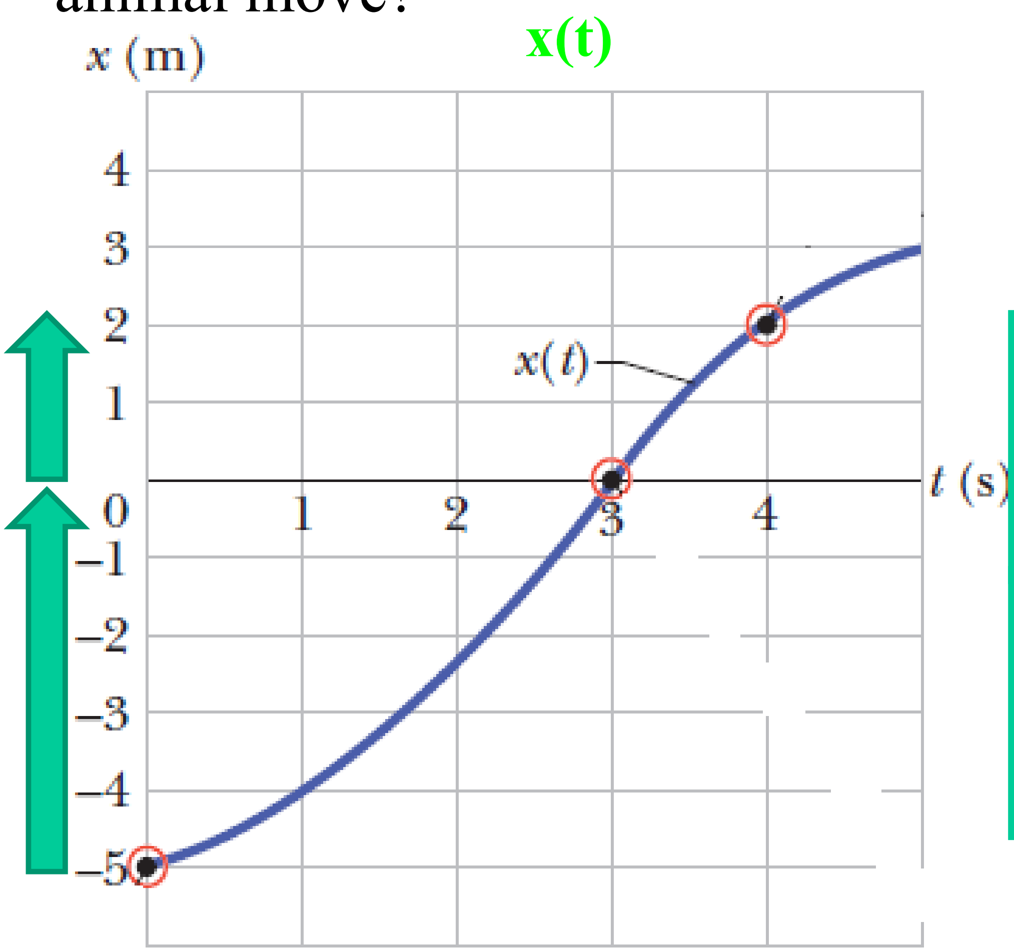
- A change from position x_1 to position x_2 is called a **displacement**
  - **Δx: Δ (delta): change in quantity (x_f-x_i)**

$$\Delta x = x_2 - x_1$$

- Displacement is a **vector quantity** (has both a direction and a magnitude). $\Delta\vec{x} = \vec{x_2} - \vec{x_1}$ & $|\Delta\vec{x}| = |\vec{x_2}| - |\vec{x_1}|$
  - An object's displacement is -4 m means that the object has moved towards decreasing x-axis by 4 m. The direction of motion, here, is toward decreasing x.
  - From x = 5 m to x = 12 m: Δx = 7 m (positive direction)
  - From x = 5 m to x = 1 m: Δx = -4 m (negative direction)
  - From x = 5 m to x = 200 m to x = 5 m: Δx = 0 m

How Fast? → Average Velocity

- How to describe position? Plot x as a function of t : graph of **x(t)**
- Ex: Straight-line motion of an animal (an object): How fast does this animal move?



Average Velocity: Unit: m/s

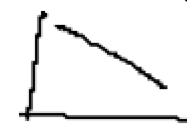
$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

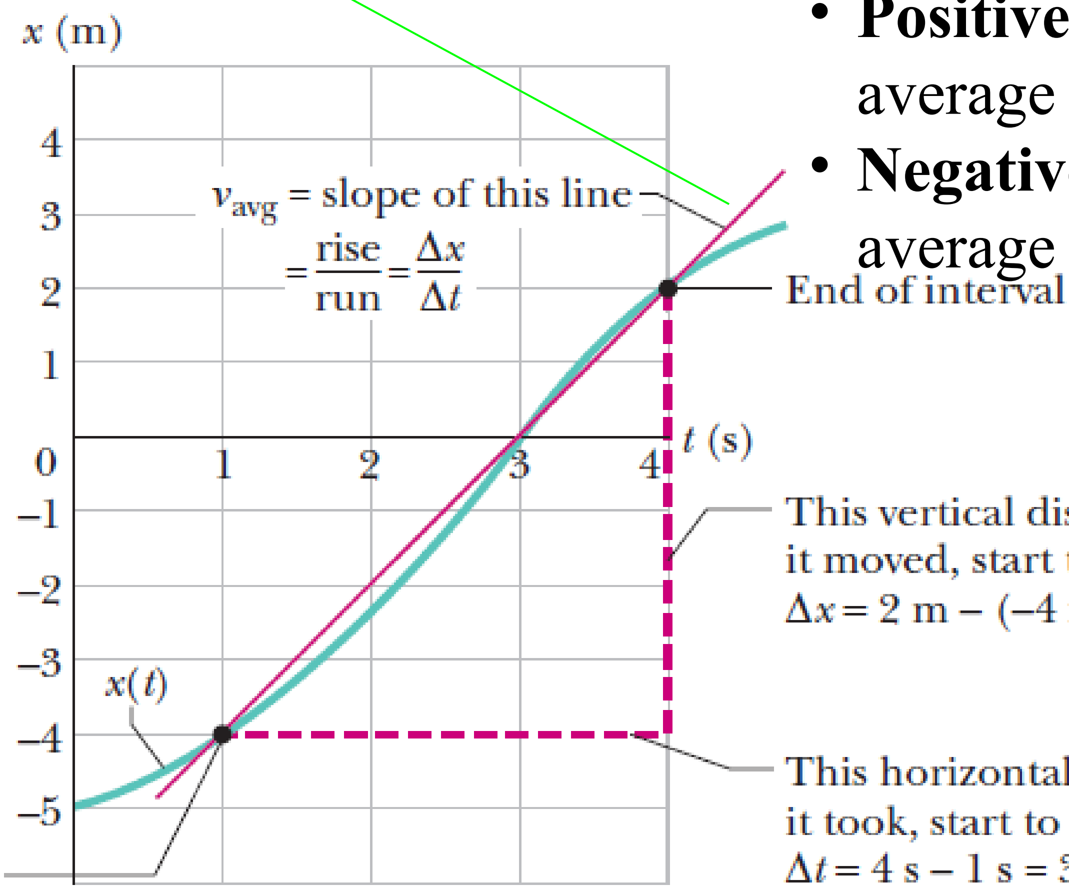
V_{avg}: The **slope** of the straight line that connects two particular points on the $x(t)$ curve.

V_{avg} has both magnitude and direction (it is another vector quantity)

How Fast? → Average Speed

- Calculation of the average velocity between t =1s and t =4s as the slope of the line that connects the points.

- **Positive slope** means positive average velocity 
- **Negative slope** means negative average velocity 



$$v_{avg} = \frac{6\ \text{m}}{3\ \text{s}} = 2\ \text{m/s}.$$

Start

How Fast? → Average Velocity & Speed !

change in direction

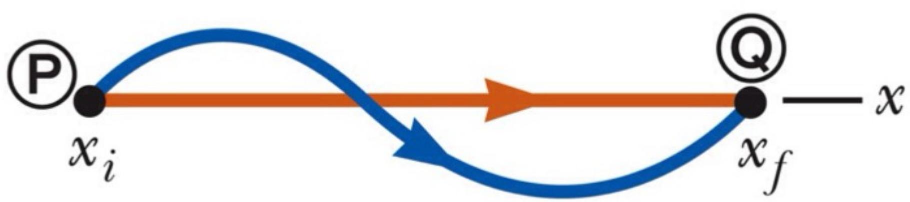
Q: What is the difference btw Ave velocity ans Ave speed?

A: Average velocity involves the particle's displacement x, the average speed involves the total distance covered *independent of direction*!

$$s_{avg} = \frac{\text{total distance}}{\Delta t}$$

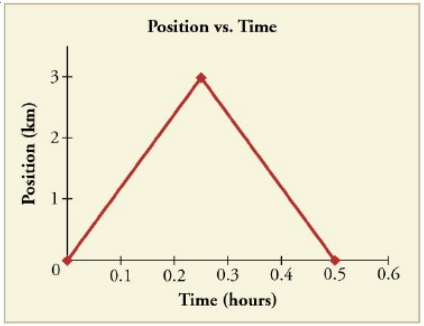
- Average speed is always positive (no direction)

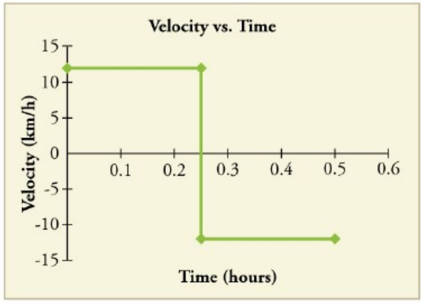
- Because average speed does *not* include direction, it lacks any algebraic sign! Average velocity = -3 m/s; average speed = 3 m/s

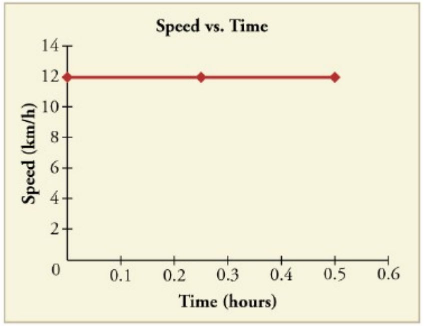


- Cars on both paths have the same average velocity since they had the same displacement in the same time interval
- The car on the blue path will have a greater average speed since the path length it traveled is larger

AK Lecture Notes







2-4 Average Velocity and Average Speed

- Example: You drive a truck along a straight road for 8.4 km at 70 km/h, at which point the truck runs out of gasoline and stops.
- Over the next 30 min, you walk another 2.0 km farther along the road to a gasoline station.

(a) What is your overall displacement from the beginning to your arrival at the station?

$$\Delta x = x_2 - x_1 = 10.4\text{ km} - 0 = 10.4\text{ km}.$$

(b) What is the time interval t from the beginning of your drive to your arrival at the station?

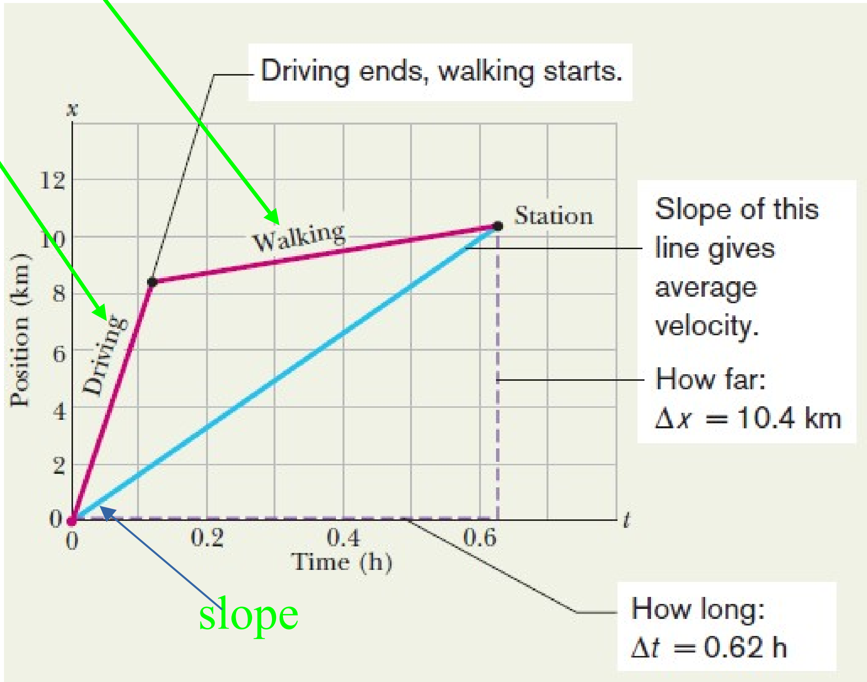
$$v_{\text{avg,dr}} = \frac{\Delta x_{\text{dr}}}{\Delta t_{\text{dr}}}.$$

Rearranging and substituting data then give us

$$\Delta t_{\text{dr}} = \frac{\Delta x_{\text{dr}}}{v_{\text{avg,dr}}} = \frac{8.4\text{ km}}{70\text{ km/h}} = 0.12\text{ h}.$$

So,

$$\Delta t = \Delta t_{\text{dr}} + \Delta t_{\text{wlk}}$$
$$= 0.12\text{ h} + 0.50\text{ h} = 0.62\text{ h}.$$



(c) What is your average velocity v_{avg} from the beginning of your drive to your arrival at the station? Find it both numerically and graphically.

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{10.4 \text{ km}}{0.62 \text{ h}} = 16.8 \text{ km/h} \approx 17 \text{ km/h}.$$

(d) Suppose that to pump the gasoline, pay for it, and walk back to the truck takes you another 45 min. What is your average speed from the beginning of your drive to your return to the truck with the gasoline?

Calculation: The total distance is 8.4 km + 2.0 km + 2.0 km = 12.4 km. The total time interval is 0.12 h + 0.50 h + 0.75 h = 1.37 h. Thus, Eq. 2-3 gives us

$$s_{avg} = \frac{12.4 \text{ km}}{1.37 \text{ h}} = 9.1 \text{ km/h}. \qquad \text{(Answer)}$$

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{\varnothing}{\Delta t} = \varnothing$$

- "How fast" more commonly refers to how fast a particle is moving at a given instant.

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- At a single moment in time, obtained from average velocity by shrinking Δt.
- v: the rate at which position x is changing with time at a given instant (*the slope of the position-time curve at the point representing that instant*).

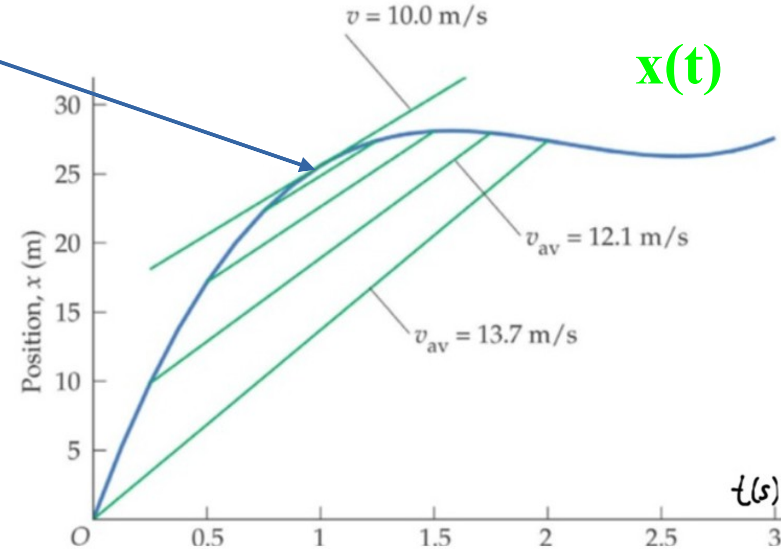
This plot shows the average velocity being measured over shorter and shorter intervals. The instantaneous velocity is tangent to the curve.

V_{max}?

V=0?

V>0?

V<0?

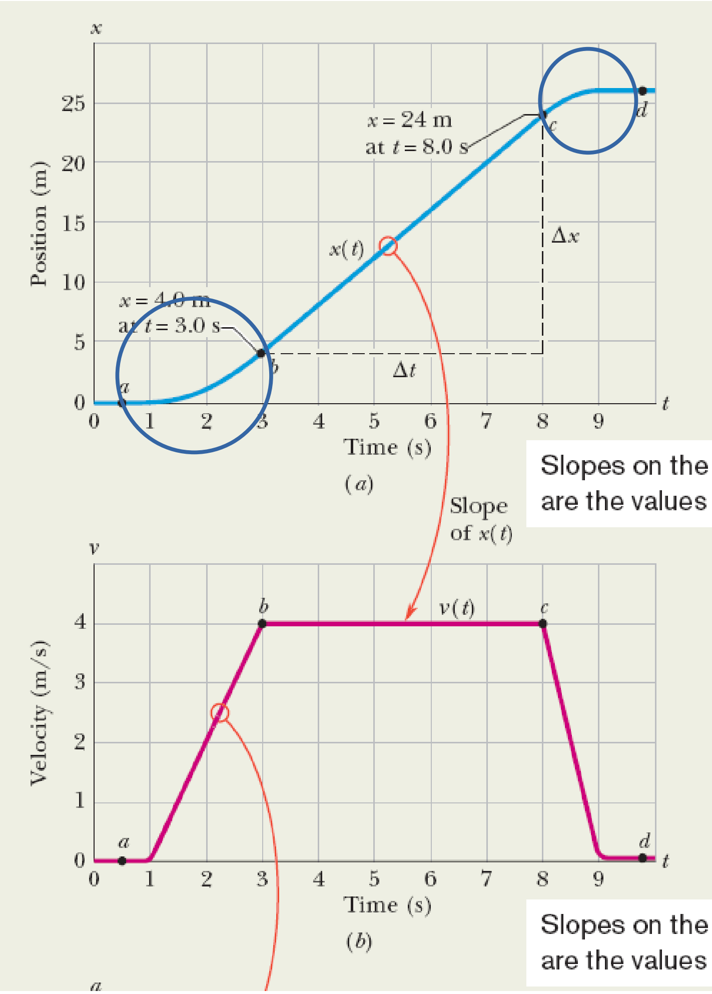


AK Lecture Notes

- **Speed** is the magnitude of velocity (velocity that has been stripped of any indication of direction).

Example:

Figure 2-6*a* is an $x(t)$ plot for an elevator cab that is initially stationary, then moves upward (which we take to be the positive direction of x), and then stops. Plot $v(t)$.



Slopes on the x versus t graph are the values on the v versus t graph.

Slopes on the v versus t graph are the values on the a versus t graph.

The slope of x(t), and so also the velocity v, is zero from 0 to 1 s, and from 9s on.

During the interval bc, the slope is constant and nonzero, so the cab moves with constant velocity (4 m/s).

KEY IDEA

We can find the velocity at any time from the slope of the $x(t)$ curve at that time.

Calculations: The slope of $x(t)$, and so also the velocity, is zero in the intervals from 0 to 1 s and from 9 s on, so then the cab is stationary. During the interval bc, the slope is constant and nonzero, so then the cab moves with constant velocity. We calculate the slope of $x(t)$ then as

$$\frac{\Delta x}{\Delta t} = v = \frac{24\text{ m} - 4.0\text{ m}}{8.0\text{ s} - 3.0\text{ s}} = +4.0\text{ m/s}. \qquad (2\text{-}5)$$

The plus sign indicates that the cab is moving in the positive x direction. These intervals (where $v = 0$ and $v = 4$ m/s) are plotted in Fig. 2-6*b*. In addition, as the cab initially begins to move and then later slows to a stop, v varies as indicated in the intervals 1 s to 3 s and 8 s to 9 s. Thus, Fig. 2-6*b* is the required plot. (Figure 2-6*c* is considered in Section 2-6.)

Given a $v(t)$ graph such as Fig. 2-6*b*, we could "work backward" to produce the shape of the associated $x(t)$ graph (Fig. 2-6*a*). However, we would not know the actual values for x at various times, because the $v(t)$ graph indicates only *changes* in x. To find such a change in x during any interval, we must, in the language of calculus, calculate the area "under the curve" on the $v(t)$ graph for that interval. For example, during the interval 3 s to 8 s in which the cab has a velocity of 4.0 m/s, the change in x is

$$\Delta x = (4.0\text{ m/s})(8.0\text{ s} - 3.0\text{ s}) = +20\text{ m}. \qquad (2\text{-}6)$$

(This area is positive because the $v(t)$ curve is above the t axis.) Figure 2-6*a* shows that x does indeed increase by 20 m in that interval. However, Fig. 2-6*b* does not tell us the *values* of x at the beginning and end of the interval. For that, we need additional information, such as the value of x at some instant.

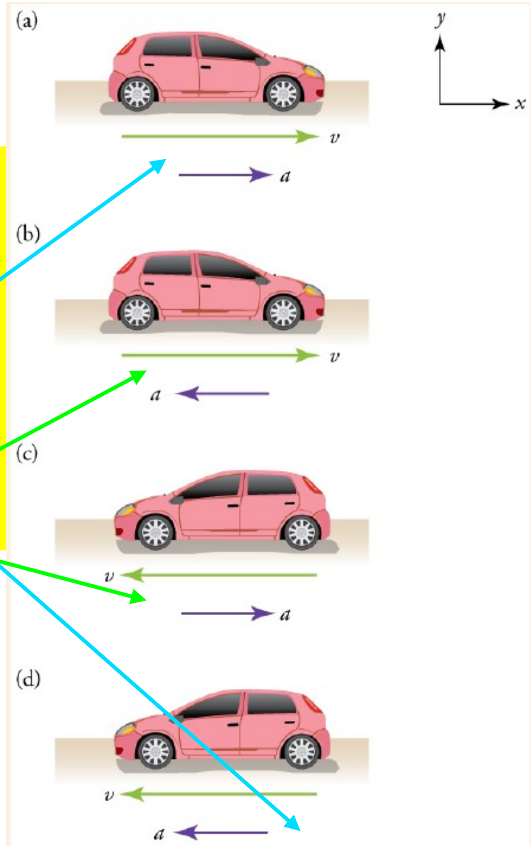
- **Average acceleration** is the change of velocity over the change of time. $\frac{\Delta v}{\Delta t}$ ↑ ↓ − $\left.\begin{array}{l} x(t) \sim v(t) \\ v(t) \sim a(t) \end{array}\right.$

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

- **Instantaneous acceleration** is the rate at which its velocity is changing at that instant.
  - In terms of the position function, the acceleration can be defined as: $a = \frac{dv}{dt}$
  - The SI unit for acceleration is **m/s²**.

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

- Acceleration has both magnitude and direction (*vector quantity*).
  - If a particle has the **same sign** for *velocity* and *acceleration*, then that particle is **speeding up**.
  - If a particle has **opposite signs** for the *velocity* and *acceleration*, then the particle is **slowing down**.



Example: If a car with velocity v = -25 m/s is braked to a stop in 5.0 s, then a = + 5.0 m/s². Acceleration is positive, but speed has decreased.

$$\frac{\Delta v}{\Delta t} = \frac{0 - (-25\,m/s)}{5s - 0s} = \frac{+25\,m/s}{5s} = +5\,m/s^2$$

opposite signs v & a ⇒ slowing down

$\Delta v = v_f - v_i = 0$

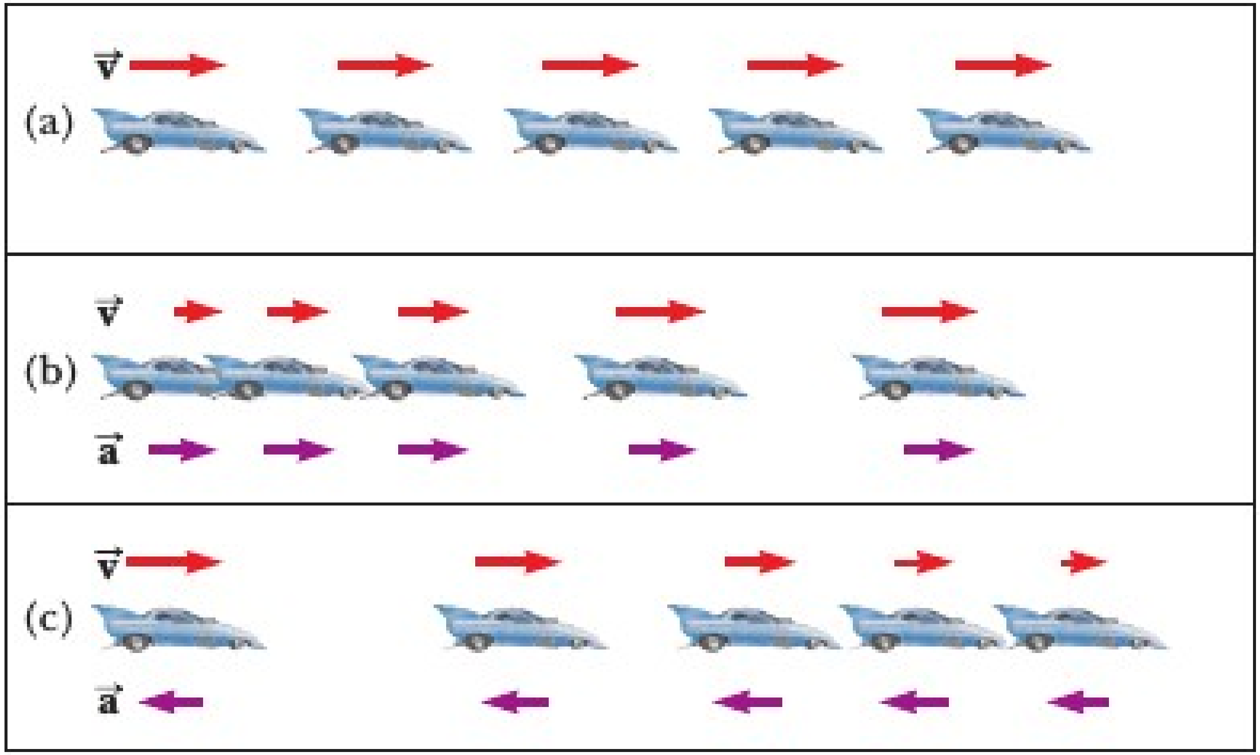
$\rightsquigarrow a = 0$

speed up
same direction
$\vec{v}$ & $\vec{a}$

same rate
of change
$\Delta v / \Delta t = a$

speed down
opposite
direction

decrease in
velocity
& same rate



(a)Motion diagram for a car moving at constant velocity (zero acceleration).

(b)Motion diagram for a car whose constant acceleration is in the direction of its velocity.

- The velocity vector at each instant is indicated by a red arrow, and the constant acceleration is indicated by a violet arrow.

(c)Motion diagram for a car whose constant acceleration is in the direction opposite the velocity at each instant.

2-6 Acceleration

- Our bodies often **react to accelerations** but <u>not to velocities</u>. A fast car often does not bother the rider, but a sudden brake is felt strongly by the rider. This is common in amusement car rides, where the rides *change velocities quickly* to thrill the riders.

- The magnitude of acceleration falling near the Earth's surface is 9.8 m/s^2, and is often referred to as ***g***.

m d

$g = 9.8\ m/s^2 \propto \left(G \frac{mM}{d^2} \right)$

EARTH M

In a rocket sled, which undergoes sudden change in velocities.




Example Continued

- When acceleration is 0 (e.g. interval *bc*) velocity is constant.
- When acceleration is positive *(ab)* upward velocity increases.
- When acceleration is negative *(cd)* upward velocity decreases.
- Steeper slope of the velocity-time graph indicates a larger magnitude of acceleration: the cab stops in half the time it takes to get up to speed.

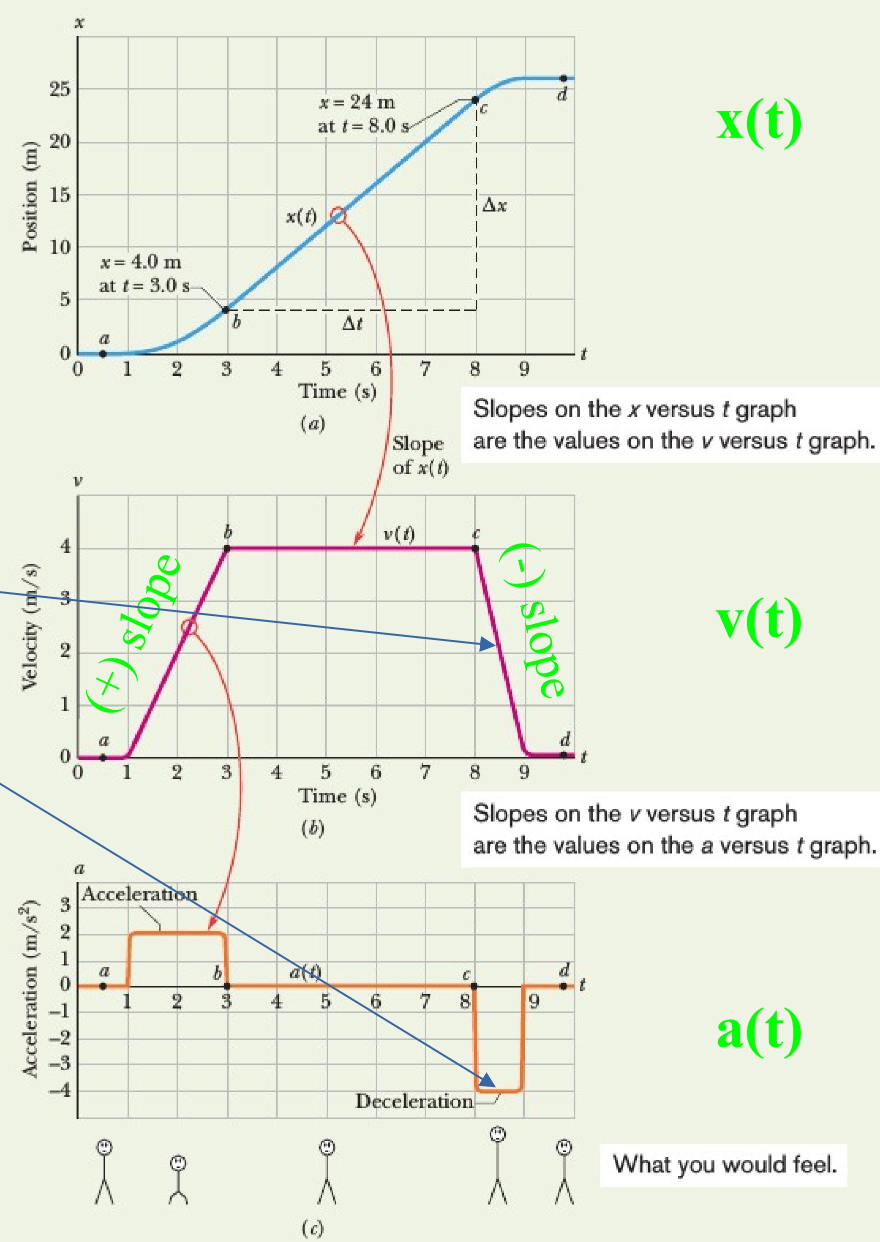


Fig. 2-6 (*a*) The $x(t)$ curve for an elevator cab that moves upward along an x axis. (*b*) The $v(t)$ curve for the cab. Note that it is the derivative of the $x(t)$ curve ($v = dx/dt$). (*c*) The $a(t)$ curve for the cab. It is the derivative of the $v(t)$ curve ($a = dv/dt$). The stick figures along the bottom suggest how a passenger's body might feel during the accelerations.

A particle's position on the x axis of Fig. 2-1 is given by

$$x(t) \rightsquigarrow x = 4 - 27t + t^3,$$

$x \propto t^3$

$v \propto t^2$

with x in meters and t in seconds.

(a) Because position x depends on time t, the particle must be moving. Find the particle's velocity function $v(t)$ and acceleration function $a(t)$.

$a \propto t$

NOT Constant $\vec{a}$!!

KEY IDEAS

(1) To get the velocity function $v(t)$, we differentiate the position function $x(t)$ with respect to time. (2) To get the acceleration function $a(t)$, we differentiate the velocity function $v(t)$ with respect to time.

Calculations: Differentiating the position function, we find

$$v = -27 + 3t^2, \qquad \text{(Answer)}$$

with v in meters per second. Differentiating the velocity function then gives us

$$a = +6t,$$

with a in meters per second squared.

Constant Acceleration

$x \propto t^2$

$v \propto t$

$a \propto$ CONSTANT

(b) Is there ever a time when $v = 0$?

Calculation: Setting $v(t) = 0$ yields

$$0 = -27 + 3t^2,$$

Example:

$x(t=0) = 4\,m$ | $-22\,m$

$v(t=0) = -27\,m/s$ | $-24\,m/s$

$a(t=0) = 0$ | $6\,m/s^2$

at $t = 1$

which has the solution

$$t = \pm 3\ \text{s}. \qquad \text{(Answer)}$$

Thus, the velocity is zero both 3 s before and 3 s after the clock reads 0.

3 s ← 0 s

→ $t > 3$

(c) Describe the particle's motion for $t \geq 0$.

Reasoning: We need to examine the expressions for $x(t)$, $v(t)$, and $a(t)$.

At $t = 0$, the particle is at $x(0) = +4$ m and is moving with a velocity of $v(0) = -27$ m/s—that is, in the negative direction of the x axis. Its acceleration is $a(0) = 0$ because just then the particle's velocity is not changing.

For $0 < t < 3$ s, the particle still has a negative velocity, so it continues to move in the negative direction. However, its acceleration is no longer 0 but is increasing and positive. Because the signs of the velocity and the acceleration are opposite, the particle must be slowing.

Indeed, we already know that it stops momentarily at $t = 3$ s. Just then the particle is as far to the left of the origin in Fig. 2-1 as it will ever get. Substituting $t = 3$ s into the expression for $x(t)$, we find that the particle's position just then is $x = -50$ m. Its acceleration is still positive.

For $t > 3$ s, the particle moves to the right on the axis. Its acceleration remains positive and grows progressively larger in magnitude. The velocity is now positive, and it too grows progressively larger in magnitude.

- **In many cases acceleration is constant, or nearly so.**
- When the acceleration is constant, its average and instantaneous acceleration values are the same.

$v \propto t$

$F_{ext} \rightsquigarrow$ object $\rightsquigarrow$ motion $\rightsquigarrow a$

$$\underset{ms}{a} = a_{avg} = \frac{v - v_0}{t - 0}. \quad \Rightarrow \quad v = v_0 + at. \quad (1)$$

$a_{avg} = a_{inst} = a$

$\Downarrow$

$\uparrow\downarrow$ in $\vec{v}_0$

Here, velocity at t=0 is v_o.

$$v_{avg} = \frac{x - x_0}{t - 0} \quad \Rightarrow \quad x = x_0 + v_{avg} t,$$

Average = ((*initial*) + (*final*)) / 2: $v_{avg} = \frac{1}{2}(v_0 + v)$

finally leading to $$x - x_0 = v_0 t + \frac{1}{2} a t^2. \quad (2)$$

Eliminating t from the Equations (1) and (2):

$$v^2 = v_0^2 + 2a(x - x_0). \quad (3)$$

$$x - x_0 = \frac{1}{2}(v_0 + v)t \quad (4)$$

$$x - x_0 = vt - \frac{1}{2} a t^2 \quad (5)$$

5 special equations

Table 2-1

Equations for Motion with Constant Acceleration[a]

Equation Number	Equation	Missing Quantity
2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

[a]Make sure that the acceleration is indeed constant before using the equations in this table.

Usually needed to solve questions

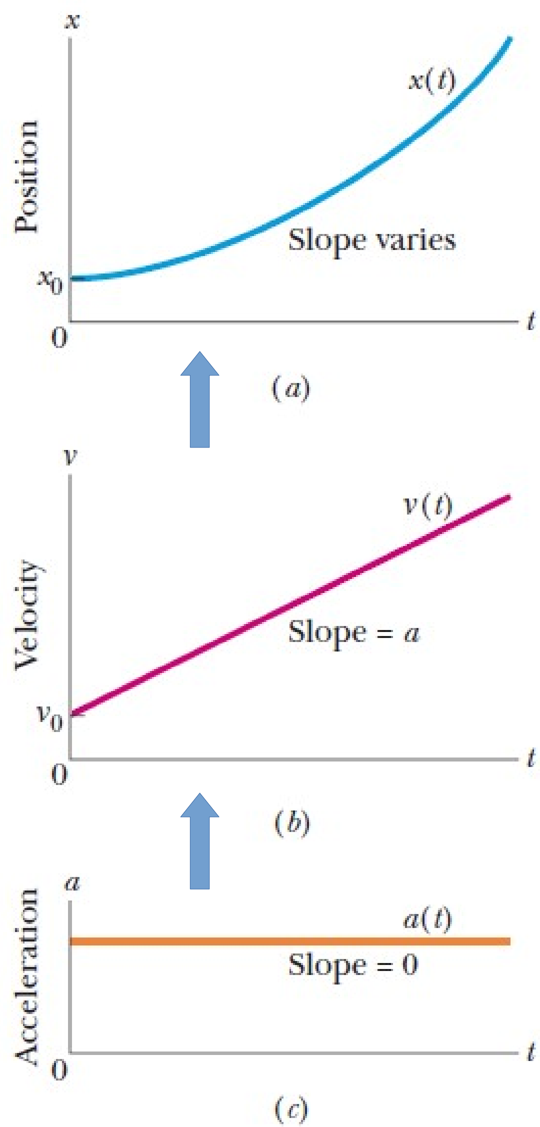
2-7 Constant Acceleration: A Special Case

- **Integrating constant acceleration** graph for a fixed time duration yields values for velocity graph during that time.
- Similarly, **integrating velocity graph** will yield values for position graph.
- Note that constant acceleration means
  - a velocity with a constant slope (v~t),
  - a position with varying slope (unless $a = 0$) ($x \sim t^2$).

$x \propto t^2$

plot of parabola

$y \propto x^2$



Slopes of the position graph are plotted on the velocity graph.

$v \propto t$

Slope of the velocity graph is plotted on the acceleration graph.

$a \propto$ constant

Figure 2-9 gives a particle's velocity v versus its position as it moves along an x axis with constant acceleration. What is its velocity at position $x = 0$? v_0

KEY IDEA

We can use the constant-acceleration equations; in particular, we can use Eq. 2-16 ($v^2 = v_0^2 + 2a(x - x_0)$), which relates velocity and position. $v(x)$

First try: Normally we want to use an equation that includes the requested variable. In Eq. 2-16, we can identify x_0 as 0 and v_0 as being the requested variable. Then we can identify a second pair of values as being v and x. From the graph, we have

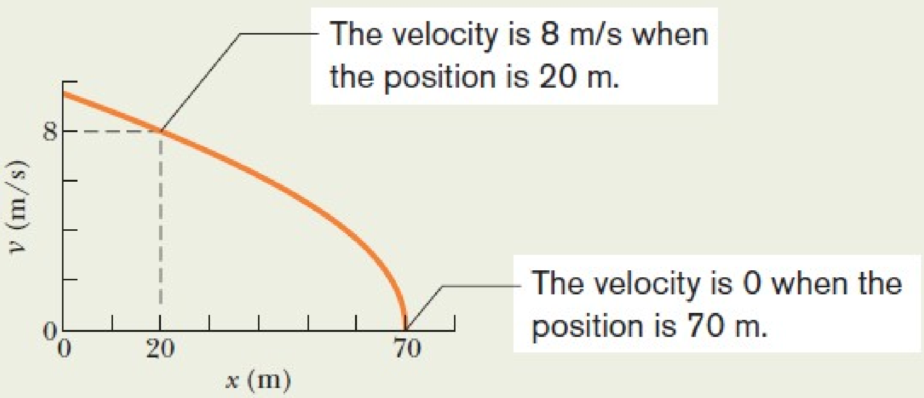


Fig. 2-9 Velocity versus position.

Example:

First try: x: 0, 20; v: v_0, 8 } 2 unknown a & v_0 X

Second try: 20, 8 ~ initial: v_i; 70, 0 ~ final: v_f

$\Rightarrow a = (-) 0.64\,m/s^2$

two such pairs: (1) $v = 8$ m/s and $x = 20$ m, and (2) $v = 0$ and $x = 70$ m. For example, we can write Eq. 2-16 as

$$(8\text{ m/s})^2 = v_0^2 + 2a(20\text{ m} - 0). \qquad (2\text{-}19)$$

However, we know neither v_0 nor a.

Second try: Instead of directly involving the requested variable, let's use Eq. 2-16 with the two pairs of known data, identifying $v_0 = 8$ m/s and $x_0 = 20$ m as the first pair and $v = 0$ m/s and $x = 70$ m as the second pair. Then we can write

$$(0\text{ m/s})^2 = (8\text{ m/s})^2 + 2a(70\text{ m} - 20\text{ m}),$$

which gives us $a = -0.64\text{ m/s}^2$. Substituting this value into Eq. 2-19 and solving for v_0 (the velocity associated with the position of $x = 0$), we find

$$v_0 = 9.5\text{ m/s}. \qquad \text{(Answer)}$$

2-9 Free-fall Acceleration

- **Free-fall acceleration** is the rate at which an object accelerates downward in the absence of air resistance.
- In this case objects close to the Earth's surface fall towards the Earth's surface with **no external forces** acting on them except for their weight.
- Use the constant acceleration model with "a" replaced by "-g", where g = 9.8 m/s^2 for motion close to the Earth's surface.
- **The equations of motion in Table 2-1 apply to objects in free-fall near Earth's surface**.

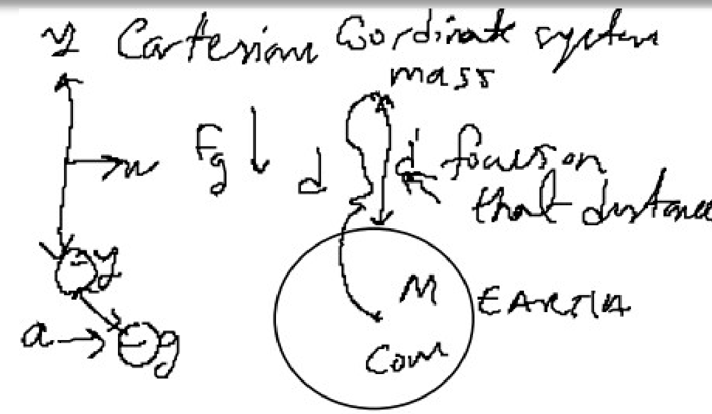


TABLE 2–5 Values of g at Different Locations on Earth (m/s^2)

Location	Latitude	g
North Pole	90° N	9.832
Oslo, Norway	60° N	9.819
Hong Kong	30° N	9.793
Quito, Ecuador	0°	9.780

AK Lecture Notes

$$v = v_0 - gt$$
$$y = y_0 + v_0 t - \frac{1}{2} g t^2$$
$$v^2 = v_0^2 - 2g(y - y_0)$$

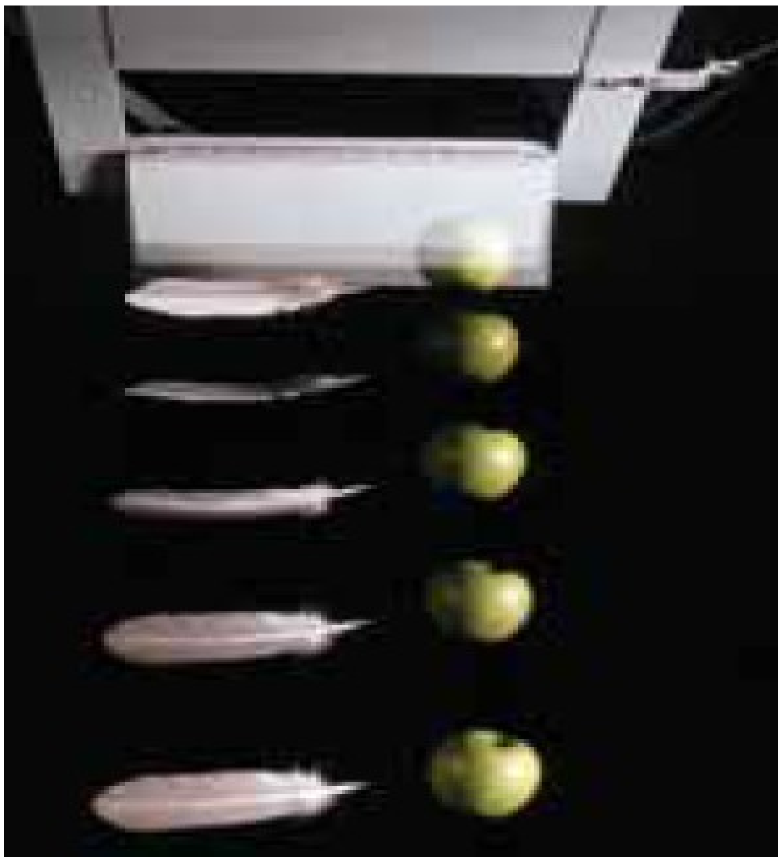
y_0 be carefull
v_0
wrt our coordinate system

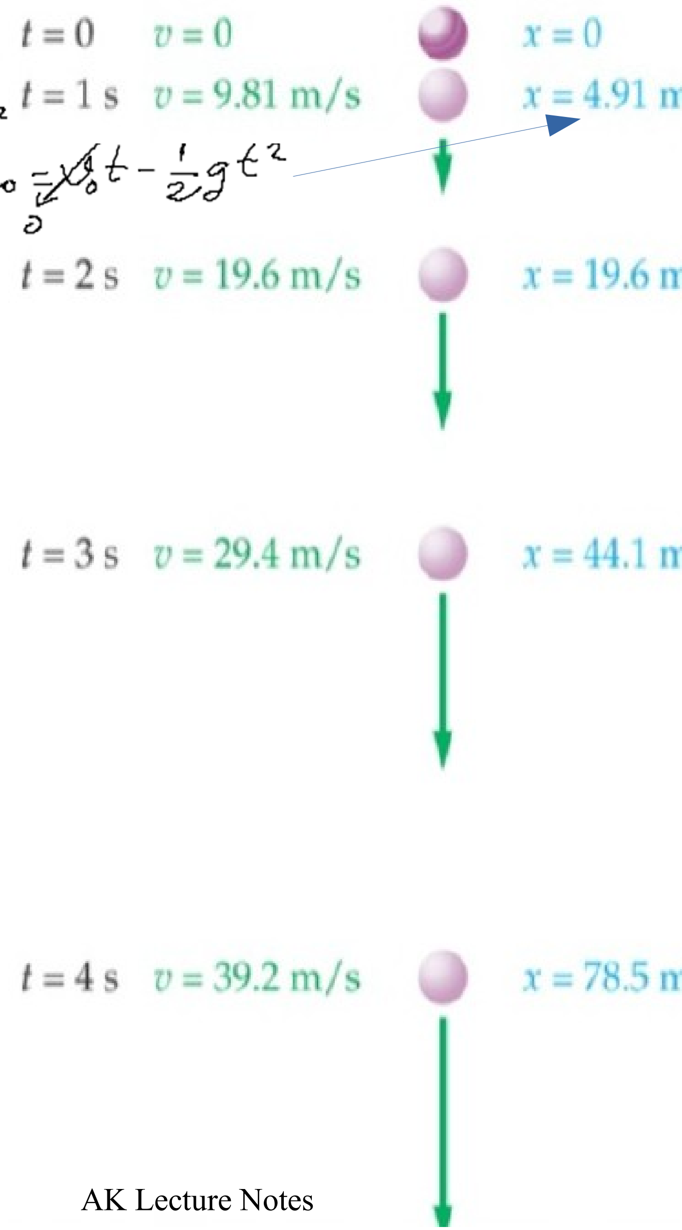
- In vacuum, a feather and an apple will fall
  no air resistance
  at the same rate.

$v = \cancel{v_0}^{0} - gt$, $9.81\ m/s^2$

$x - x_0 = \cancel{v_0}^{0} t - \frac{1}{2} g t^2$

Independent of the properties of the object (mass, density, shape, see Figure)





AK Lecture Notes

Example:

In Fig. 2-11, a pitcher tosses a baseball up along a y axis, with an initial speed of 12 m/s.

(a) How long does the ball take to reach its maximum height? $y_{max} \rightarrow v = 0 \quad t = ?$

KEY IDEAS

(1) Once the ball leaves the pitcher and before it returns to his hand, its acceleration is the free-fall acceleration $a = -g$. Because this is constant, Table 2-1 applies to the motion. (2) The velocity v at the maximum height must be 0.

Calculation: Knowing v, a, and the initial velocity $v_0 = 12$ m/s, and seeking t, we solve Eq. 2-11, which contains

$$v = v_0 - gt \qquad \uparrow \vec{v}_0 \quad \downarrow \vec{F}_g$$

$$t = ?$$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2 \quad ??$$

$$v^2 = v_0^2 - 2g(y - y_0)$$

$$v = 0 \text{ at } y \rightarrow y_{max}$$

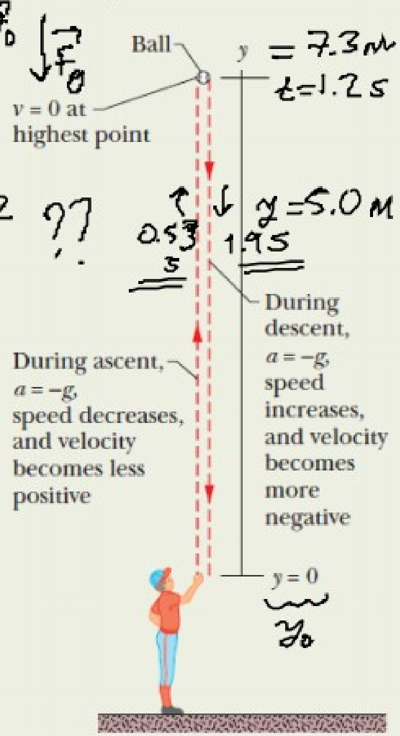


Fig. 2-11 A pitcher tosses a baseball straight up into the air. The equations of free fall apply for rising as well as for falling objects, provided any effects from the air can be neglected.

those four variables. This yields

$$t = \frac{v - v_0}{a} = \frac{0 - 12\text{ m/s}}{-9.8\text{ m/s}^2} = 1.2\text{ s.} \qquad \text{(Answer)}$$

(b) What is the ball's maximum height above its release point? $y_{max} = ?$

Calculation: We can take the ball's release point to be $y_0 = 0$. We can then write Eq. 2-16 in y notation, set $y - y_0 = y$ and $v = 0$ (at the maximum height), and solve for y. We get

$$y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (12\text{ m/s})^2}{2(-9.8\text{ m/s}^2)} = 7.3\text{ m.} \qquad \text{(Answer)}$$

(c) How long does the ball take to reach a point 5.0 m above its release point?

Calculations: We know v_0, $a = -g$, and displacement $y - y_0 = 5.0$ m, and we want t, so we choose Eq. 2-15. Rewriting it for y and setting $y_0 = 0$ give us

$$\left.\begin{matrix} y_i \rightarrow 0 \\ y_f \rightarrow 5 \end{matrix}\right\} \Delta y = y = v_0 t - \tfrac{1}{2} g t^2,$$

or

$$5.0\text{ m} = (12\text{ m/s})t - (\tfrac{1}{2})(9.8\text{ m/s}^2)t^2.$$

If we temporarily omit the units (having noted that they are consistent), we can rewrite this as

$$t_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$4.9t^2 - 12t + 5.0 = 0.$$

$$a t^2 + b t + c = 0$$

Solving this quadratic equation for t yields

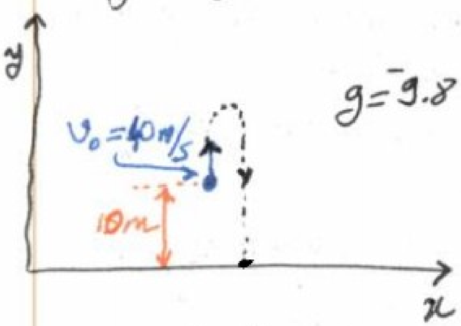
$$t = 0.53\text{ s} \quad \text{and} \quad t = 1.9\text{ s.} \qquad \text{(Answer)}$$

There are two such times! This is not really surprising because the ball passes twice through $y = 5.0$ m, once on the way up and once on the way down.

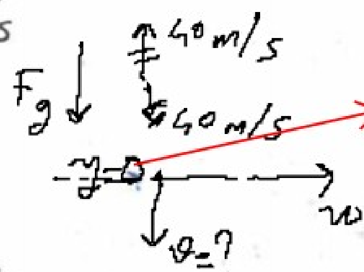
Example: A ball is thrown with an initial velocity of 40 m/s from a height of 10 m.

i) Determine the maximum height of the ball

ii) When will it hit the ground? And also determine the velocity of ball at the time of hit?



$g = 9.8$ m/s



maximum height $\Rightarrow v = 0$

$v = v_0 - gt \rightarrow 0 = 40 m/s - 9.8 m/s^2 \, t$

$y_i \rightsquigarrow +10$ m

$y_f \rightsquigarrow y_{max}$ $\Rightarrow \boxed{t = 4.08 \text{ sec}}$

$y - y_0 = v_0 t - \frac{1}{2} g t^2$

$y - 10m = 40 m/s (4.08 s) - \frac{1}{2} 9.8 m/s^2 (4.08 s)^2$

$\Rightarrow \boxed{y_{max} = \cancel{80.1 \text{ m}} \ 91.63 \text{ m}}$

t	0	4.08	8.16	8.40
v	40 m/s	0	40 m/s	42.38 m/s

$y_f \quad y_i$

$y - y_0 = v_0 t - \frac{1}{2} g t^2$

$0 - 10m = (40 m/s) t - \frac{1}{2}(9.8 m/s^2) t^2$

2nd → release at y_{max} $v_0 = 0$

3rd → $h = 10$ m, $\vec{v}_0 = 40(-\hat{j})$

$4.9 t^2 - 40 t - 10 = 0$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{40 \pm \sqrt{1600 - 4(4.9)(-10)}}{2 \cdot (4.9)}$

~~3.05 sec~~

$\boxed{t = 8.40 \text{ sec}}$

✗ -0.24 sec not physical

$v^2 = v_0^2 - 2g(y - y_0)$

$v^2 = (40 m/s)^2 - 2(9.8 m/s^2)(0 - 10 m)$

$\boxed{v = 42.38 \text{ m/s}}$

$\vec{v} = 42.39 m/s (-\hat{j})$

1. During a hard sneeze, your eyes might shut for 0.50 s. If you are driving a car at 90 km/h during such a sneeze, how far does the car move during that time?

1)

$$v = 90\,\frac{km}{h}\left(\frac{1000\,m}{1\,km}\right)\left(\frac{1\,h}{60\,min}\right)\left(\frac{1\,min}{60\,s}\right) = 25\,m/s = \frac{x}{t} = \frac{x}{0.50\,s}$$

$$x = 12.5\,m \simeq \underline{\underline{13\,m}}$$

x(t)

2. An electron moving along the x-axis has a position given by $x=16te^{-t}$ m, where t is in seconds. How far is the electron from the origin when it momentarily stops?

x=0 v=0

14) An e^- is moving along x-axis with $x(t)=16te^{-t}$
How far away e^- stops from the center of coordinate system & momentarily?

$a(t)$ & $b(t)$ $\quad 16t, \quad e^{-t}$ $\qquad \frac{d(ab)}{dt} = a'b + ab'$

stops → velocity is zero ⇒ $\frac{dx(t)}{dt} = v(t) = 16e^{-t} - 16te^{-t}$

$\boxed{v(t)=16e^{-t}(1-t)}$

$0=16e^{-t}(1-t) \Rightarrow t=1\,sn$
↳ $v=0$

we have found when. Now, find where $x(t=1sn)=16(1s)e^{-1}$
$\boxed{=5.9m}$

x(t)

3.The position of a particle moving along an axis is given by $x=12t^2-2t^3$, where x is in meters and t is in seconds. Determine (a) the position, (b) the velocity, and (c) the acceleration of the particle at $t=3.0$ s . (d) What is the maximum positive coordinate reached by the particle and (e) at what time is it reached? (f) What is the maximum positive velocity reached by the particle and (g) at what time is it reached? (h) What is the acceleration of the particle at the instant the particle is not moving (other than at $t=0$)? (i) Determine the average velocity of the particle between $t=0$ and $t=3$ s.

	0s	3s	4s
x	0m	54m	64m
v	0	18m/s	0

18) $x(t)=12t^2-2t^3$ @ $t=3$ s

i) $x(t=3s)=12\times(3s)^2-2(3s)^3=54m$

b) $v(t)=24t-6t^2$

$v(t=3s)=24\times(3s)-6\times(3s)^2=18m/s$

opposite signs

c) $a(t)=24-12t$

$a(t=3s)=24-12\times(3)=-12\,m/s^2$

iii) At max $x \rightarrow v=0$

$0=24t-6t^2 \rightarrow t=4s$

$\Rightarrow x(t=4s)=12\times16-2\times64=64m$ (x_{max})

v) $t=4s$

vi) At max $v \rightarrow a=0$

$0=24-12t \rightarrow t=2s$

$v(t=2s)=24\times2-6\times4=24m/s$ (v_{max})

vii) $t=2s$

3. (Continued) The position of a particle moving along an axis is given by $x=12t^2-2t^3$, where x is in meters and t is in seconds. Determine (a) the position, (b) the velocity, and (c) the acceleration of the particle at $t=3.0$ s . (d) What is the maximum positive coordinate reached by the particle and (e) at what time is it reached? (f) What is the maximum positive velocity reached by the particle and (g) at what time is it reached? (h) What is the acceleration of the particle at the instant the particle is not moving (other than at $t=0$)? (i) Determine the average velocity of the particle between $t=0$ and $t=3$ s.

viii) $a=?$ when $v=0$ momentarily no motion $\Leftrightarrow$ see parts iv & v

$\Rightarrow t=4s \longrightarrow a(t=4s)=24-12\times4 \boxed{=-24\, m/s^2}$

ix) v_{avg} when $t=0$, $t=3$ $\left\{ \begin{array}{l} v(t=0)=0\ m/s \\ v(t=3s)=18\ m/s \end{array} \right\}$ $v_{avg}=\dfrac{v_{t=0}+v_{t=3}}{2}$

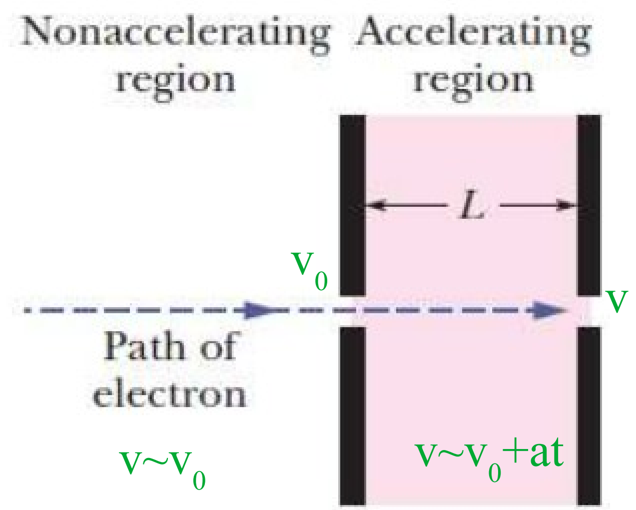
$= 9\, m/s$

which is not correct!!

These are instantaneous velocities.

For the concept of v_{avg}, we need $\dfrac{\Delta x}{\Delta t}=\dfrac{54m-0m}{3s\ -0s}\boxed{=18\, m/s}$

4. An electron with an initial velocity $v_0 = 1.50\times10^5$ m/s enters a region of length L=1.00 cm where it is electrically accelerated (see Figure). It emerges with $v = 5.70\times10^6$ m/s . What is its acceleration, assumed constant?

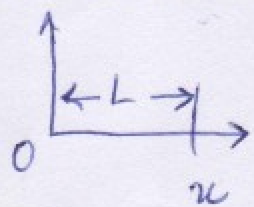


23) Constant acceleration. 2 basics + 3 derived eqns

$v_0 = 1.50\times10^5\, m/s = v_i$

$v = 5.70\times10^6\, m/s = v_f$

$L = 1.00\, cm = 0.01\, m$



$$v^2 = v_0^2 + 2a(x - x_0)$$

$$(5.70\times10^6\, m/s)^2 = (1.50\times10^5\, m/s)^2 + 2a(0.01\, m)$$

$$\Rightarrow a = \frac{(5.70\times10^6\, m/s)^2 - (1.50\times10^5\, m/s)^2}{2(0.01\, m)} = 1.62\times10^{15}\, m/s^2$$

OR

$x - x_0 = v_0 t + \frac{1}{2} a t^2$ & $v = v_0 + at$ X

2 unknowns (t,a)

assume as “a” constant!

5. The brakes on your car can slow you at a rate of 5.2 m/s2. (a) If you are going 137 km/h and suddenly see a state trooper, what is the minimum time in which you can get your car under the 90 km/h speed limit? (b) Graph x versus t and v versus t for such a slowing.

30) i) Rate of slowing: $\frac{\Delta v}{\Delta t} = a_{avg} = a$; constant acceleration $\boxed{v = v_0 + at}$

$v_0 = 137\ km/h = 137\ \frac{km}{h} \cdot \frac{1000\,m}{1\,km} \cdot \frac{1h}{60\,min} \cdot \frac{1\,min}{60\,s} = 38.06\ m/s$

$v = 90\ km/h = 90\ km/h$ " $= 25\ m/s$

$a = -5.2\ m/s^2$

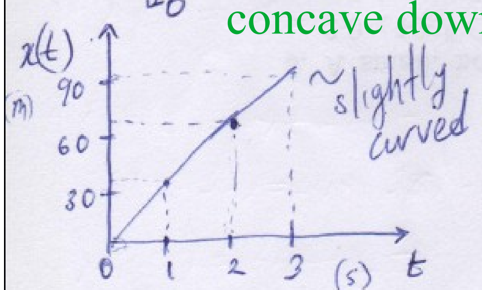
slowing

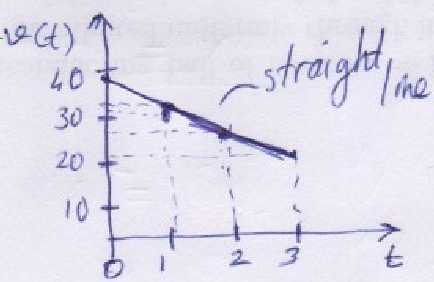
$v = v_0 + at \rightarrow t = \frac{v - v_0}{a}$

$t = \frac{25\,m/s - 38.06\,m/s}{-5.2\,m/s^2} = 2.51\,s$

ii) $x - x_0 = v_0 t + \frac{1}{2} a t^2$ (x_0: 0) $\quad v = v_0 + at$

concave down

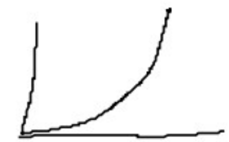




negative slope ⇒ slowing down

$x(t) = (38.06\,m/s)t + \frac{1}{2}(-5.2\,m/s^2)t^2 \quad v(t) = 38.06\,m/s + (-5.2\,m/s^2)t$

$x \propto t^2$

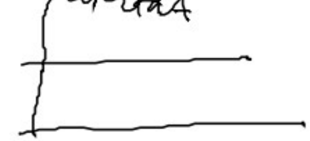


concave up

$v \propto t$

positive slope

a; constant

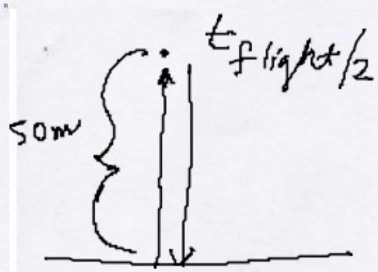


6. (a) With what speed must a ball be thrown vertically from ground level to rise to a maximum height of 50 m? (b) How long will it be in the air? (c) Sketch graphs of *y*, *v*, and *a* versus *t* for the ball. On the first two graphs, indicate the time at which 50 m is reached.

45) $a = -g$ $\quad y \uparrow \rightarrow x$

i) maximum height → stops → $v = 0$
50 m

$$v = v_0 - gt = 0 \rightarrow t = \frac{v_0}{g} = \frac{v_0}{9.8\,m/s^2} \; ?$$



$$v^2 = v_0^2 - 2g\underbrace{(y - \overset{\sim 0}{y_0})}_{50m} = 0^2 \rightarrow v_0 = \sqrt{2g(y - y_0)}$$

$$= \sqrt{2 \times 9.8\,m/s^2 \; 50\,m}$$

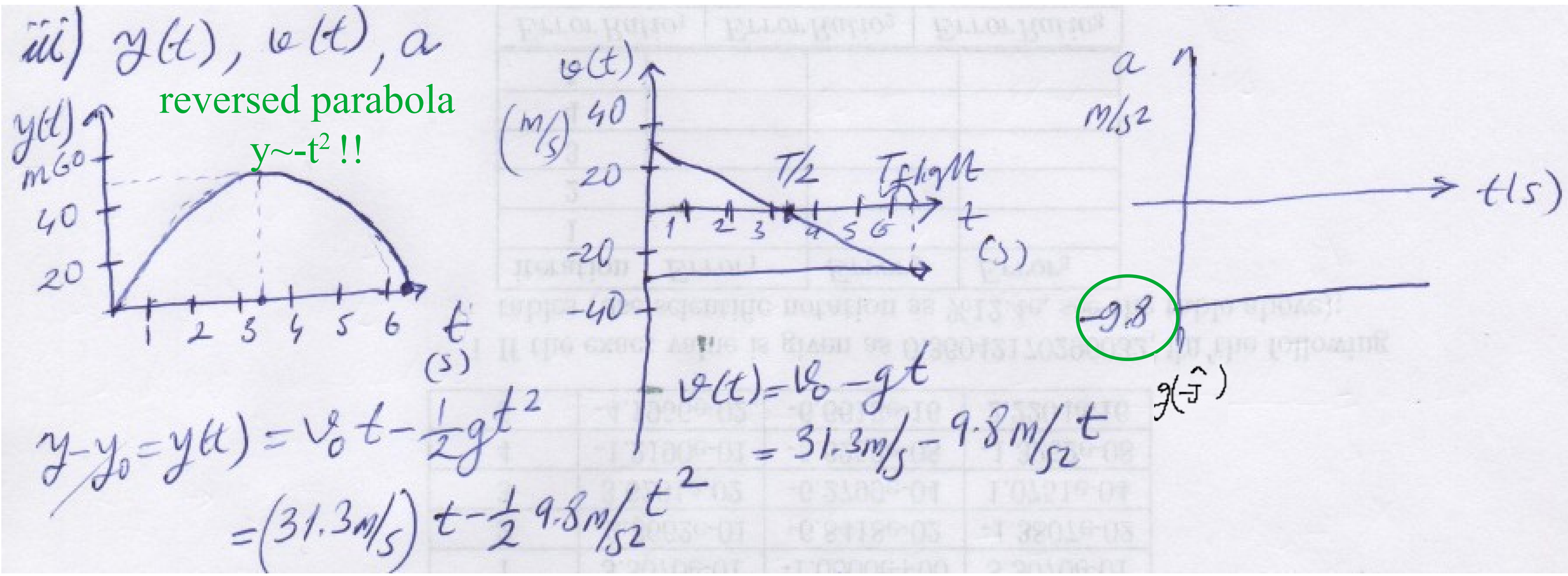
$$= 31.3\,m/s$$

ii) $t = \frac{v_0}{g}$; for maximum height

$$\underset{up}{\uparrow} + \underset{down}{\downarrow} = T_{flight} = 2t = \frac{2 v_0}{g} = \frac{2 \times 31.3\,m/s}{9.8\,m/s^2}$$

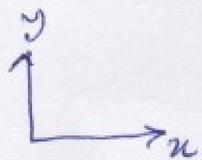
$$= 6.39\,s \sim 6.4\,s$$

6. (a) With what speed must a ball be thrown vertically from ground level to rise to a maximum height of 50 m? (b) How long will it be in the air? (c) Sketch graphs of *y*, *v*, and *a* versus *t* for the ball. On the first two graphs, indicate the time at which 50 m is reached.



7. To test the quality of a tennis ball, you drop it onto the floor from a height of 4.00 m. It rebounds to a height of 2.00 m. If the ball is in contact with the floor for 12.0 ms, (a) what is the magnitude of its average acceleration during that contact and (b) is the average acceleration up or down?

57)



i) $v^2 = v_0^2 - 2g(y-y_0)$

$v^2 = 0 - 2\cdot 9.8\,m/s^2\,(0-4m) \Rightarrow 8.85\,m/s = v \rightarrow \vec{v}_i = 8.85\,m/s\,(-\hat{j})$

$(y_f - y_i)$

velocity just before hitting

drop

$v^2 = v_0^2 - 2g(y-y_0)$

$(y_f - y_i)$

$0 = v_0^2 - 2\times 9.8\,m/s^2\,(2m-0m) \Rightarrow 6.26\,m/s\ v \rightarrow \vec{v}_f = 6.26\,m/s\,(\hat{j})$

rebounds to height

velocity just after hitting

$\Rightarrow \vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{12.0\times10^{-3}\,s} = \frac{[6.26\,m/s - (-8.85\,m/s)]\,\hat{j}}{12.0\times10^{-3}\,s}$

$= 1.26\times10^{3}\,m/s^2\,\hat{j}$

ii) positive $\hat{j} \Rightarrow$ upward

Position

- Relative to origin
- Positive and negative directions

Displacement

- Change in position (vector)

$$\Delta x = x_2 - x_1$$ **Eq. (2-1)**

Average Velocity

- Displacement / time (vector)

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$ **Eq. (2-2)**

Average Speed

- Distance traveled / time

$$s_{avg} = \frac{\text{total distance}}{\Delta t}$$ **Eq. (2-3)**

Instantaneous Velocity

- At a moment in time
- Speed is its magnitude

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$ **Eq. (2-4)**

Average Acceleration

- Ratio of change in velocity to change in time

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$ **Eq. (2-7)**

Instantaneous Acceleration

- First derivative of velocity
- Second derivative of position

$$a = \frac{dv}{dt}$$

Eq. (2-8)

Constant Acceleration

- Includes free-fall, where
  a = -*g* along the vertical axis

Tab. (2-1)

Equation Number	Equation	Missing Quantity
2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

Additional Materials

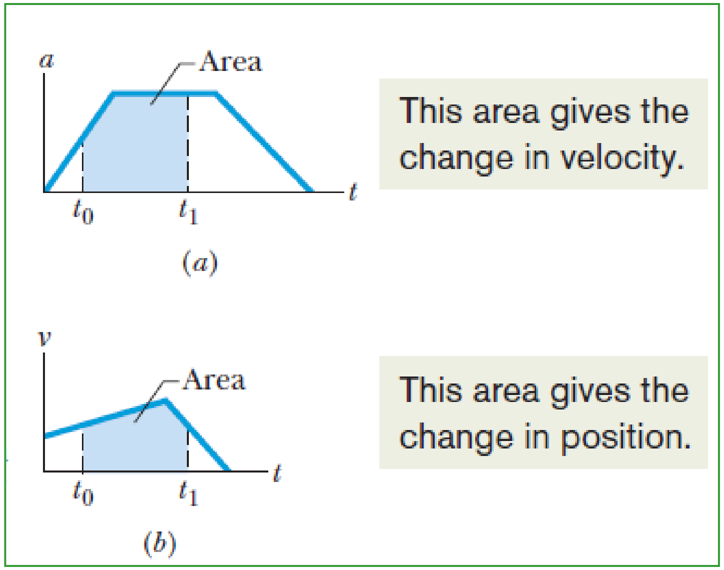
Integrating acceleration:

Starting from $a = dv/dt$

we obtain $v_1 - v_0 = \int_{t_0}^{t_1} a\, dt.$

(v_o= velocity at time t=0, and v_1= velocity at time t = t_1).

Note that $\int_{t_0}^{t_1} a\, dt = \left(\begin{array}{c}\text{area between acceleration curve} \\ \text{and time axis, from } t_0 \text{ to } t_1\end{array}\right).$



Integrating velocity

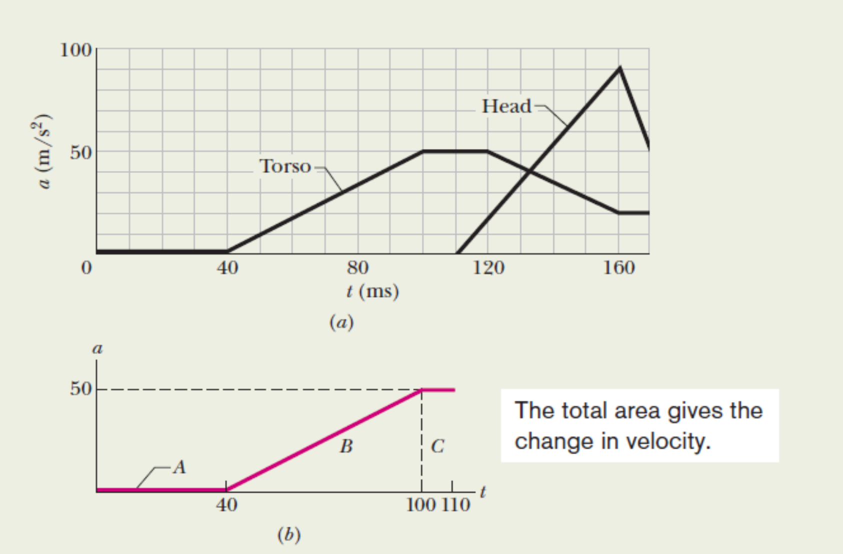
Similarly, we obtain $x_1 - x_0 = \int_{t_0}^{t_1} v\, dt,$

(x_o= position at time t = 0, and x_1 = position at time t=t_1), and $\int_{t_0}^{t_1} v\, dt = \left(\begin{array}{c}\text{area between velocity curve} \\ \text{and time axis, from } t_0 \text{ to } t_1\end{array}\right).$

Example:

"Whiplash injury" commonly occurs in a rear-end collision where a front car is hit from behind by a second car. In the 1970s, researchers concluded that the injury was due to the occupant's head being whipped back over the top of the seat as the car was slammed forward. As a result of this finding, head restraints were built into cars, yet neck injuries in rear-end collisions continued to occur.

In a recent test to study neck injury in rear-end collisions, a volunteer was strapped to a seat that was then moved abruptly to simulate a collision by a rear car moving at 10.5 km/h. Figure 2-13*a* gives the accelerations of the volunteer's torso and head during the collision, which began at time $t = 0$. The torso acceleration was delayed by 40 ms because during that time interval the seat back had to compress against the volunteer. The head acceleration was delayed by an additional 70 ms. What was the torso speed when the head began to accelerate?



KEY IDEA

We can calculate the torso speed at any time by finding an area on the torso $a(t)$ graph.

Calculations: We know that the initial torso speed is $v_0 = 0$ at time $t_0 = 0$, at the start of the "collision." We want the torso speed v_1 at time $t_1 = 110$ ms, which is when the head begins to accelerate.

$$v_1 - v_0 = \left(\begin{array}{c}\text{area between acceleration curve}\\ \text{and time axis, from } t_0 \text{ to } t_1\end{array}\right).$$

For convenience, let us separate the area into three regions (Fig. 2-13*b*). From 0 to 40 ms, region *A* has no area:

$$\text{area}_A = 0.$$

From 40 ms to 100 ms, region *B* has the shape of a triangle, with area

$$\text{area}_B = \tfrac{1}{2}(0.060\text{ s})(50\text{ m/s}^2) = 1.5\text{ m/s}.$$

From 100 ms to 110 ms, region *C* has the shape of a rectangle, with area

$$\text{area}_C = (0.010\text{ s})(50\text{ m/s}^2) = 0.50\text{ m/s}.$$

Substituting these values and $v_0 = 0$ into Eq. 2-26 gives us

$$v_1 - 0 = 0 + 1.5\text{ m/s} + 0.50\text{ m/s},$$

or

$$v_1 = 2.0\text{ m/s} = 7.2\text{ km/h}. \qquad \text{(Answer)}$$