

1. A cave rescue team lifts an injured spelunker directly upward and out of a sinkhole by means of a motor-driven cable. The lift is performed in three stages, each requiring a vertical distance of 10.0 m: (a) the initially stationary spelunker is accelerated to a speed of 5.00 m/s; (b) he is then lifted at the constant speed of 5.00 m/s; (c) finally he is decelerated to zero speed. How much work is done on the 80.0 kg rescuee by the force lifting him during each stage?

Ch 7.5 (22)

Three stages
lifting.
each is 10m.
 $m = 80.0 \text{ kg}$



i) Accelerated to a speed of 5.00 m/s

$$F_i - mg = ma = F_{\text{net}}$$

$$F_i d - mgd = F_{\text{net}} d$$

$$W_1 - mgd = W = \Delta K_1$$

$$W_1 - mgd = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$\Rightarrow W_1 = \frac{1}{2} (80.0 \text{ kg}) (5 \text{ m/s})^2 + (80.0 \text{ kg}) (9.8 \text{ m/s}^2) (10 \text{ m})$$

$$W_1 = 8.84 \text{ kJ}$$

ii) Constant speed of 5.00 m/s

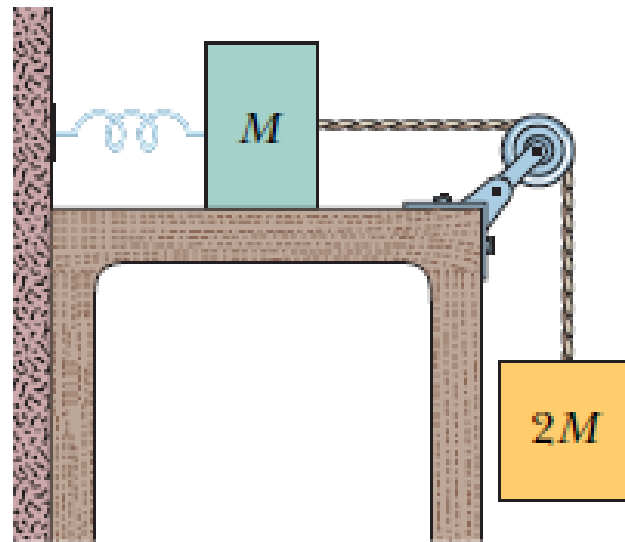
$$W_2 - mgd = \Delta K_2 = 0 \rightarrow W_2 = mgd = 7.84 \text{ kJ}$$

iii) Decelerated to zero speed.

$$W_3 - mgd = \Delta K_3 = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$W_3 = mgd - \frac{1}{2} m v_i^2 = 6.84 \text{ kJ}$$

2. Two blocks, of masses $M=2.0$ kg and $2M$, are connected to a spring of spring constant $k=200$ N/m that has one end fixed, as shown in Figure. The horizontal surface and the pulley are frictionless, and the pulley has negligible mass. The blocks are released from rest with the spring relaxed. (a) What is the combined kinetic energy of the two blocks when the hanging block has fallen 0.090 m? (b) What is the kinetic energy of the hanging block when it has fallen that 0.090 m? (c) What maximum distance does the hanging block fall before momentarily stopping?



Ch 8.10 (91)

$M = 2.0 \text{ kg}$

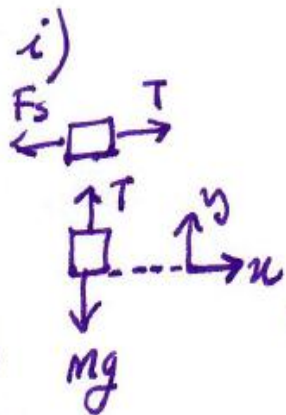
$2M$

$k = 200 \text{ N/m}$

• released from rest

• spring is relaxed

$d = 0.090 \text{ m}$



Conservation of energy

$\Delta U + \Delta K = 0$

$U_f - U_i + K_f - K_i = 0$

$\Rightarrow \frac{1}{2} kx^2 + mg(-0.090 \text{ m}) + \frac{1}{2} m v^2 = 0$

$K_{\text{sys}} = 2.7 \text{ J}$

$K_{\text{sys}} = K_M + K_{2M} = \frac{1}{2} M v^2 + \frac{1}{2} (2M) v^2 = \frac{3}{2} M v^2 = 2.7 \text{ J}$

$\Rightarrow K_{2M} = 1.8 \text{ J}$

$d_{\text{max}} = ? \Rightarrow K_f = 0 \rightarrow \frac{1}{2} k d^2 + mg(d) + 0 = 0 \rightarrow \frac{1}{2} k d^2 - mgd = 0$

$\Rightarrow d = \frac{2mg}{k} = 0.392 \text{ m}$

3×10^{-3}

3. A soccer player kicks a soccer ball of mass 0.45 kg that is initially at rest. The foot of the player is in contact with the ball for 3.0×10^{-3} s, and the force of the kick is given by

$$F(t) = [(6.0 \times 10^6)t - (2.0 \times 10^9)t^2] \text{ N}$$

for $0 \leq t \leq 3.0 \times 10^{-3}$ s, where t is in seconds. Find the magnitudes of

- the impulse on the ball due to the kick,
- the average force on the ball from the player's foot during the period of contact,
- the maximum force on the ball from the player's foot during the period of contact,
- the ball's velocity immediately after it loses contact with the player's foot.

$\Rightarrow d = \text{spring } K = 0.2 \dots$

Ch 9.7 (37)

$m = 0.45 \text{ kg}$
Initially at rest
Contact time = $3.0 \times 10^{-3} \text{ s}$

$F(t) = [6.0 \times 10^6 t - 2.0 \times 10^9 t^2] \text{ N}$
 $0 \leq t \leq 3.0 \times 10^{-3} \text{ s}$

i) $J = ?$ impulse $\vec{F}_{av} = \frac{\vec{J}}{\Delta t}$ OR $J = \int F(t) dt = \int_0^{3 \times 10^{-3}} (6 \times 10^6 t - 2 \times 10^9 t^2) dt$
 $\rightarrow J = 3 \times 10^6 t^2 - \frac{2 \times 10^9}{3} t^3 \Big|_0^{3 \times 10^{-3}} = 3 \times 10^6 (9 \times 10^{-6}) - \frac{2}{3} \times 10^9 (27 \times 10^{-9}) = 9 \text{ N s}$

ii) $F_{av} = \frac{J}{\Delta t} = \frac{9 \text{ N s}}{3.0 \times 10^{-3} \text{ s}} = 3 \times 10^3 \text{ N}$

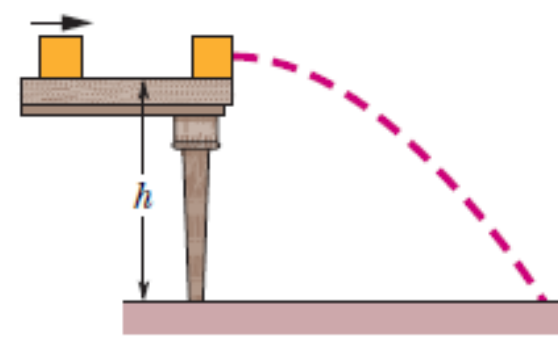
iii) $F_{max} = ?$ during the period of contact
 $\frac{dF(t)}{dt} = 0 \rightarrow 6 \times 10^6 - 4 \times 10^9 t = 0 \Rightarrow t = 1.5 \times 10^{-3} \text{ s}$

$\Rightarrow F(t = 1.5 \times 10^{-3} \text{ s}) = F_{max} = 6 \times 10^6 (1.5 \times 10^{-3}) - 2 \times 10^9 (1.5 \times 10^{-3})^2$
 $F_{max} = 4.5 \times 10^3 \text{ N}$

iv) $v = ?$ when contact is lost.

$\Delta \vec{p} = \vec{p}_f - \vec{p}_i = m \vec{v}_f - m \vec{v}_i \rightarrow \Delta p = m v = J \Rightarrow v = \frac{J}{m} = \frac{9 \text{ N s}}{0.45 \text{ kg}} = 20 \text{ kg m/s} \cdot \frac{1}{52 \text{ kg}}$
 $v = 20 \text{ m/s}$

4. In Figure, a 3.2 kg box of running shoes slides on a horizontal frictionless table and collides with a 2.0 kg box of ballet slippers initially at rest on the edge of the table, at height $h=0.40$ m. The speed of the 3.2 kg box is 3.0 m/s just before the collision. If the two boxes stick together because of packing tape on their sides, what is their kinetic energy just before they strike the floor?



Ch 9.11 (101)
 $m_1 = 3.2$ kg v_{1i}
 $m_2 = 2.0$ kg $v_{2i} = 0$
 $h = 0.40$ m
 $v_{1i} = 3.0$ m/s
 stick together
 \Rightarrow completely
 inelastic
 collision
 $K = ?$ before impact
 to floor

Conservation of momentum $\uparrow \vec{v} \rightarrow x$

$$\vec{P}_i = \vec{P}_f \text{ in } x\text{-direction} \quad P_i = P_f \rightarrow m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v$$

$$\Rightarrow v = \frac{m_1}{m_1 + m_2} v_{1i} = \frac{3.2 \text{ kg}}{(3.2 \text{ kg} + 2.0 \text{ kg})} 3.0 \text{ m/s} = 1.8 \text{ m/s } x\text{-direction}$$

Now, we have projectile motion

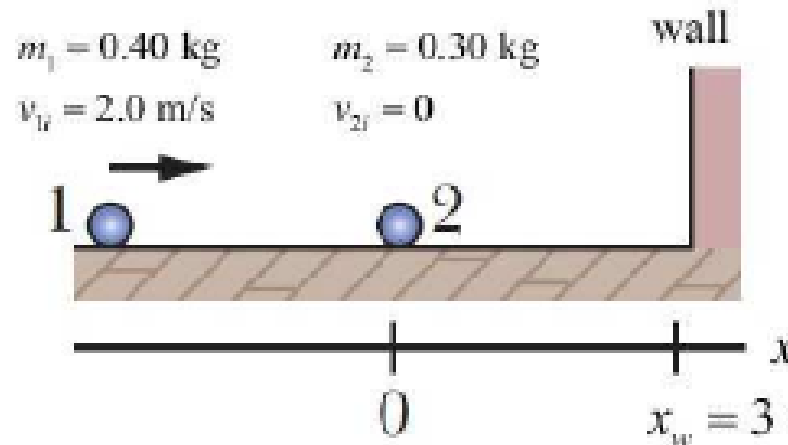
$$K_2 + U_2 = K_1 + U_1$$

? 0 known known

$$K_2 = \frac{1}{2} (3.2 \text{ kg} + 2.0 \text{ kg}) (1.8 \text{ m/s})^2 + (5.2 \text{ kg}) (9.8 \text{ m/s}^2) (0.40 \text{ m})$$

$$\Rightarrow \boxed{K_2 = 29 \text{ J}}$$

5. In figure below, particle 1 of mass $m_1 = 0.40$ kg slides rightward along an x axis on a frictionless floor with a speed of $v_{1i} = 2.0$ m/s. When it reaches $x = 0$, it undergoes a one-dimensional elastic collision with stationary particle 2 ($v_{2i} = 0$) of mass $m_2 = 0.30$ kg.
- Calculate the particle velocities v_{1f} and v_{2f} after the elastic collision.
 - After the collision, particle 2 reaches a wall at $x_w = 3$ m, it bounces from the wall during which 36% of its kinetic energy is lost (turned into thermal energy). At what position on the x axis does particle 2 collide again with particle 1?
- Use 3 decimal digits in your calculations.



i) Collision at $x=0$ Elastic collision \rightarrow KE is conserved

$$\vec{P}_i = \vec{P}_f \quad (1)$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$K_i = K_f \quad (1)$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{(0.4 \text{ kg}) - (0.3 \text{ kg})}{(0.4 \text{ kg}) + (0.3 \text{ kg})} 2.0 \text{ m/s}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2(0.4 \text{ kg})}{(0.4 \text{ kg}) + (0.3 \text{ kg})} 2.0 \text{ m/s}$$

$v_{1f} = 0.286 \text{ m/s}$	$v_{2f} = 2.286 \text{ m/s}$
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ii) $x_w = x_2 = v_{2f} t \quad (1)$

$$t = \frac{3 \text{ m}}{2.286 \text{ m/s}} = 1.3125$$

$$x_1 = v_{1f} t = (0.286 \text{ m/s}) \times (1.3125)$$

$x_1 = 0.375 \text{ m}$

When particle 2 reaches the wall

36% KE is lost \rightarrow bounce

$$K_{2b} = 0.64$$

$$\frac{K_2}{K_1} = \frac{v_{2b}^2}{v_{2f}^2} = 0.64$$

$$v_{2b} = \sqrt{0.64} (2.286 \text{ m/s})$$

Second Collision Point

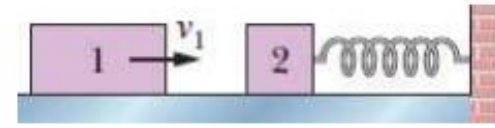
$$0.375 \text{ m} + v_{1f} t = 3 \text{ m} - v_{2b} t$$

$$\Rightarrow t = \frac{3 \text{ m} - 0.375 \text{ m}}{1.839 \text{ m/s} + 0.286 \text{ m/s}}$$

$t = 1.2415$

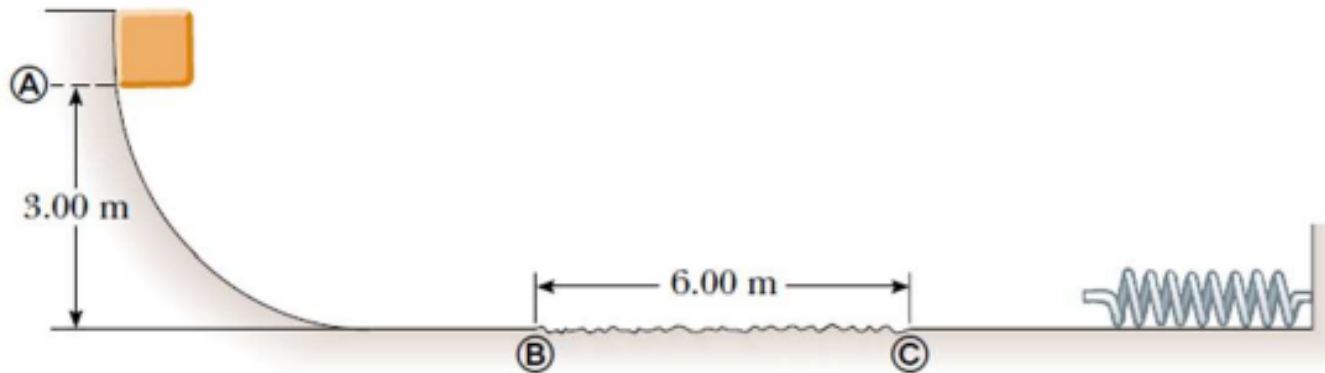
$$x = 0.375 \text{ m} + 1.839 \text{ m/s} \times 1.2415 \text{ s} = 0.73 \text{ m}$$

6. In Figure, block 2 (mass 1.0 kg) is at rest on a frictionless surface and touching the end of an unstretched spring of spring constant 200 N/m. The other end of the spring is fixed to a wall. Block 1 (mass 2.0 kg), traveling at speed $v_1 = 4.0$ m/s, collides with block 2, and the two blocks **stick** together.



Answer: $x = 0.33$ m

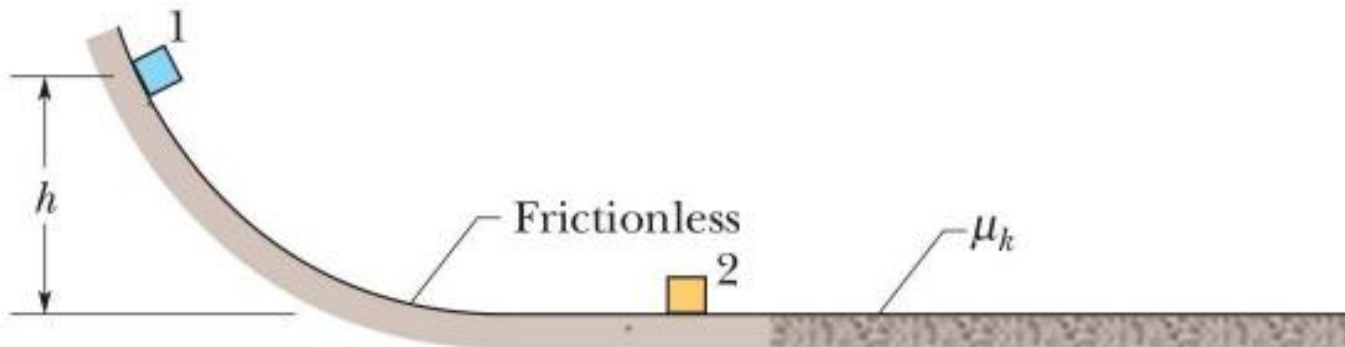
7. A 10.0 kg block is released from point A in figure. The track is frictionless except for the portion between B and C, which has a length of 6.00 m. The block travels down the track, hits a spring of force constant $k = 2250 \text{ N/m}$, and compresses the spring 0.25 m from its equilibrium position before coming to rest momentarily. Determine the coefficient of kinetic friction between the block and the rough surface between B and C. **Answer:** $\mu_k = 0.38$.



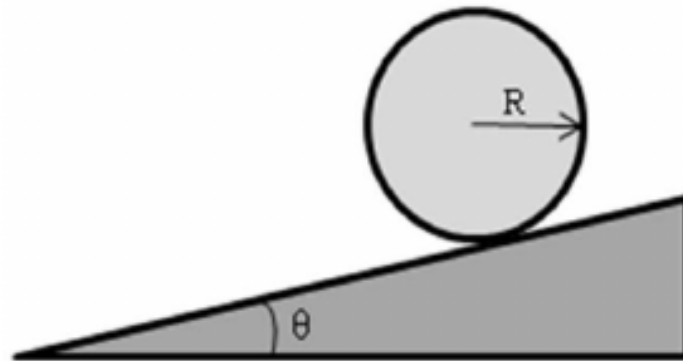
8. In the figure, block 1 of mass m_1 slides from rest along a frictionless ramp from height $h = 2.50$ m and then collides with stationary block 2, which has mass $m_2 = 2.00m_1$. After the collision, block 2 slides into a region where the coefficient of kinetic friction μ_k is 0.500 and comes to a stop in distance d within that region. What is the value of distance d if the collision is

- a) elastic
b) completely inelastic?

Answers: a) $d = 2.23$ m b) $d = 0.55$ m.



9. A uniform ball, of mass $M = 6.0 \text{ kg}$ and radius R , rolls smoothly from rest down a ramp at angle $\theta = 30.0^\circ$ (see Figure). ($I = \frac{2}{5} MR^2$) a) The ball descends a vertical height $h = 1.20 \text{ m}$ to reach the bottom of the ramp. What is its speed at the bottom? b) What are the magnitude and direction of the frictional force (f_s) on the ball as it rolls down the ramp? **Answers: a) $v_{com} = 4.1 \text{ m/s}$ b) $f_s = 8.4 \text{ N}$ upward.**



10. A block of mass $M = 4.0 \text{ kg}$ is released from a height of $H = 12.0 \text{ m}$ on an inclined plane with an angle of $\theta = 37^\circ$. The block slides down the inclined plane and travels a distance of L on the horizontal surface and stops at C. The kinetic coefficient of friction for the inclined plane is $\mu_1 = 0.3$ and the kinetic coefficient for horizontal surface is $\mu_2 = 0.4$.
- Find the speed of the particle at point B.
 - Find the work done by the friction force from B to C.
 - Find L the distance traveled on the horizontal surface.

Answer: (a) $v \approx 12 \text{ m/s}$ (b) $W_f = -288 \text{ J}$ (c) $L \cong 18.4 \text{ m}$