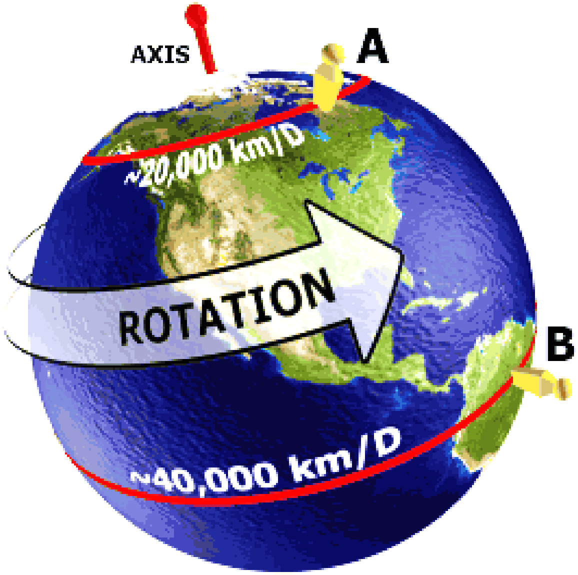
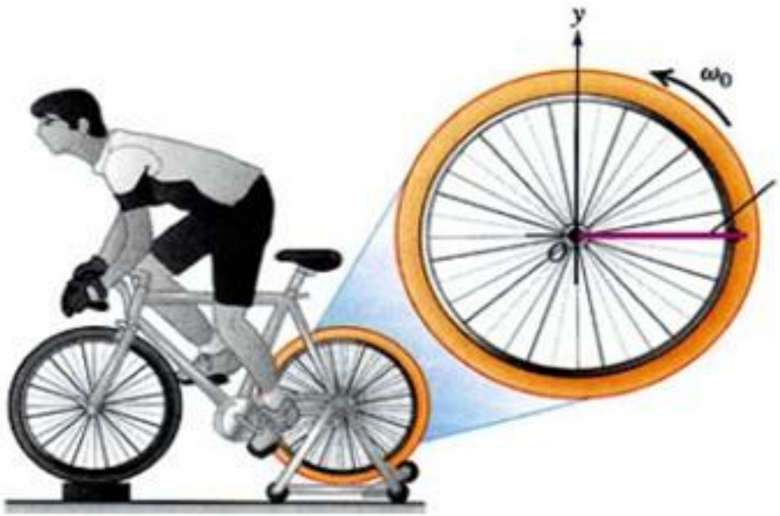
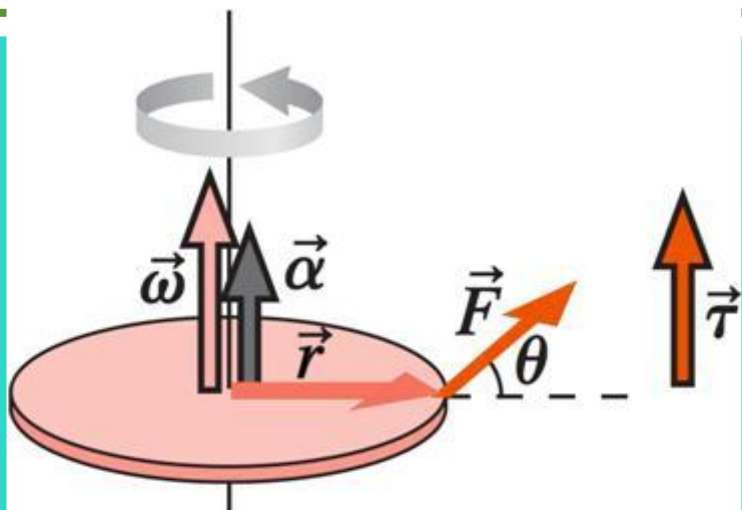




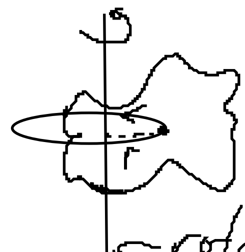
Chapter 10

Rotation



10 ROTATION 241

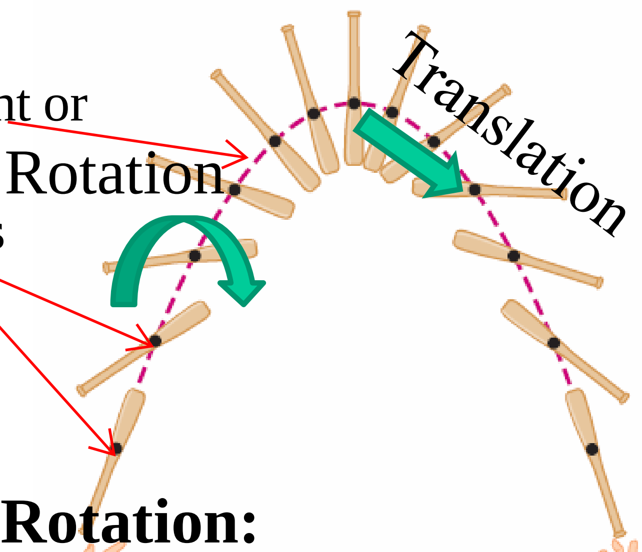
- We now look at motion of **rotation**.
- We will find the same laws apply.
- But we will need new quantities to express them
  - Torque , τ (if an applied force causes to a rotation)
  - Rotational inertia , I (since it has mass)
- A **rigid body** rotates as a unit, locked together.



- We look at rotation about a **fixed axis.**
- These requirements exclude from consideration: (not a rigid body)
  - The Sun, where layers of gas rotate separately
  - A rolling bowling ball, where rotation and translation occur & happens together (next chapter)

Translation and Rotation

- **Motion of Translation :** Object moves along a straight or curved line.
- **Motion of Rotation:** Object rotates about a fixed axis (object rotates about center of mass) .



Motion of Translation:

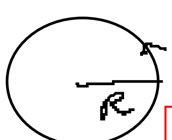
i.e. Motion along a straight line (along x-axis)

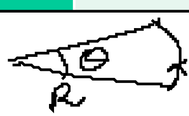
Motion of Rotation:

i.e. rotation of tires about a fixed axis

Variable	Symbol	Unit
Position	**x**	**meter (m)**
Displacement	**Δx**	**meter (m)**
Velocity	**V**	**meters/sec (m/s)**
Acceleration	**A**	**meters/sec^2(m/s^2)**

Variable	Symbol	Unit
Angular Position	θ	radians (rad)
Angular Displacement	$\Delta\theta$	radians (rad)
Angular Velocity	ω	radians/sec (rad/s)
Angular Acceleration	α	radians/sec^2 (rad/s^2)





$s \sim s = R\theta$

$\boxed{1 \text{ rad} = 2\pi = 1 \text{ revolution}}$

2π

$\theta = 2\pi$

1 rad ? one complete turn

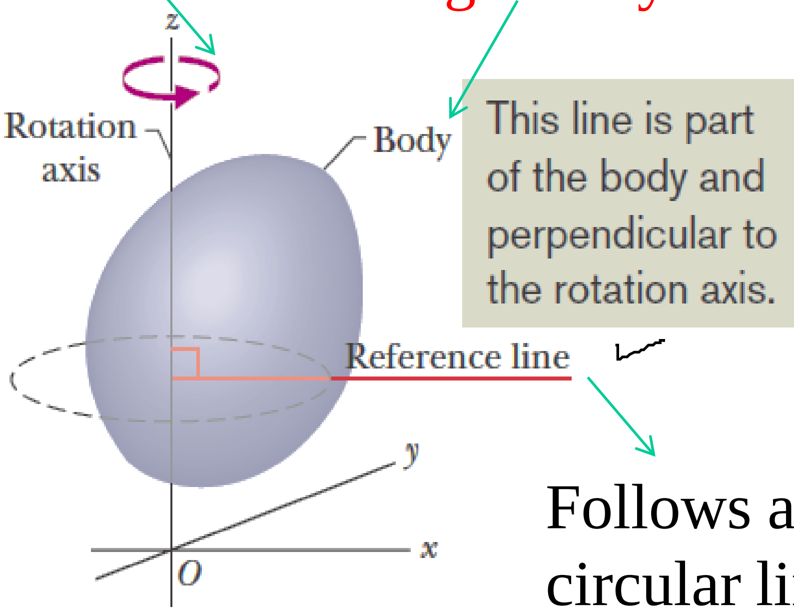
10.1_2 The Rotation & Rotational Variables

- A **rigid body** is a body that can rotate with all its parts locked together and without any change in its shape.
- A **fixed axis** means that the rotation occurs about an axis that does not move. The fixed axis is called the **axis of rotation.**

Fixed axes

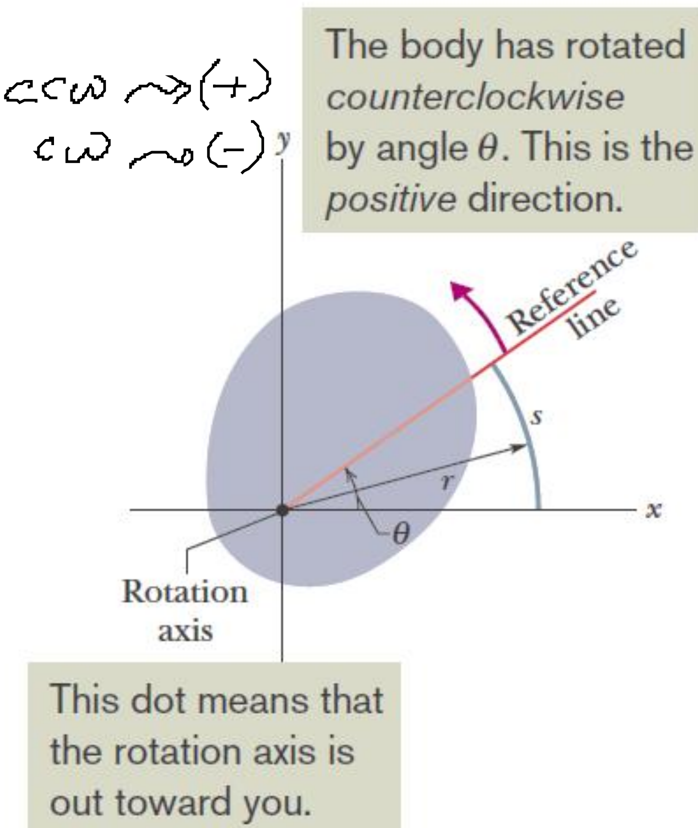
Rigid body

TOP VIEW



Follows a circular line

ccw → (+)
cw → (−)



- Reference Line: The **angular position** of this line (and of the object) is taken relative to a fixed direction, the **zero angular position.**

1- Angular Position, θ

(s = θR)

$$\theta = \frac{s}{r} \qquad \text{(radian measure)}$$

s: length of a circular arc that extends from the x axis reference line.

r: radius of the circle.

circumference of a circle : arc length

$$1 \text{ rev} = 360° = \frac{2\pi r}{r} = 2\pi \text{ rad},$$

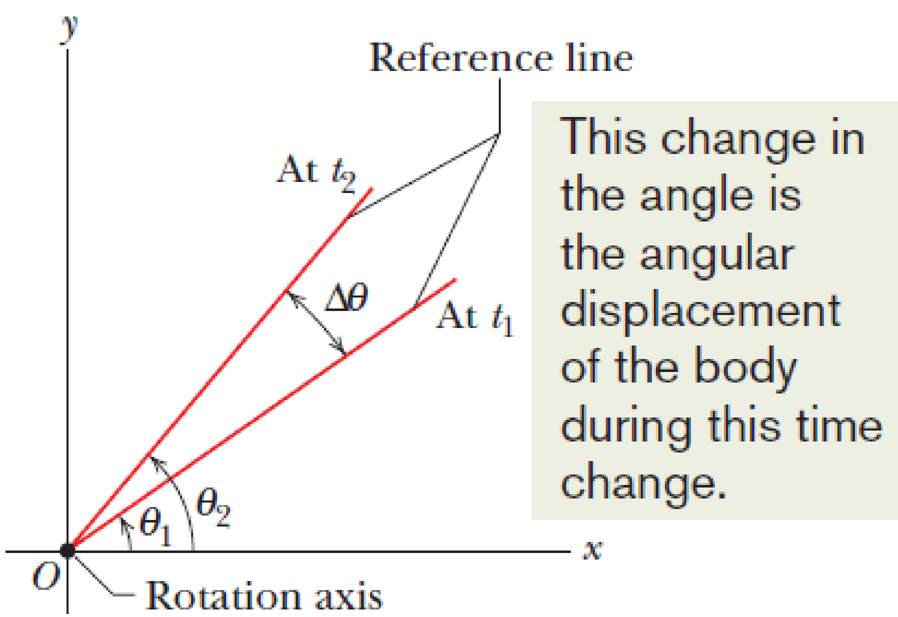
360° 2π rad
30° x

(as π → 3.14)

$$1 \text{ rad} = 57.3° = 0.159 \text{ rev}.$$

An angle (θ) measured in **radians (rad)!**

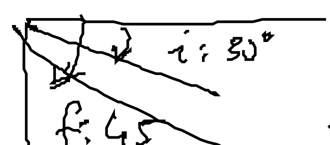
2- Angular Displacement (Δθ)



- Changing the angular position of the reference line from θ_1 to θ_2, the body undergoes an angular displacement Δθ given by;

$$\Delta\theta = \theta_2 - \theta_1.$$

- An angular displacement in the counterclockwise direction is positive, and one in the clockwise direction is negative.
- Clocks are negative!



Δθ = -45 - (-30°)

3- Angular Velocity (ω)

Average angular velocity: angular displacement during a time interval

$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t},$$

Instantenous angular velocity: limit as $\Delta t \to 0$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}.$$

- If the body is rigid, these equations hold for all points on the body.
- Magnitude of angular velocity = **angular speed.**

4- Angular Acceleration (α): If the angular velocity of a rotating body is not constant, then the body has an angular acceleration.

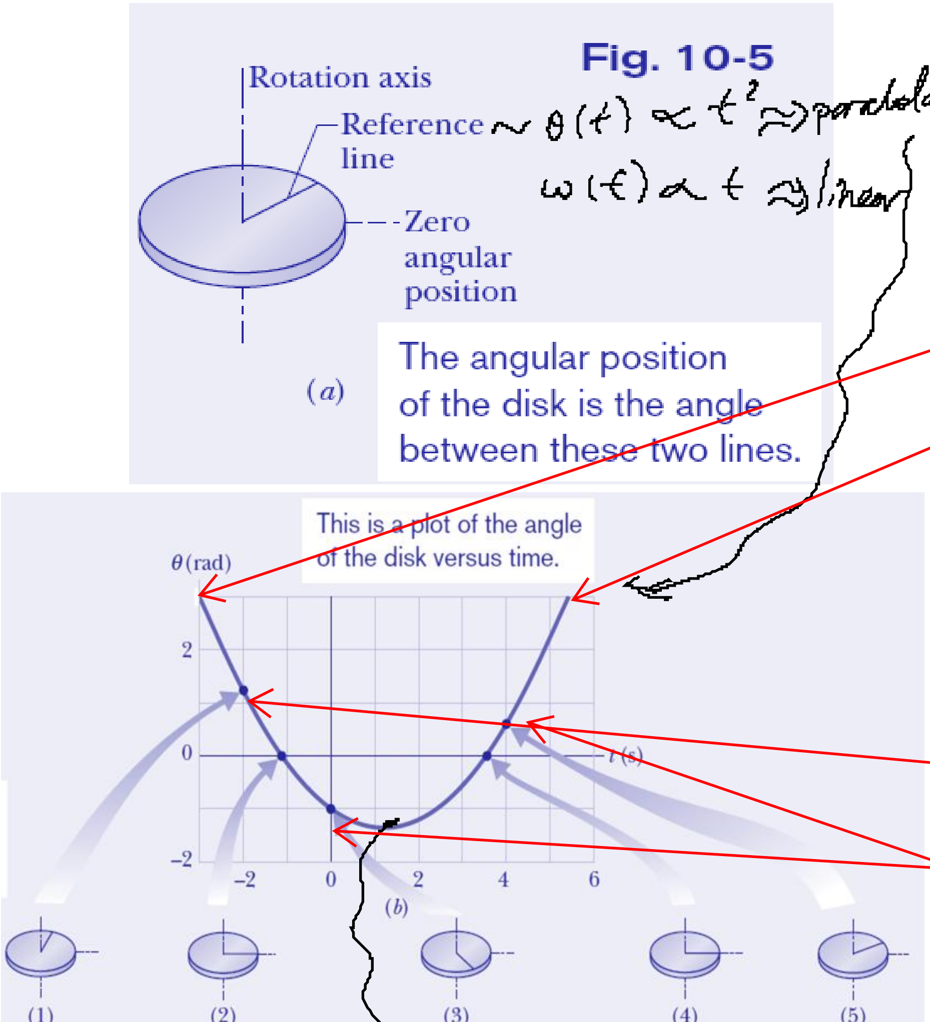
Average angular acceleration: angular velocity change during a time interval

$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t},$$

Intantenous angular acceleration: limit as $\Delta t \to 0$

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}.$$

Sample problem:



~ $\theta(t) \propto t^2$ ⇝ parabola

$\omega(t) \propto t$ ⇝ linear

min value ⇝ next page

The disk in Fig. 10-5*a* is rotating about its central axis like a merry-go-round. The angular position $\theta(t)$ of a reference line on the disk is given by

$$\theta = -1.00 - 0.600t + 0.250t^2, \qquad (10\text{-}9)$$

with t in seconds, θ in radians, and the zero angular position as indicated in the figure.

(a) Graph the angular position of the disk versus time from $t = -3.0$ s to $t = 5.4$ s. Sketch the disk and its angular position reference line at $t = -2.0$ s, 0 s, and 4.0 s, and when the curve crosses the t axis.

Calculations: To sketch the disk and its reference line at a particular time, we need to determine θ for that time. To do so, we substitute the time into Eq. 10-9. For $t = -2.0$ s, we get

$$\theta = -1.00 - (0.600)(-2.0) + (0.250)(-2.0)^2$$

$$= 1.2 \text{ rad} = 1.2 \text{ rad}\,\frac{360°}{2\pi \text{ rad}} = 69°.$$

360° 2π rad

1.2 rad x

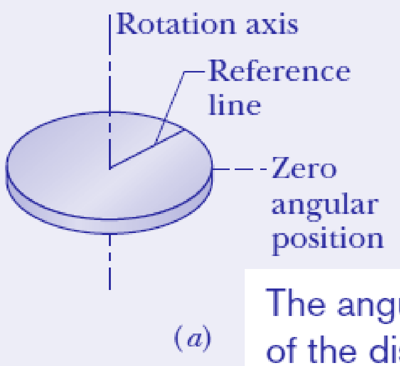
(+ counterccw)

This means that at $t = -2.0$ s the reference line on the disk is rotated counterclockwise from the zero position by 1.2 rad = 69° (counterclockwise because θ is positive). Sketch 1 in Fig. 10-5*b* shows this position of the reference line.

Similarly, for $t = 0$, we find $\theta = -1.00$ rad $= -57°$, which means that the reference line is rotated clockwise from the zero angular position by 1.0 rad, or 57°, as shown in sketch 3. For $t = 4.0$ s, we find $\theta = 0.60$ rad $= 34°$ (sketch 5). Drawing sketches for when the curve crosses the t axis is easy, because then $\theta = 0$ and the reference line is momentarily aligned with the zero angular position (sketches 2 and 4).

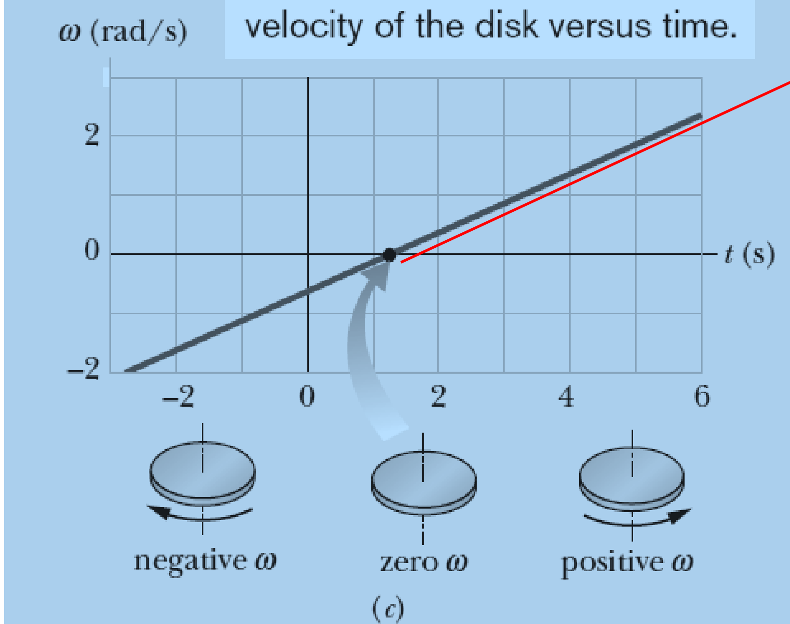
Sample problem contd.:

Fig. 10-5



The angular position of the disk is the angle between these two lines.

This is a plot of the angular velocity of the disk versus time.



(b) At what time t_{min} does $\theta(t)$ reach the minimum value shown in Fig. 10-5b? What is that minimum value?

Calculations: The first derivative of $\theta(t)$ is

$$\frac{d\theta}{dt} = -0.600 + 0.500t. \doteq 0 \qquad (10\text{-}10)$$

Setting this to zero and solving for t give us the time at which $\theta(t)$ is minimum:

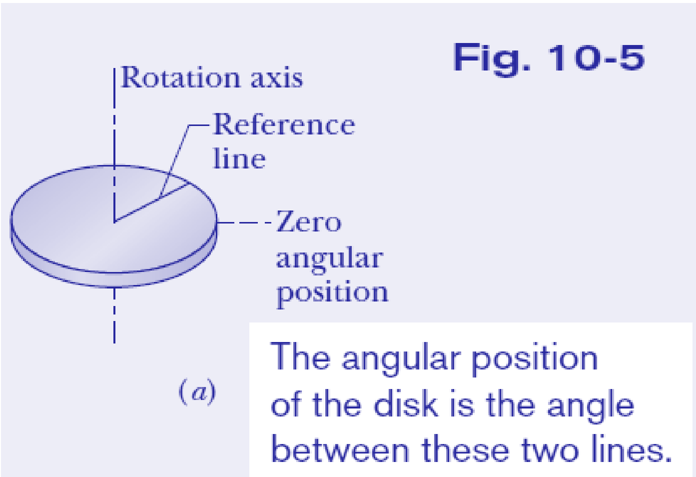
$$t_{min} = 1.20 \text{ s.} \qquad \text{(Answer)}$$

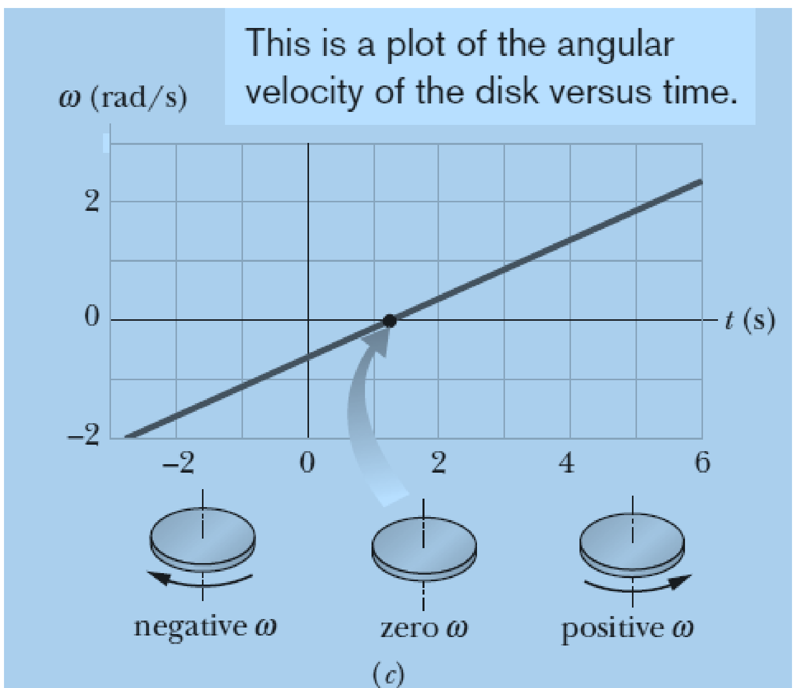
To get the minimum value of θ, we next substitute t_{min} into Eq. 10-9, finding

$$\theta = -1.36 \text{ rad} \approx \underline{\underline{-77.9°}}. \qquad \text{(Answer)}$$

$$\omega(t) = -0.6 + 0.5\,t$$

Sample problem contd.:





(c) Graph the angular velocity ω of the disk versus time from $t = -3.0$ s to $t = 6.0$ s. Sketch the disk and indicate the direction of turning and the sign of ω at $t = -2.0$ s, 4.0 s, and t_{min}.

Calculations: To sketch the disk at $t = -2.0$ s, we substitute that value into Eq. 10-11, obtaining

$$\omega = -1.6 \text{ rad/s.} \qquad \text{(Answer)}$$

The minus sign here tells us that at $t = -2.0$ s, the disk is turning clockwise (the left-hand sketch in Fig. 10-5*c*).

Substituting $t = 4.0$ s into Eq. 10-11 gives us

$$\omega = 1.4 \text{ rad/s.} \qquad \text{(Answer)}$$

The implied plus sign tells us that now the disk is turning counterclockwise (the righthand sketch in Fig. 10-5*c*).

For t_{min}, we already know that $d\theta/dt = 0$. So, we must also have $\omega = 0$. That is, the disk momentarily stops when the reference line reaches the minimum value of θ in Fig. 10-5*b*, as suggested by the center sketch in Fig. 10-5*c*. On the graph, this momentary stop is the zero point where the plot changes from the negative clockwise motion to the positive counterclockwise motion.

(d) Use the results in parts (a) through (c) to describe the motion of the disk from $t = -3.0$ s to $t = 6.0$ s.

Description: When we first observe the disk at $t = -3.0$ s, it has a positive angular position and is turning clockwise but slowing. It stops at angular position $\theta = -1.36$ rad and then begins to turn counterclockwise, with its angular position eventually becoming positive again.

Sample problem: Angular Velocity and Acceleration

A child's top is spun with angular acceleration

$$\alpha = 5t^3 - 4t,$$

with t in seconds and α in radians per second-squared. At $t = 0$, the top has angular velocity 5 rad/s, and a reference line on it is at angular position $\theta = 2$ rad.

(a) Obtain an expression for the angular velocity $\omega(t)$ of the top. That is, find an expression that explicitly indicates how the angular velocity depends on time. (We can tell that there *is* such a dependence because the top is undergoing an angular acceleration, which means that its angular velocity *is* changing.)

Calculations: Equation 10-8 tells us

$$d\omega = \alpha\, dt,$$

so

$$\int d\omega = \int \alpha\, dt.$$

From this we find

$$\omega = \int (5t^3 - 4t)\, dt = \tfrac{5}{4}t^4 - \tfrac{4}{2}t^2 + C.$$

To evaluate the constant of integration C, we note that $\omega = 5$ rad/s at $t = 0$. Substituting these values in our expression for ω yields

$$5 \text{ rad/s} = 0 - 0 + C,$$

so $C = 5$ rad/s. Then

$$\omega = \tfrac{5}{4}t^4 - 2t^2 + 5. \qquad \text{(Answer)}$$

(b) Obtain an expression for the angular position $\theta(t)$ of the top.

Calculations: Since Eq. 10-6 tells us that

$$d\theta = \omega\, dt,$$

we can write

$$\theta = \int \omega\, dt = \int (\tfrac{5}{4}t^4 - 2t^2 + 5)\, dt$$

$$= \tfrac{1}{4}t^5 - \tfrac{2}{3}t^3 + 5t + C'$$

$$= \tfrac{1}{4}t^5 - \tfrac{2}{3}t^3 + 5t + 2, \qquad \text{(Answer)}$$

where C' has been evaluated by noting that $\theta = 2$ rad at $t = 0$.

$$\theta(t) \sim \omega(t) \sim \alpha(t)$$

$$\frac{d\theta}{dt} \qquad \frac{d\omega}{dt}$$

$$\int d\omega = \int \alpha(t)\, dt$$

$$\int d\theta = \int \omega(t)\, dt$$

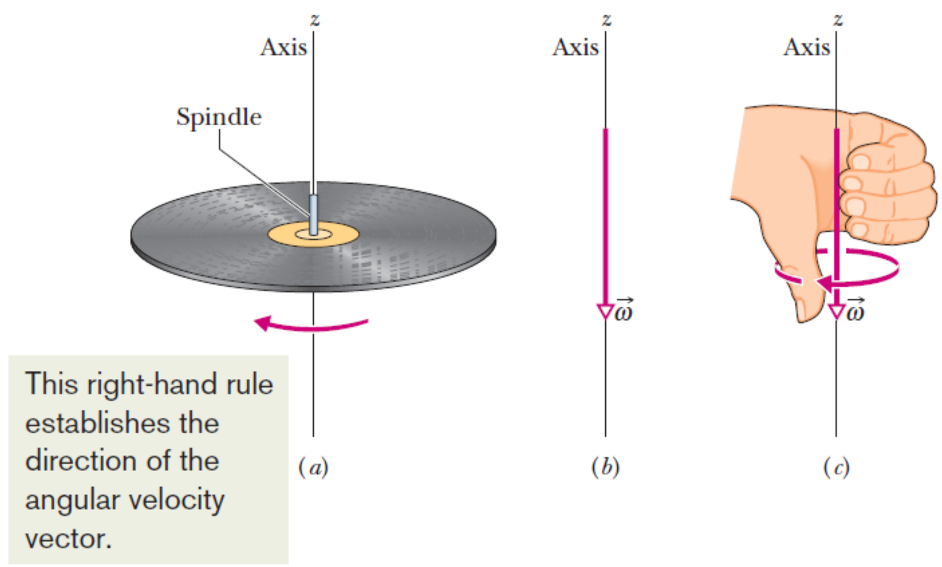
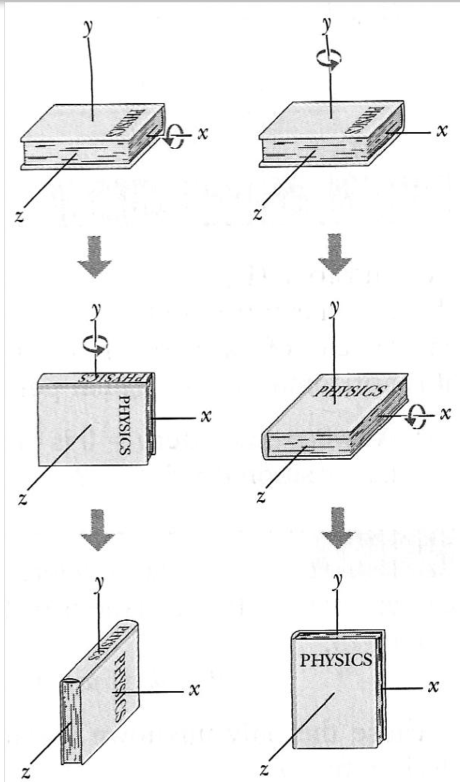


Fig. 10-6 (*a*) A record rotating about a vertical axis that coincides with the axis of the spindle. (*b*) The angular velocity of the rotating record can be represented by the vector $\vec{\omega}$, lying along the axis and pointing down, as shown. (*c*) We establish the direction of the angular velocity vector as downward by using a right-hand rule. When the fingers of the right hand curl around the record and point the way it is moving, the extended thumb points in the direction of $\vec{\omega}$.



- With right-hand rule to determine direction, angular velocity & acceleration can be written as vectors.
- If the body rotates around the vector, then the vector points along the axis of rotation,
- Angular displacements are *not* vectors, because the order of rotation matters for rotations around different axes.

TABLE 10-1

Equations of Motion for Constant Linear Acceleration and for Constant Angular Acceleration

Equation Number	Linear Equation	Missing Variable		Angular Equation
(2-11)	$v = v_0 + at$	$x - x_0$	$\theta - \theta_0$	$\omega = \omega_0 + \alpha t$
(2-15)	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
(2-16)	$v^2 = v_0^2 + 2a(x - x_0)$	t	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
(2-17)	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	α	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
(2-18)	$x - x_0 = vt - \frac{1}{2}at^2$	v_0	ω_0	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$

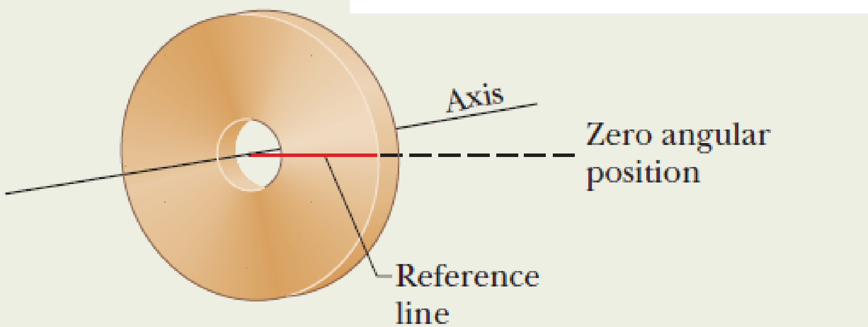
- Just as in the basic equations for constant linear acceleration, the basic equations for constant angular acceleration can be derived in a similar manner.
- The constant angular acceleration equations are similar to the constant linear acceleration equations.
  - **We simply change linear quantities to angular ones.**

10.4 Rotation with Constant Angular Acceleration

Sample problem: Constant Angular Acceleration

Fig. 10-8

We measure rotation by using this reference line.
Clockwise = negative
Counterclockwise = positive



A grindstone (Fig. 10-8) rotates at constant angular acceleration $\alpha = 0.35\ \text{rad/s}^2$. (+) At time $t = 0$, it has an angular velocity of $\omega_0 = -4.6$ rad/s and a reference line on it is horizontal, at the angular position $\theta_0 = 0$.

(a) At what time after $t = 0$ is the reference line at the angular position $\theta = 5.0$ rev? ~ 1 rev ~ 2π rad

The angular acceleration is constant, so we can use the rotation equation:

$$\theta - \theta_0 = \omega_0 t + \tfrac{1}{2}\alpha t^2,$$

Substituting known values and setting $\theta_0 = 0$ and $\theta = 5.0$ rev $= 10\pi$ rad give us

$$10\pi\ \text{rad} = (-4.6\ \text{rad/s})t + \tfrac{1}{2}(0.35\ \text{rad/s}^2)t^2.$$

Solving this quadratic equation for *t, we find t =32 s.*

(b) Describe the grindstone's rotation between t =0 and t =32 s.

Description: The wheel is initially rotating in the negative (clockwise) direction with angular velocity w_0=4.6 rad/s, but its angular acceleration a is positive.

The initial opposite signs of angular velocity and angular acceleration means that the wheel slows in its rotation in the negative direction, stops, and then reverses to rotate in the positive direction.

After the reference line comes back through its initial orientation of q= 0, the wheel turns an additional 5.0 rev by time t =32 s. θ = 0

(c) At what time t does the grindstone momentarily stop?

Calculation: With w =0, we solve for the corresponding time t.

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - (-4.6\ \text{rad/s})}{0.35\ \text{rad/s}^2} = 13\ \text{s}.$$

İZMİR KÂTİP ÇELEBİ ÜNİVERSİTESİ

Sample problem: Constant Angular Acceleration

Constant angular acceleration, riding a Rotor

While you are operating a Rotor (a large, vertical, rotating cylinder found in amusement parks), you spot a passenger in acute distress and decrease the angular velocity of the cylinder from 3.40 rad/s to 2.00 rad/s in 20.0 rev, at constant angular acceleration. (The passenger is obviously more of a "translation person" than a "rotation person.")

(a) What is the constant angular acceleration during this decrease in angular speed?

KEY IDEA

Because the cylinder's angular acceleration is constant, we can relate it to the angular velocity and angular displacement via the basic equations for constant angular acceleration (Eqs. 10-12 and 10-13).

Calculations: The initial angular velocity is $\omega_0 = 3.40$ rad/s, the angular displacement is $\theta - \theta_0 = 20.0$ rev, and the angular velocity at the end of that displacement is $\omega = 2.00$ rad/s. But we do not know the angular acceleration α and time t, which are in both basic equations.

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

2 unknowns (α, t) $\qquad \omega = \omega_0 + \alpha t$

To eliminate the unknown t, we use Eq. 10-12 to write

$$t = \frac{\omega - \omega_0}{\alpha},$$

which we then substitute into Eq. 10-13 to write

$$\theta - \theta_0 = \omega_0\left(\frac{\omega - \omega_0}{\alpha}\right) + \tfrac{1}{2}\alpha\left(\frac{\omega - \omega_0}{\alpha}\right)^2.$$

Solving for α, substituting known data, and converting 20 rev to 125.7 rad, we find

$$\alpha = \frac{\omega^2 - \omega_0^2}{2(\theta - \theta_0)} = \frac{(2.00 \text{ rad/s})^2 - (3.40 \text{ rad/s})^2}{2(125.7 \text{ rad})}$$

$$= -0.0301 \text{ rad/s}^2. \qquad \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \qquad \text{(Answer)}$$

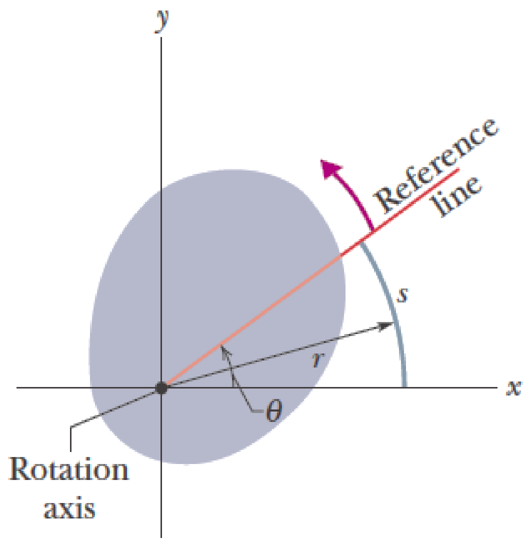
(b) How much time did the speed decrease take?

Calculation: Now that we know α, we can use Eq. 10-12 to solve for t:

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{2.00 \text{ rad/s} - 3.40 \text{ rad/s}}{-0.0301 \text{ rad/s}^2}$$

$$= 46.5 \text{ s.} \qquad \text{(Answer)}$$

- Linear and angular variables are related by *r* (perpendicular distance from the rotational axis)



r: radius of the circle travelled by particle

Period of Revolution

$$T = \frac{2\pi r}{v}$$

$$T = \frac{2\pi}{\omega} \qquad \text{(radian measure)}$$

- If a reference line rotates through an angle θ, a point within the body at a position ***r*** from the rotation axis moves a distance ***s*** along a circular arc, where ***s*** is given by:

$$s = \theta r \qquad \text{(radian measure).}$$

Differentiating the above equation with respect to time

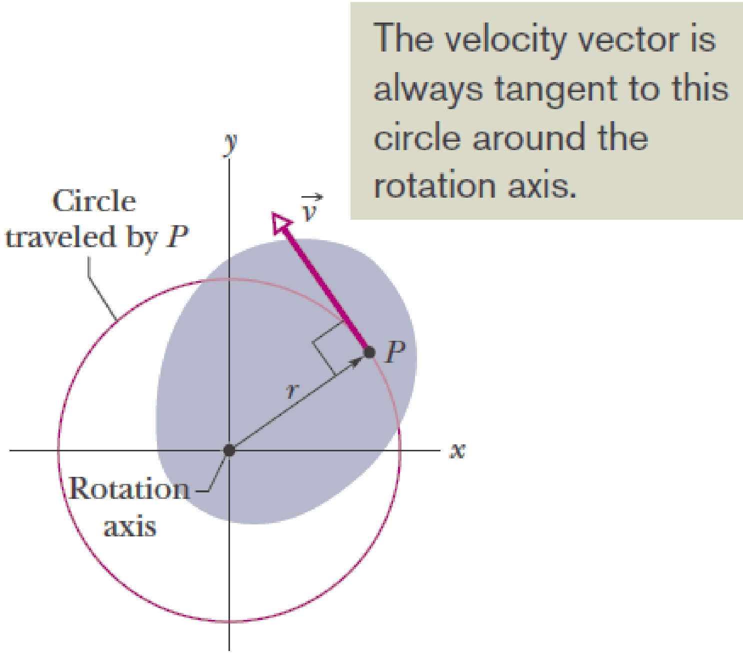
$$v = \omega r \qquad \text{(radian measure)}$$

Angular speed of particles are the same but linear speed increases as going to outside of the rotation axis!

ω

$v \rightsquigarrow v(r) = \omega r$

Tangential acceleration



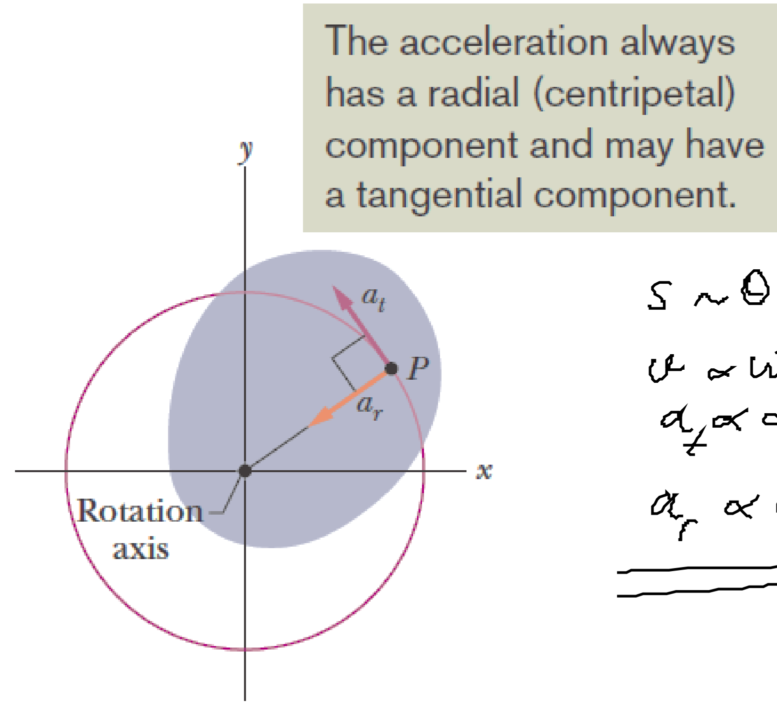
$$v = \omega r \qquad \text{(radian measure)}$$

Differentiating with respect to time

$$a_t = \alpha r \qquad \text{(radian measure)}$$

a_t : *tangential* acceleration

Radial acceleration (in terms of angular velocity)



$s \sim \theta r$

$v \propto \omega r$

$a_t \propto \alpha r$

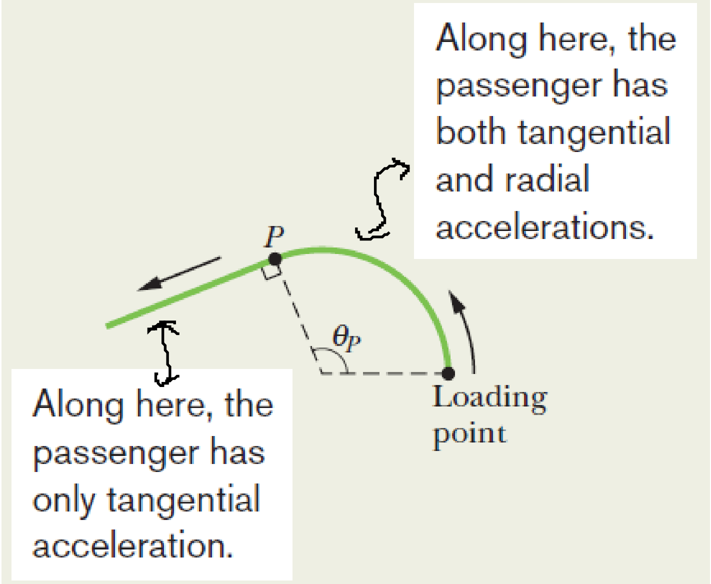
$a_r \propto \omega^2 r$

$$a_r = \frac{v^2}{r} = \omega^2 r$$

a_r :The radial part of the acceleration is the *centripetal* acceleration

Sample problem

Consider an induction roller coaster (which can be accelerated by magnetic forces even on a horizontal track). Each passenger is to leave the loading point with acceleration g along the horizontal track.
That first section of track forms a circular arc (see Figure), so that the passenger also experiences a centripetal acceleration. As the passenger accelerates along the arc, the magnitude of this centripetal acceleration increases alarmingly. When the magnitude ***a*** of the net acceleration reaches 4g at some point P and angle θ_P along the arc, the passenger moves in a straight line, along a tangent to the arc.



(a)What angle θ_P should the arc subtend so that a is 4g at point P?

<u>Calculations:</u>

Substituting ω_o=0, and θ_o=0, and we find:

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0).$$

$a_t = \alpha r$

But $$\omega^2 = \frac{2a_t\theta}{r}.$$

which gives: $a_r = \omega^2 r$ $\quad a_r = 2a_t\theta.$

we know

This leads us to a total acceleration:

$$a = \sqrt{a_t^2 + a_r^2}.$$

Substituting for a_r, and solving for θ lead to:

$$\theta = \tfrac{1}{2}\sqrt{\frac{a^2}{a_t^2} - 1}.$$

When ***a*** reaches the design value of 4g, angle θ is the angle θ_P . Substituting a =4g, θ= θ_P, and a_t= g, we find:

$$\theta_P = \tfrac{1}{2}\sqrt{\frac{(4g)^2}{g^2} - 1} = 1.94 \text{ rad} = 111°.$$

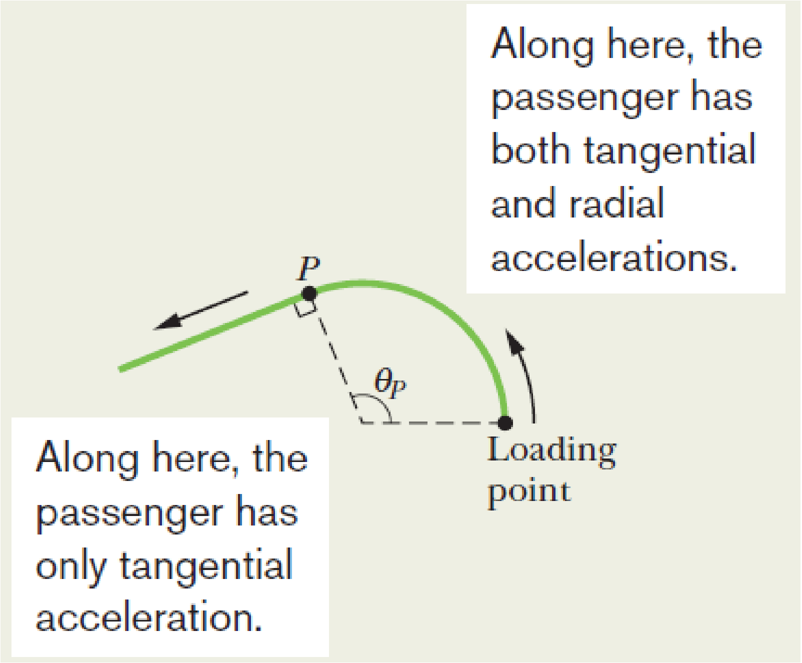
Sample problem cont.

(b) What is the magnitude a of the passenger's net acceleration at point P and after point P?

Reasoning: At P, a has the design value of 4g. Just after P is reached, the passenger moves in a straight line and no longer has centripetal acceleration.

Thus, the passenger has only the acceleration magnitude g along the track.

Hence, a =4g at P and a =g after P.



10.6 Kinetic Energy of Rotation

- For an extended rotating rigid body,
  - treat the body as a collection of particles with different linear velocities (same angular velocity for all particles but possibly different radii),
  - and add up the kinetic energies of all the particles to find the total kinetic energy of the body: $K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 + \cdots = \sum \frac{1}{2}m_iv_i^2$

(m_i is the mass of the i^{th} particle and v_i is its speed).

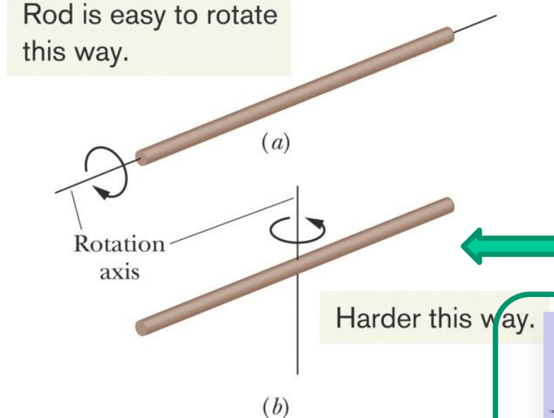
$$(v = \omega r) \Rightarrow K = \sum \frac{1}{2}m_i(\omega r_i)^2 = \frac{1}{2}\underbrace{\left(\sum m_i r_i^2\right)}\omega^2,$$

$mr^2 \sim I$: moment of inertia

$$I = \sum m_i r_i^2 \qquad \text{(rotational inertia)}$$

Rotational Inertia(or moment of inertia) I

(With respect to axis of rotation)



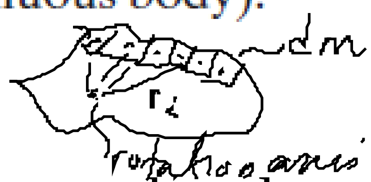
$$K = \frac{1}{2}I\omega^2 \qquad \text{(radian measure)}$$

Kinetic Energy of Rotation

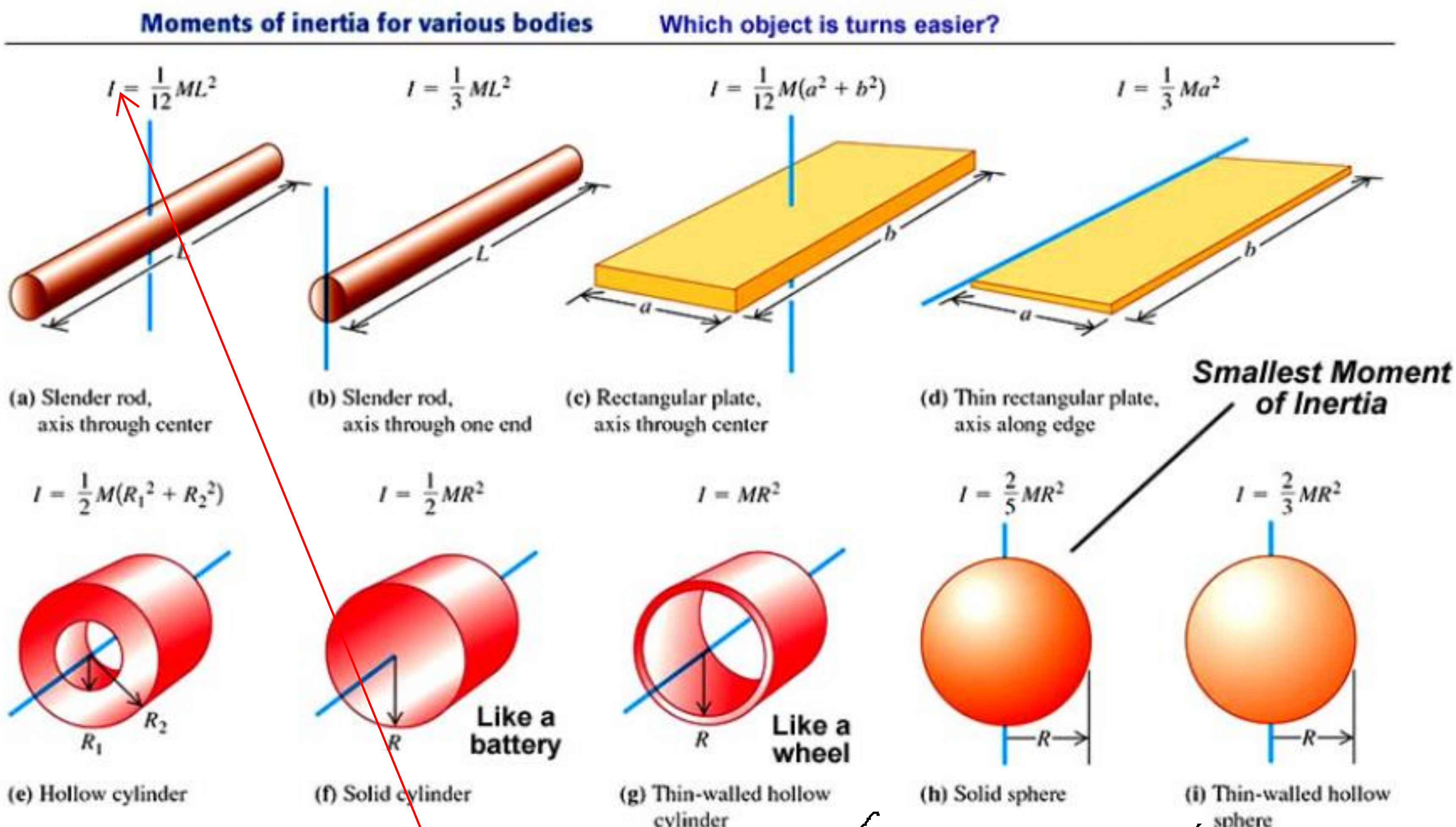
10.7 Calculating the Rotational Inertia

- I is a constant for a rigid object and given rotational axis.
- Caution: the axis for I must always be specified.
- Use these equations for a finite set of rotating particles.
- Rotational inertia corresponds to how difficult it is to change the state of rotation (speed up, slow down or change the axis of rotation)
- If a rigid body consists of a great many adjacent particles (it is continuous), we consider an integral and define the rotational inertia of the body as

$$I = \int r^2\, dm \qquad \text{(rotational inertia, continuous body).}$$



- In principle we can always use this equation.
- But there is a set of common shapes for which values have already been calculated (see Table) for common axes.



I for (a) $\quad \frac{M}{L} = \lambda = \frac{dm}{dx}$: uniform density $\left\{ I = \int r^2 dm \sim \int_{-L/2}^{L/2} x^2 \lambda \, dx = \lambda \frac{x^3}{3} \Big|_{-L/2}^{L/2} \right.$

$\underline{\frac{1}{12} M L^2} \quad \frac{M}{L} \frac{L^3}{12} \sim \lambda \frac{L^3}{12} \sim \lambda \left(\frac{L^3}{24} - \left(-\frac{L^3}{24} \right) \right)$

- If a rigid body consists of a great many adjacent particles

The Moment of Inertia about the center of the rod

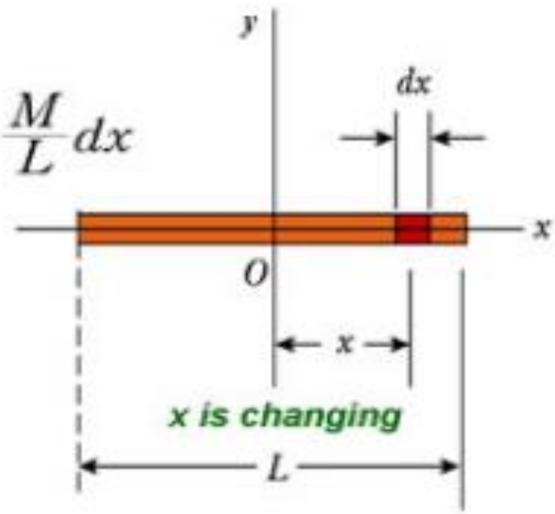
$$I = \lim_{\Delta m_i \to 0} \sum_i r_i^2 \Delta m_i = \int r^2 dm$$

Example:

a mass element

$$dm = \lambda dx = \frac{M}{L} dx$$

mass per unit length



from the definition

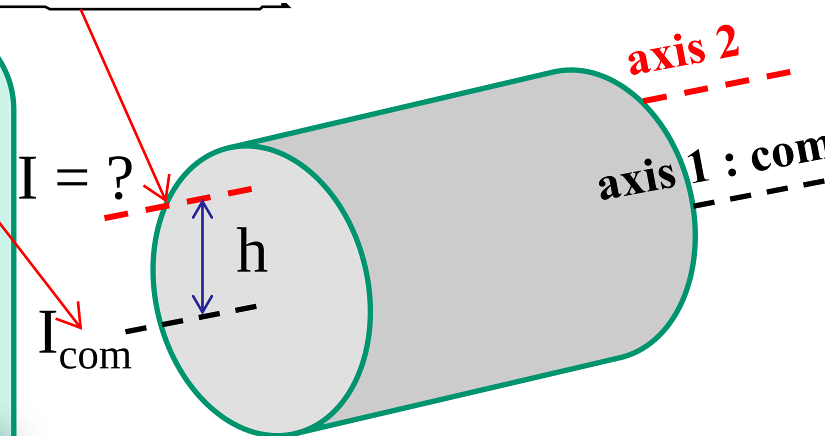
$$I = \int r^2 dm = \int_{-L/2}^{L/2} x^2 \frac{M}{L} dx = \frac{M}{L} \int_{-L/2}^{L/2} x^2 dx$$

$$I = \frac{M}{L} \left[\frac{x^3}{3}\right]_{-L/2}^{L/2} = \frac{1}{12} ML^2$$

- If we know the rotational inertia about an axis that extends through the body's center of mass (I_{com}), then we can calculate rotational inertia about an another axis ***parallel to the first one***.

Parallel Axis Theorem:
If h is a perpendicular distance between a given axis and the axis through the center of mass (these two axes being parallel). Then the rotational inertia I about the given axis is

$$I = I_{com} + Mh^2 \quad \text{(parallel-axis theorem)}$$



h: perpendicular distance to com

- Note the axes *must* be parallel, and the first *must* go through the center of mass.

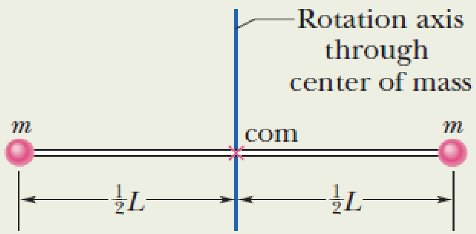
- This does *not* relate the moment of inertia for two arbitrary axes.

one should be com one
(I_{com})

Sample problem: Rotational Inertia

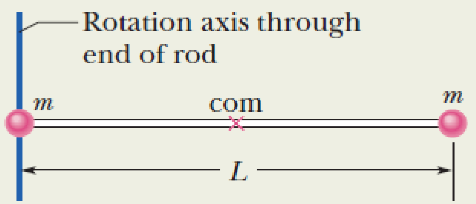
Figure 10-13*a* shows a rigid body consisting of two particles of mass m connected by a rod of length L and negligible mass.

(a) What is the rotational inertia I_{com} about an axis through the center of mass, perpendicular to the rod as shown?



(*a*)

Here the rotation axis is through the com.



(*b*)

Here it has been shifted from the com without changing the orientation. We can use the parallel-axis theorem.

Calculations: For the two particles, each at perpendicular distance $\frac{1}{2}L$ from the rotation axis, we have

$$I = \sum m_i r_i^2 = (m)(\tfrac{1}{2}L)^2 + (m)(\tfrac{1}{2}L)^2$$

$$= \tfrac{1}{2}mL^2. \qquad \text{(Answer)}$$

I_{com}

(b) What is the rotational inertia I of the body about an axis through the left end of the rod and parallel to the first axis (Fig. 10-13*b*)?

First technique: We calculate I as in part (a), except here the perpendicular distance r_i is zero for the particle on the left and L for the particle on the right. Now Eq. 10-33 gives us

$$I = m(0)^2 + mL^2 = mL^2. \qquad \text{(Answer)}$$

Second technique: Because we already know I_{com} about an axis through the center of mass and because the axis here is parallel to that "com axis," we can apply the parallel-axis theorem (Eq. 10-36). We find

$$I = I_{com} + Mh^2 = \tfrac{1}{2}mL^2 + (2m)(\tfrac{1}{2}L)^2$$

$$= mL^2. \qquad \text{(Answer)}$$

h

total mass !!

Sample problem: Rotational Inertia

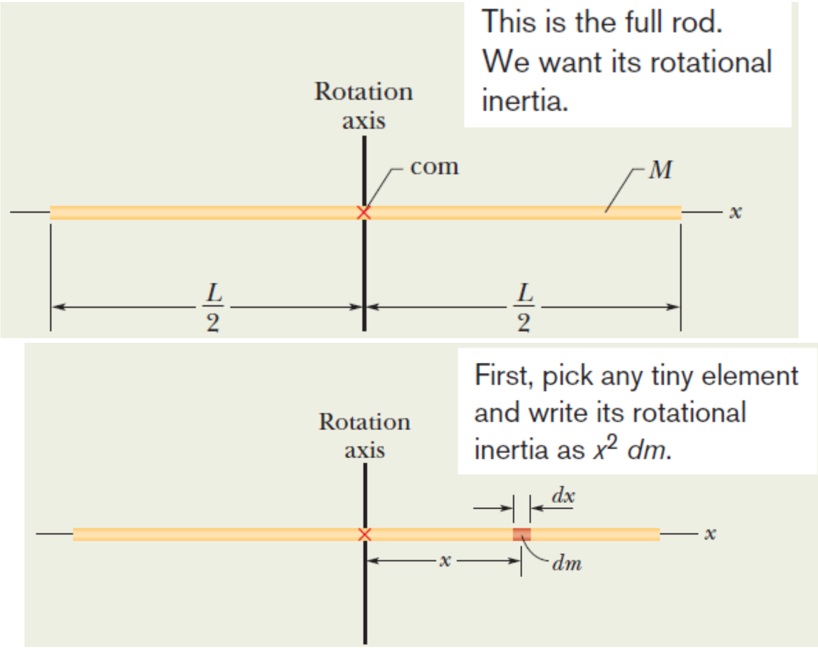


Figure 10-14 shows a thin, uniform rod of mass M and length L, on an x axis with the origin at the rod's center.

(a) What is the rotational inertia of the rod about the perpendicular rotation axis through the center?

KEY IDEAS

(1) Because the rod is uniform, its center of mass is at its center. Therefore, we are looking for I_{com}. (2) Because the rod is a continuous object, we must use the integral of Eq. 10-35,

$$I = \int r^2\,dm, \qquad (10\text{-}38)$$

to find the rotational inertia.

Calculations: We want to integrate with respect to coordinate x (not mass m as indicated in the integral), so we must relate the mass dm of an element of the rod to its length dx along the rod. (Such an element is shown in Fig. 10-14.) Because the rod is uniform, the ratio of mass to length is the same for all the elements and for the rod as a whole. Thus, we can write

$$\frac{\text{element's mass } dm}{\text{element's length } dx} = \frac{\text{rod's mass } M}{\text{rod's length } L}$$

or $$dm = \frac{M}{L}\,dx.$$

We can now substitute this result for dm and x for r in Eq. 10-38. Then we integrate from end to end of the rod (from $x = -L/2$ to $x = L/2$) to include all the elements. We find

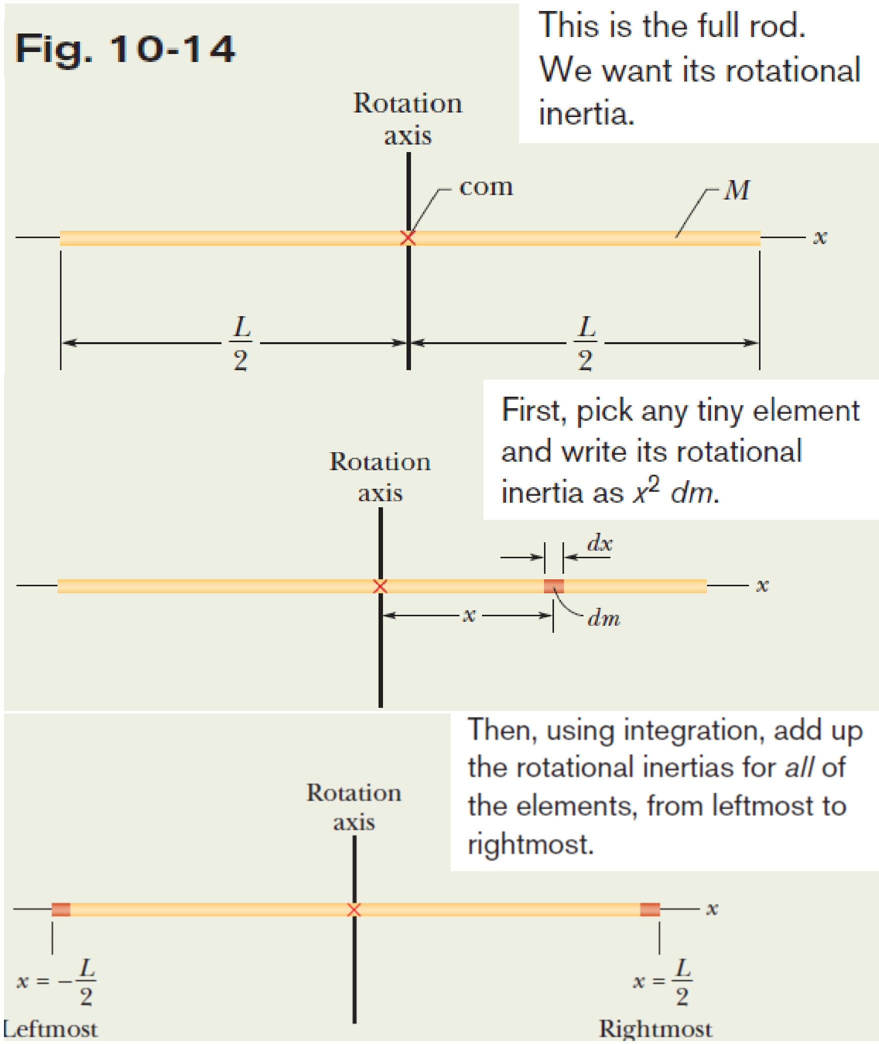
$$I = \int_{x=-L/2}^{x=+L/2} x^2 \left(\frac{M}{L}\right) dx$$

$$= \frac{M}{3L}\left[x^3\right]_{-L/2}^{+L/2} = \frac{M}{3L}\left[\left(\frac{L}{2}\right)^3 - \left(-\frac{L}{2}\right)^3\right]$$

$$= \tfrac{1}{12}ML^2. \qquad \text{(Answer)}$$

Sample problem: Rotational Inertia cont.

Fig. 10-14



(b) What is the rod's rotational inertia I about a new rotation axis that is perpendicular to the rod and through the left end?

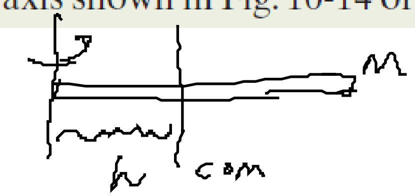
KEY IDEAS

We can find I by shifting the origin of the x axis to the left end of the rod and then integrating from $x = 0$ to $x = L$. However, here we shall use a more powerful (and easier) technique by applying the parallel-axis theorem (Eq. 10-36), in which we shift the rotation axis without changing its orientation.

Calculations: If we place the axis at the rod's end so that it is parallel to the axis through the center of mass, then we can use the parallel-axis theorem (Eq. 10-36). We know from part (a) that I_{com} is $\frac{1}{12}ML^2$. From Fig. 10-14, the perpendicular distance h between the new rotation axis and the center of mass is $\frac{1}{2}L$. Equation 10-36 then gives us

$$I = I_{com} + Mh^2 = \frac{1}{12}ML^2 + (M)(\tfrac{1}{2}L)^2$$
$$= \frac{1}{3}ML^2. \qquad \text{(Answer)}$$

Actually, this result holds for any axis through the left or right end that is perpendicular to the rod, whether it is parallel to the axis shown in Fig. 10-14 or not.



- The ability of a force F to rotate the body depends on both the magnitude of its tangential component F_t, and also on just how far from O, the pivot point, the force is applied.
- To include both these factors, a quantity called **torque τ** is defined as:

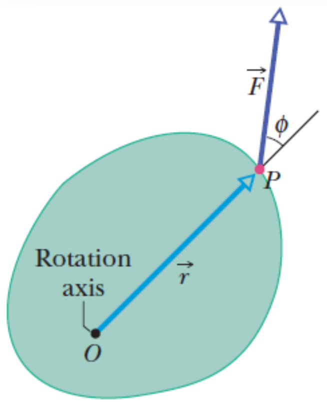
$$\tau = (r)(F\sin\phi).$$

$$\tau = (r)(F\sin\phi) = rF_t$$

$$\tau = (r\sin\phi)(F) = r_\perp F,$$

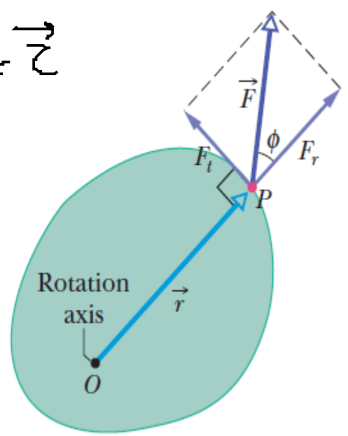
where $r_\perp$ is called the moment arm of F.

$\vec{r} \times \vec{F} = \vec{\tau}$



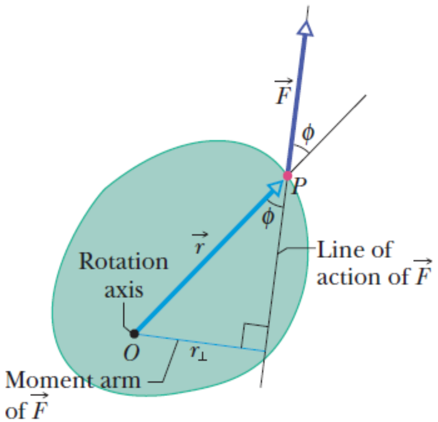
The torque due to this force causes rotation around this axis (which extends out toward you).

(a)



But actually only the *tangential* component of the force causes the rotation.

(b)



You calculate the same torque by using this moment arm distance and the full force magnitude.

(c)

Again, torque is positive if it would cause a counterclockwise rotation, otherwise negative. (cw)

- **Torque** takes these factors into account:
  1. A line extended through the applied force is called the **line of action** of the force.
  2. The perpendicular distance from the line of action to the axis is called the **moment arm.**
- The unit of torque is the newton-meter, N m.
- Note that 1 J = 1 N m, but torques are never expressed in joules, torque is not energy.

- Rewrite $F = ma$ with rotational variables:

$$F_t = ma_t.$$

$$\Downarrow$$

$$\tau = F_t r = m a_t r.$$

$$\Downarrow$$

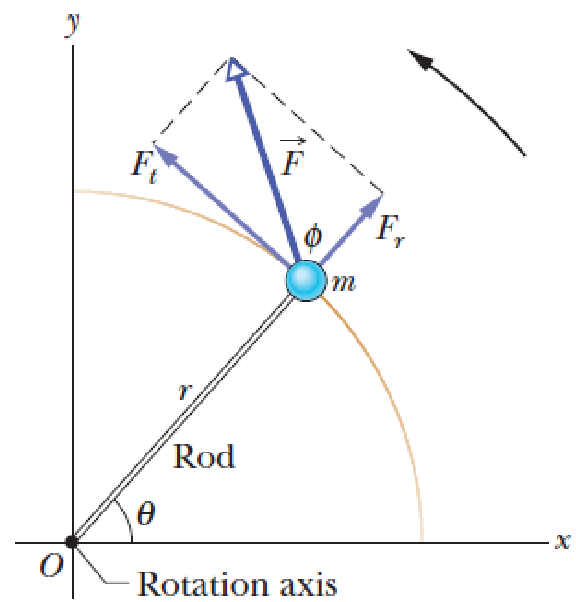
$$\tau = m(\alpha r)r = (mr^2)\alpha.$$

$$\Downarrow$$

$$\tau = I\alpha$$

$rF \leftrightsquigarrow I\alpha$

The torque due to the tangential component of the force causes an angular acceleration around the rotation axis.



- It is torque that causes angular acceleration

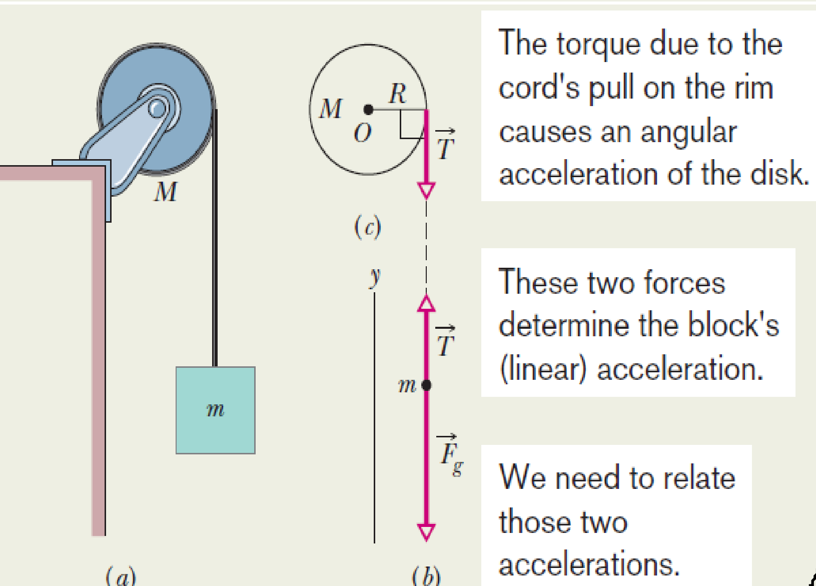
For more than one force, we can generalize:

$$\tau_{\text{net}} = I\alpha \qquad \text{(radian measure)}$$

10.9 Newton's 2nd Law of Rotation

Sample problem: Newton's 2nd Law in Rotational Moition

Figure 10-18*a* shows a uniform disk, with mass $M = 2.5$ kg and radius $R = 20$ cm, mounted on a fixed horizontal axle. A block with mass $m = 1.2$ kg hangs from a massless cord that is wrapped around the rim of the disk. Find the acceleration of the falling block, the angular acceleration of the disk, and the tension in the cord. The cord does not slip, and there is no friction at the axle.



Forces on block:

From the block's freebody, we can write Newton's second law for components along a vertical y axis as: T –mg= ma.

- The torque from the tension force, **T**, is -RT, negative because the torque rotates the disk clockwise from rest. $\tau = I\alpha = -RT$
- The rotational inertia I of the disk is ½ MR².
- But $\tau_{net} = I\alpha = -RT = 1/2\, MR^2\alpha$.
- Because the cord does not slip, the linear acceleration a of the block and the (tangential) linear acceleration a_t of the rim of the disk are equal. We now have: T=-1/2 Ma. $a = a_t = \alpha R$

Combining results:

$$a = -g\frac{2m}{M + 2m} = -(9.8\text{ m/s}^2)\frac{(2)(1.2\text{ kg})}{2.5\text{ kg} + (2)(1.2\text{ kg})}$$
$$= -4.8\text{ m/s}^2. \qquad \text{(Answer)}$$

We then find T:

$$T = -\tfrac{1}{2}Ma = -\tfrac{1}{2}(2.5\text{ kg})(-4.8\text{ m/s}^2)$$
$$= 6.0\text{ N}. \qquad \text{(Answer)}$$

The angular acceleration of the disk is:

$$\alpha = \frac{a}{R} = \frac{-4.8\text{ m/s}^2}{0.20\text{ m}} = -24\text{ rad/s}^2. \qquad \text{(Answer)}$$

Note that the acceleration a of the falling block is less than g, and tension T in the cord (=6.0 N) is less than the gravitational force on the hanging block (mg =11.8 N).

The rotational work-kinetic energy theorem states:

$$\Delta K = K_f - K_i = \tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_i^2 = W \qquad \text{(work–kinetic energy theorem).}$$

The work done in a rotation about a fixed axis can be calculated by:

$w = \int F\,dw$

$$W = \int_{\theta_i}^{\theta_f} \tau\, d\theta \qquad \text{(work, rotation about fixed axis),}$$

where τ is the torque doing the work *W, and* θ_i *and* θ_f *are the body's angular* positions before and after the work is done, respectively

When τ is constant

$$W = \tau(\theta_f - \theta_i) \qquad \text{(work, constant torque)}$$

The rate at which the work is done is the power

$$P = \frac{dW}{dt} = \tau\omega \qquad \text{(power, rotation about fixed axis)}$$

10.10 Work and Rotational Kinetic Energy

TABLE 10-3

Some Corresponding Relations for Translational and Rotational Motion

Pure Translation (Fixed Direction)		Pure Rotation (Fixed Axis)	
Position	x	Angular position	θ
Velocity	$v = dx/dt$	Angular velocity	$\omega = d\theta/dt$
Acceleration	$a = dv/dt$	Angular acceleration	$\alpha = d\omega/dt$
Mass	m	Rotational inertia	I
Newton's second law	$F_{net} = ma$	Newton's second law	$\tau_{net} = I\alpha$
Work	$W = \int F\,dx$	Work	$W = \int \tau\,d\theta$
Kinetic energy	$K = \frac{1}{2}mv^2$	Kinetic energy	$K = \frac{1}{2}I\omega^2$
Power (constant force)	$P = Fv$	Power (constant torque)	$P = \tau\omega$
Work–kinetic energy theorem	$W = \Delta K$	Work–kinetic energy theorem	$W = \Delta K$

Sample problem: Work, Rotational KE,

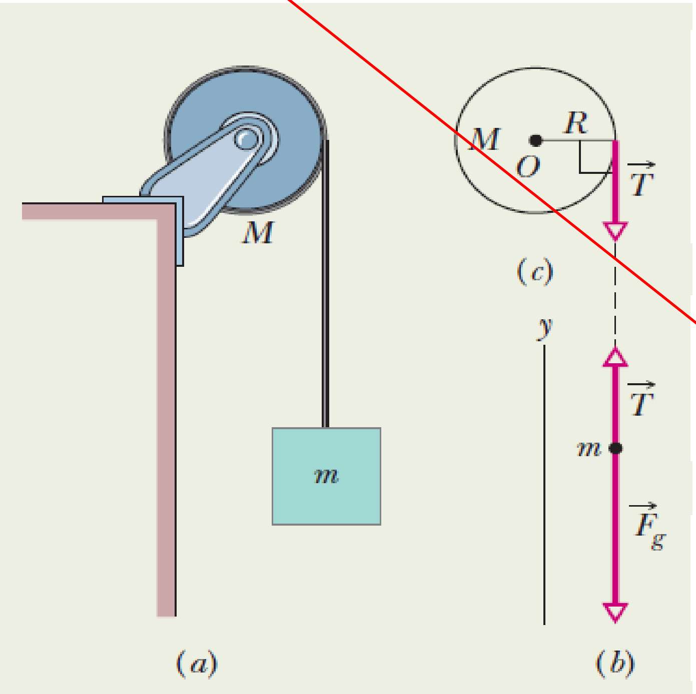
Let the disk in Fig. 10-18 start from rest at time $t = 0$ and also let the tension in the massless cord be 6.0 N and the angular acceleration of the disk be –24 rad/s². What is its rotational kinetic energy K at $t = 2.5$ s?

Calculations: Because we want ω and know α and ω_0 $(= 0)$, we use Eq. 10-12:

$$\omega = \omega_0 + \alpha t = 0 + \alpha t = \alpha t.$$

Substituting $\omega = \alpha t$ and $I = \frac{1}{2}MR^2$ into Eq. 10-34, we find

$$\begin{aligned} K &= \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(\tfrac{1}{2}MR^2)(\alpha t)^2 = \tfrac{1}{4}M(R\alpha t)^2 \\ &= \tfrac{1}{4}(2.5\text{ kg})[(0.20\text{ m})(-24\text{ rad/s}^2)(2.5\text{ s})]^2 \\ &= 90\text{ J}. \qquad \text{(Answer)} \end{aligned}$$



We can also get this answer by finding the disk's kinetic energy from the work done on the disk.

Calculations: First, we relate the *change* in the kinetic energy of the disk to the net work W done on the disk, using the work–kinetic energy theorem of Eq. 10-52 $(K_f - K_i = W)$. With K substituted for K_f and 0 for K_i, we get

$$K = K_i + W = 0 + W = W. \qquad (10\text{-}60)$$

Next we want to find the work W. We can relate W to the torques acting on the disk with Eq. 10-53 or 10-54. The only torque causing angular acceleration and doing work is the torque due to force $\vec{T}$ on the disk from the cord, which is equal to $-TR$. Because α is constant, this torque also must be constant. Thus, we can use Eq. 10-54 to write

$$W = \tau(\theta_f - \theta_i) = \underset{\text{(cw)}}{-TR}(\theta_f - \theta_i). \qquad (10\text{-}61)$$

Because α is constant, we can use Eq. 10-13 to find $\theta_f - \theta_i$. With $\omega_i = 0$, we have

$$\theta_f - \theta_i = \omega_i t + \tfrac{1}{2}\alpha t^2 = 0 + \tfrac{1}{2}\alpha t^2 = \tfrac{1}{2}\alpha t^2.$$

Now we substitute this into Eq. 10-61 and then substitute the result into Eq. 10-60. Inserting the given values $T = 6.0$ N and $\alpha = -24$ rad/s², we have

$$\begin{aligned} K = W &= -TR(\theta_f - \theta_i) = -TR(\tfrac{1}{2}\alpha t^2) = -\tfrac{1}{2}TR\alpha t^2 \\ &= -\tfrac{1}{2}(6.0\text{ N})(0.20\text{ m})(-24\text{ rad/s}^2)(2.5\text{ s})^2 \\ &= 90\text{ J}. \qquad \text{(Answer)} \end{aligned}$$

Angular Position

Measured around a **rotation axis**, relative to a **reference line**:

$$\theta = \frac{s}{r}$$

Eq. (10-1)

Angular Velocity and Speed

Average and instantaneous values:

$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t},$$

Eq. (10-5)

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}.$$

Eq. (10-6)

Angular Displacement

A change in angular position

$$\Delta\theta = \theta_2 - \theta_1.$$

Eq. (10-4)

Angular Acceleration

Average and instantaneous values:

$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t},$$

Eq. (10-7)

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}.$$

Eq. (10-8)

Kinematic Equations

Given in Table 10-1 for constant acceleration

Match the linear case

Rotational Kinetic Energy and Rotational Inertia

$$K = \tfrac{1}{2} I\omega^2 \quad \text{(radian measure)}$$

Eq. (10-34)

$$I = \sum m_i r_i^2 \quad \text{(rotational inertia)}$$

Eq. (10-33)

Linear and Angular Variables Related

Linear and angular displacement, velocity, and acceleration are related by *r*

The Parallel-Axis Theorem

Relate moment of inertia around any parallel axis to value around com axis

$$I = I_{\mathrm{com}} + Mh^2$$ **Eq. (10-36)**

Torque

Force applied at distance from an axis:

$$\tau = (r)(F \sin \phi).$$ **Eq. (10-39)**

Moment arm: perpendicular distance to the rotation axis

Work and Rotational Kinetic Energy

$$W = \int_{\theta_i}^{\theta_f} \tau \, d\theta$$ **Eq. (10-53)**

$$P = \frac{dW}{dt} = \tau\omega$$ **Eq. (10-55)**

Newton's Second Law in Angular Form

$$\tau_{\mathrm{net}} = I\alpha$$ **Eq. (10-42)**

1. A disk rotates about its central axis starting from rest and accelerates with constant angular acceleration. At one time it is rotating at 10 rev/s; 60 revolutions later, its angular speed is 15 rev/s. Calculate (a) the angular acceleration, (b) the time required to complete the 60 revolutions, (c) the time required to reach the 10 rev/s angular speed, and (d) the number of revolutions from rest until the time the disk reaches the 10 rev/s angular speed.

3 (14)

Disc rotates around its central axis
α : constant
$\omega_1 = 10$ rev/s $t = t_1$
$\omega_2 = 15$ rev/s $t = t_2$
60 revolutions

i) $\omega = \omega_0 + \alpha t$; $\omega_2^2 = \omega_1^2 + 2\alpha(\theta_2 - \theta_1)$ 60 rev

$\rightarrow (15\ \text{rev/s})^2 = (10\ \text{rev/s})^2 + 2\alpha\ 60\ \text{rev} \rightarrow \boxed{\alpha = 1.04\ \text{rev/s}^2}$ (+) ccw

ii) $t_2 - t_1 = \Delta t = ?$ Now use $\omega_2 = \omega_1 + \alpha t$; $15\ \text{rev/s} = 10\ \text{rev/s} + 1.04\ \text{rev/s}^2\ t$

$\Rightarrow \boxed{t = 4.81\ \text{s}}$ $t_1 = 0 \rightarrow t_2 = \Delta t = t$ starting time OR $\Delta\theta = \frac{1}{2}(\omega_1 + \omega_2)\Delta t$

iii) $\omega_1 = \omega_0 + \alpha t \rightarrow t = \dfrac{10\ \text{rev/s} - 0}{1.04\ \text{rev/s}^2}$ $\boxed{t = 9.6\ \text{s}}$

iv) # of revolutions = ? from $t = 0$ to $t = 9.6$ s. Use $\omega_1^2 = \omega_0^2 + 2\alpha\Delta\theta$

$\Rightarrow (10\ \text{rev/s})^2 = 0^2 + 2 \times 1.04\ \text{rev/s}^2\ \Delta\theta \rightarrow \boxed{\Delta\theta = 48\ \text{rev}}$

2. If a 32.0 N·m torque on a wheel causes angular acceleration 25.0 rad/s^2, what is the wheel's rotational inertia?

(50) $\tau = I\alpha \quad \left(\tau = r \times F = rF\sin 90 = rF_t = r m a_t = r m \alpha r = \underbrace{mr^2}_{I}\alpha \right)$

$$\Rightarrow I = \frac{\tau}{\alpha} = \frac{32.0\ \mathrm{N.m}}{25.0\ \mathrm{rad/s^2}} = 1.28\ \mathrm{kg\,m^2}$$

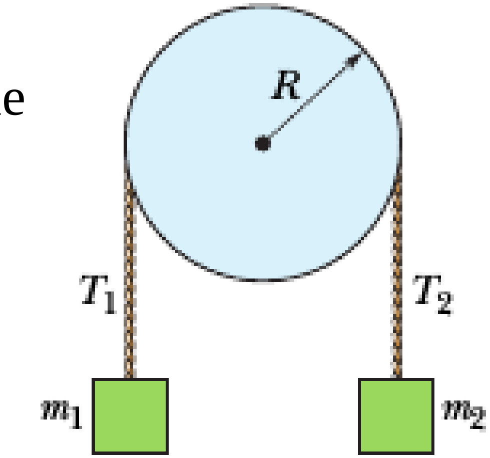
$$\frac{\mathrm{kg\ m/s^2\ m}}{\mathrm{rad/s^2}}$$

$I \propto m r^2$

$\mathrm{kg} \quad \mathrm{m^2}$

10 Solved Problems

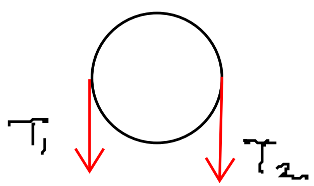
3.In Figure, block 1 has mass m_1=460 g, block 2 has mass m_2=500 g, and the pulley, which is mounted on a horizontal axle with negligible friction, has radius R=5.00 cm. When released from rest, block 2 falls 75.0 cm in 5.00 s without the cord slipping on the pulley. (a) What is the magnitude of the acceleration of the blocks? What are (b) tension T_2 and (*c*) tension T_1? (d) What is the magnitude of the pulley's angular acceleration? (e) What is its rotational inertia?



↓(+) y

$m_2 g - T_2 = m_2 a$

$m_1 g - T_1 = -m_1 a$



$(T_2 - T_1)R = \tau$

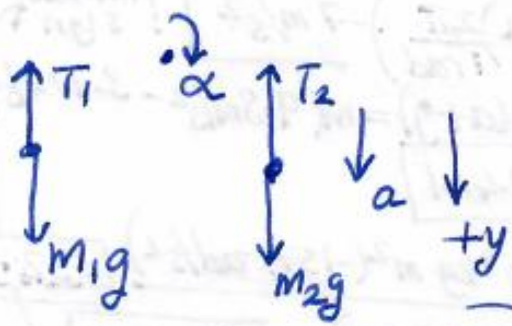
$a = \alpha R$

$y = y_0 + \cancel{v_0 t} + \frac{1}{2} a t^2$

$\Delta y = 0.75\ m$ $t = \Delta t = 5\ s$

$\frac{2y}{t^2} = a$

(51) $m_1 = 460g$
$m_2 = 500g$
pully $R = 5.00cm$
released from rest
block 2 → 75.0cm
falls in 5.00s



$a_t = \alpha R$

$(T_2 - T_1)R = \tau = I\alpha$

i) $m_2 g - T_2 = m_2 a$
$m_1 g - T_1 = -m_1 a$

Unknowns T_1, T_2, a?
need one more equation
$\tau = -TR = I\alpha$ & $a_t = a = \alpha R$

$y - y_0 = v_0 t + \frac{1}{2}at^2$, $v_0 = 0$

$0.75m - 0m = \frac{1}{2}a(5s)^2$
$\rightarrow a = 6 \times 10^{-2} m/s^2$
of the system

ii) $T_2 = m_2(g - a)$
$= 0.5kg(9.8 - 6\times10^{-2}) m/s^2$
$= 4.87N$

iii) $T_1 = m_1(g + a)$
$= 0.46kg(9.8 + 6\times10^{-2}) m/s^2$
$= 4.54N$

iv) $\alpha = ?$ $\quad \alpha = \frac{a_t}{R} = \frac{a}{R}$

$\alpha = \frac{6\times10^{-2} m/s^2}{5\times10^{-2} m} = 1.20 \ rad/s^2$

v) $I = ?$ $\quad -\tau = I\alpha \rightarrow -(T_1 - T_2)R = I\alpha$
cw
$T_2 - T_1 = 4.87N - 4.54N$

$I = \frac{(4.87 - 4.54)N \ 0.05m}{1.20 \ rad/s^2} = 1.38\times10^{-2} kg m^2$

4. A 32.0 kg wheel, essentially a thin hoop with radius 1.20 m, is rotating at 280 rev/min. It must be brought to a stop in 15.0 s. (a) How much work must be done to stop it? (b) What is the required average power?

$I = MR^2$: see Table

$K_f - K_i = \Delta K$, with $K_f = 0$

8(61)

$m = 32.0\ \text{kg}$

thin hoop

$R = 1.2\ \text{m}$

$\omega = 280\ \text{rev/min}$

$\Delta t = 15.0$ to stop

Table 10-2a

$I = MR^2$

i) $W = -\Delta U$ X

$W = \Delta K$ ✓

$K = \frac{1}{2} I \omega^2$

$\Delta K = \cancel{K_2} - K_1$, with $K_2 = 0$

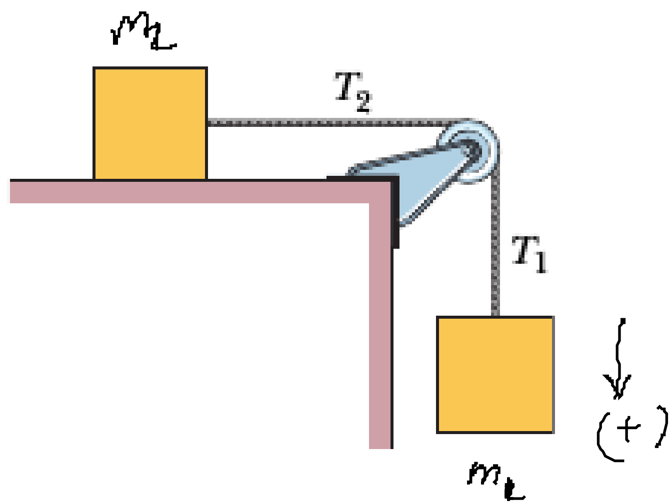
$$W = \Delta K = -\frac{1}{2} I \omega^2 = -\frac{1}{2}(32.0\,\text{kg})(1.2\,\text{m})^2 (29.3\,\text{rad/s})^2$$

$$\boxed{W = -1.98 \times 10^4\ \text{J}}$$

ii) $$P = \frac{|W|}{\Delta t} = \frac{1.98 \times 10^4\ \text{J}}{15.0\ \text{s}} \boxed{= 1.32 \times 10^3\ \text{W}}$$

$$\omega = 280 \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi\,\text{rad}}{\text{rev}} \cdot \frac{1\,\text{min}}{60\,\text{s}} = 29.3\ \text{rad/s}$$

5. In Figure, two 6.20 kg blocks are connected by a massless string over a pulley of radius 2.40 cm and rotational inertia $7.40\text{x}10^{-4}$ kg·m^2. The string does not slip on the pulley; it is not known whether there is friction between the table and the sliding block; the pulley's axis is frictionless. When this system is released from rest, the pulley turns through 0.650 rad in 91.0 ms and the acceleration of the blocks is constant. What are (a) the magnitude of the pulley's angular acceleration, (b) the magnitude of either block's acceleration, (c) string tension T_1, and (d) string tension T_2?



$T_2 - f = m_2 a_2$

$m_1 g - T_1 = m_1 a$

$(T_1 - T_2) R = I\alpha$

$\Delta\theta = 0.650$ rad

$\Delta t = t = 91.$ ms

9 (71)

$m_1 = m_2 = 6.2\,kg = m$

pulley radius $= 2.40\,cm$

$I = 7.40 \times 10^{-4}\,kg\,m^2$

Friction? not known if exists

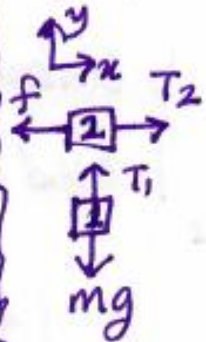
Released from rest

$\Delta\theta = 0.650\,rad$

$\Delta t = 91.0\,ms$

acceleration: constant

i) $\alpha = ?$



possible friction

unknowns

$T_2 - f = m a_{2x}$

$-T_1 + mg = m a_{1y}$

$\tau_{net} = I \alpha_{pulley}$

$T_2 R - T_1 R = I \alpha_{pulley}$

$$\boxed{a_{2x} = a_{1y} = \alpha_{pulley} R}$$

constant acceleration

$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$

0

$\Delta\theta = 0.650\,rad = \frac{1}{2}\alpha (91\,ms)^2$

$$\boxed{\alpha = 157\,rad/s^2}$$

ii) $a = \alpha R = (0.024\,m)(157\,rad/s^2) \boxed{= 3.77\,m/s^2}$

iii) $T_1 = ?$ $\quad -T_1 + mg = ma$ $\quad T_1 = m(a+g) = m(9.8\,m/s^2 - 3.77\,m/s^2)$

$$\boxed{T_1 = 37.4\,N}$$

iv) $T_2 R - T_1 R = I\alpha$

$T_2 = \frac{T_1 R + I\alpha}{R} = T_1 + \frac{I\alpha}{R} = 37.4\,N + \frac{7.40\times10^{-4}\,kg\,m^2 (-157\,rad/s^2)}{(0.024\,m)} \boxed{= 32.56\,N}$

?

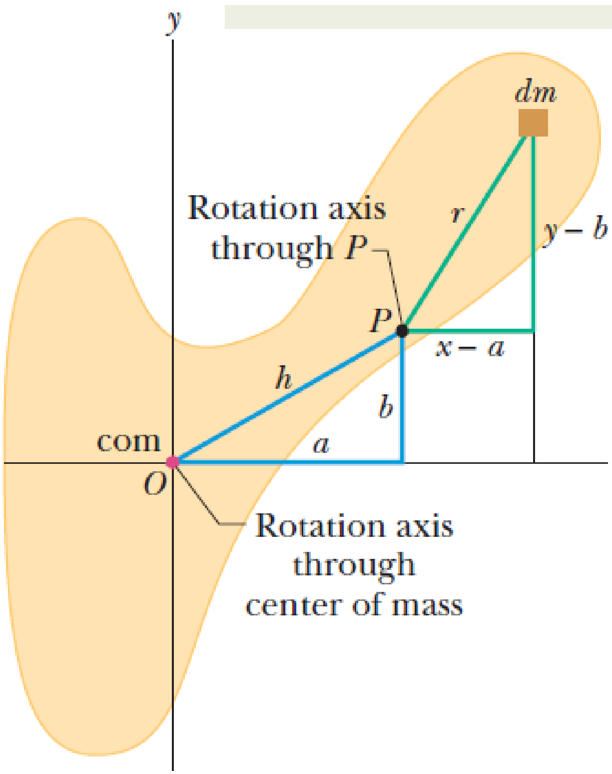
$$\boxed{T_2 < T_1}$$

$(T_1 - T_2) = \frac{I\alpha}{R}$

$T_2 = T_1 - \frac{I\alpha}{R}$

Additional Materials

Proof of the Parallel-Axis Theorem

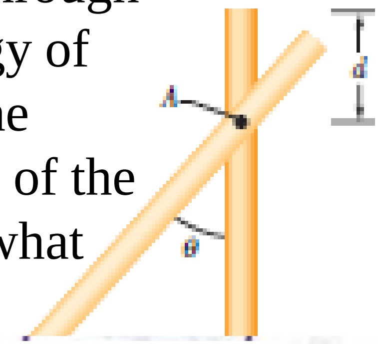


- Let O be the center of mass (and also the origin of the coordinate system) of the arbitrarily shaped body shown in cross section.
- Consider an axis through O perpendicular to the plane of the figure, and another axis through point P parallel to the first axis.
- Let the x and y coordinates of P be a and b.
- Let dm be a mass element with the general coordinates x and y. The rotational inertia of the body about the axis through P is:

$$I = \int r^2\, dm = \int [(x-a)^2 + (y-b)^2]\, dm$$
$$= \int (x^2+y^2)\, dm - 2a\int x\, dm - 2b\int y\, dm + \int (a^2+b^2)\, dm$$

- But $x^2 + y^2 = R^2$, where R is the distance from O to dm, the first integral is simply I_{com}, the rotational inertia of the body about an axis through its center of mass.
- The last term in is Mh^2, where M is the body's total mass.

6. In Figure, a thin uniform rod (mass 3.0 kg, length 4.0 m) rotates freely about a horizontal axis *A* that is perpendicular to the rod and passes through a point at distance d=1.0 m from the end of the rod. The kinetic energy of the rod as it passes through the vertical position is 20 J. (a) What is the rotational inertia of the rod about axis ? (b) What is the (linear) speed of the end of the rod as the rod passes through the vertical position? (c) At what angle ϴ will the rod momentarily stop in its upward swing?



10 (10 3)

uniform thin rod
$m = 3.0\ kg$
$l = 4.0\ m$
rotates
$d = 1.0\ m$
$K = 20\ J$ at vertical position

i) $I = I_{com} + mh^2$: Parallel Axis theorem

$= \frac{1}{12}ML^2 + mh^2$ { h: distance to com

$= \frac{1}{2}(3.0kg)(4.0m)^2 + (3.0kg)(1.0m)^2 = 7.0\ kgm^2$

ii) $K_{rot} = \frac{1}{2} I\omega^2 = \frac{1}{2}(7.0\ kgm^2)(\omega^2) \rightarrow \omega = \sqrt{\frac{2 \times 20J}{7.0\ kgm^2}} = 2.4\ rad/s$

$v = \omega r$ { $v_B = \omega r_{AB} = (2.4\ rad/s)(3.00m) \boxed{= 7.2\ m/s}$

iii) $\theta_{max} = ?$

$(1m)\cos\theta = d$

$h_{max} = 3m - (1m)\cos\theta$

$\Delta h = h_{max} - h_0$

$= 3m - 1m\cos\theta - 2m$

$\boxed{\Delta h = 1m(1 - \cos\theta)}$

com locations at
$\theta = 0 \rightarrow h_0 = 2\ m$
$\theta = \theta_{max} \rightarrow h_{max} = ?$

com motion
$K_2 + \cancel{U_2} = \cancel{K_1} + U_1$
$\frac{1}{2}mv^2$, 0, 0, $mg\Delta h$
$20J = (3.0kg)(9.8m/s^2)\,1m(1 - \cos\theta)$
$20J \rightarrow \cos\theta \approx 0.32$, $\theta \approx 71°$