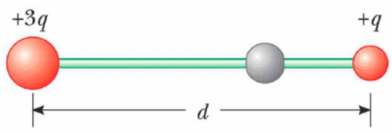
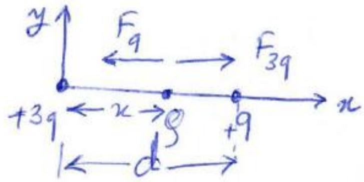
Open Book Quiz - Ch21 Electric Charge
(Duration: 30 minutes)

1. Two small beads having positive charges 3q and q are fixed at the opposite ends of a horizontal, insulating rod, extending from the origin to the point x=d. As shown in Figure, a third small charged bead is free to slide on the rod. At what position is the third bead in equilibrium? Can it be in stable equilibrium?





in equilibrium

$|\vec{F_q}| = |\vec{F_{3q}}|$

$k\frac{|Q||+q|}{(d-x)^2} = k\frac{|Q||+3q|}{x^2} \rightarrow x^2 = 3(d-x)^2$

$\rightarrow x^2 = 3d^2 - 6dx + x^2 3$ OR $x = \sqrt{3}(d-x)$

$2x^2 - 6dx + 3d^2 = 0$ $\qquad x = \sqrt{3}d - \sqrt{3}x$

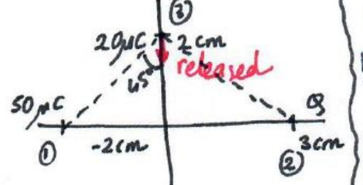
$x_{1,2} = \frac{6d \pm \sqrt{36d^2 - 4 \times 2 \times 3d^2}}{4}$ $\qquad x + \sqrt{3}x = \sqrt{3}d$

$= \frac{6d \pm 2\sqrt{3}d}{4} = \frac{3d \pm \sqrt{3}d}{2}$ $\qquad x = \frac{\sqrt{3}}{1+\sqrt{3}}d$

$= \left(\frac{3 \pm \sqrt{3}}{2}\right)d \rightarrow \boxed{x = \left(\frac{3-\sqrt{3}}{2}\right)d} \Leftrightarrow x = \left(\frac{3-\sqrt{3}}{2}\right)d$

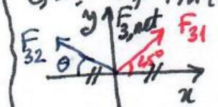
physical one as $x < d$

2. Two particles are fixed on an *x* axis. Particle 1 of charge 50 μC is located at *x*=-2.0 cm; particle 2 of charge *Q* is located at *x*=3.0 cm. Particle 3 of charge magnitude 20 μC is released from rest on the *y* axis at *y*=2.0 cm. What is the value of *Q* if the initial acceleration of particle 3 is in the positive direction of the *y* axis?



①&② are fixed
③ is released from rest

Q=? if initial acceleration of ③ is in +y-axis



$F_{31,x} = F_{32,x}$

$|F_{31}| = k\frac{|20\times10^{-6}C||50\times10^{-6}C|}{(2\times10^{-2}m)^2 + (2\times10^{-2}m)^2}$, $F_{31,x} = |F_{31}|\cos 45$

Q should be positive

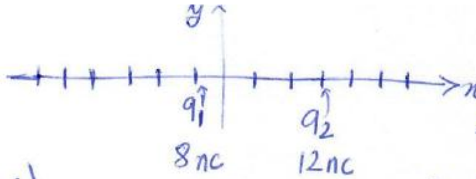
$|F_{32}| = k\frac{|20\times10^{-6}C||Q|}{(2\times10^{-2}m)^2 + (3\times10^{-2}m)^2}$, $F_{32,x} = |F_{32}|\cos\theta$

$\theta' = \tan^{-1}\frac{3\times10^{-2}m}{2\times10^{-2}m} = 56.31°$ $\qquad \theta = 90 - \theta'$

$\Rightarrow k\frac{|20\times10^{-6}C||50\times10^{-6}C|}{8\times10^{-4}m^2}\underset{\cos 45°}{\wedge} = k\frac{|20\times10^{-6}C|\,Q}{13\times10^{-4}m^2}\cos 33.69° \rightarrow |Q| = 69\times10^{-6}C$

Open Book Quiz - Ch22 Electric Fields
(Duration: 30 minutes)

1. A positive point charge q_1=8 nC is on the axis at x_1= -1 m, a second positive point charge q_2=12 nC is on the c axis at x_2=3 m. Find the net electric field (a) at point A on the x-axis at x=6 m, and (b) at point B on the x-axis at x=2 m.



i) $E(x=6m)=?$

ii) $E(x=2m)=?$

i) $\longrightarrow E_{q_1}$, $\longrightarrow E_{q_2}$

$$E(x=6m)=\sum_{i=1}^{2} E_{q_i}(x=6m)=E_{q_1}(x=6m)+E_2(x=6m)$$

$$=k\frac{|q_1|}{(7m)^2}+k\frac{|q_2|}{(3m)^2}=\left(8.99\times10^9\frac{N}{C^2}m^2\right)\left(\frac{8\times10^{-9}C}{49m^2}+\frac{12\times10^{-9}C}{9m^2}\right)$$

$$=13.45\ N/C \rightsquigarrow \boxed{\vec{E}(x=6m)=13.45\ N/C\ \hat{i}}$$

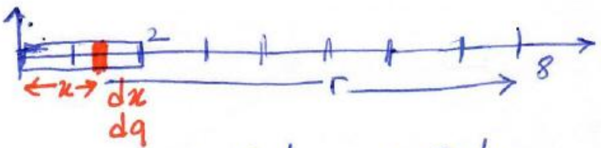
ii) $E(x=2m)=?$

$\longrightarrow E_{q_1}$, $\longleftarrow E_{q_2}$

$$E(x=2m)=k\frac{|q_1|}{(9m^2)}-k\frac{|q_2|}{1m^2}=8.99\times10^9\frac{N}{C^2}m^2\left(\frac{8\times10^{-9}C}{9m^2}-\frac{12\times10^{-9}C}{1m^2}\right)$$

$$=-99.9\ N/C \rightsquigarrow \boxed{\vec{E}(x=2m)=99.9\ N/C\ (-\hat{i})}$$

2. A charge of 40 nC is uniformly distributed along the axis from x = 0 to x=2 m. Determine the magnitude of the electric field at a point on the x-axis with x =8 m.



$E(x=8)=?$

$$E=k\frac{q}{r^2}\rightsquigarrow dE=k\frac{dq}{r^2}\ \&\ \lambda=\frac{Q}{L}=\frac{dq}{dx}$$

$$\rightarrow dE=k\frac{\lambda dx}{r^2}=k\frac{\lambda dx}{(8-x)^2}=k\lambda\frac{dx}{(8-x)^2}\qquad \lambda=\frac{40\times10^{-9}C}{2m}$$

$$\rightarrow \int dE=k\lambda\int_0^2\frac{dx}{(8-x)^2}=k\lambda\left.\frac{1}{8-x}\right|_0^2=k\lambda\left(\frac{1}{8-2}-\frac{1}{8-0}\right)=k\lambda\left(\frac{1}{6}-\frac{1}{8}\right)$$

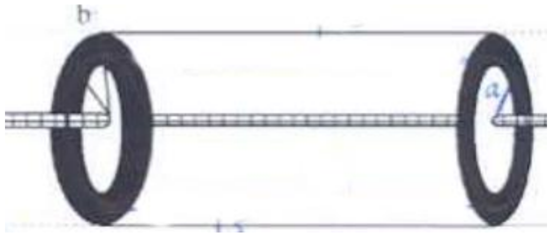
$$\rightarrow E(x=8)=\int dE=\left(8.99\times10^9\frac{N m^2}{C^2}\right)\frac{40\times10^{-9}C}{2m}\frac{(8-6)m}{48m^2}=\boxed{7.49\ N/C}$$

magnitude

Take Home Quiz - Ch23 Gauss'Law
(Due to: Oct 30, 2018 (Class Hour))

1. An infinitely long cylindrical insulating shell of inner radius *a* and outer radius *b* has a uniform volume charge density ρ. A line of uniform linear charge density λ, is placed along the axis of the shell. Determine the electric field in the following regions:

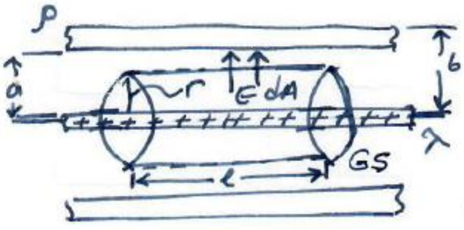
a) r<a
b) a<r<b
c) r> b



i) $r<a$

$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$

$E(2\pi r l) = \frac{Q_{enc}}{\varepsilon_0}$



$Q_{line} = \lambda l$

$Q_{cylinder} = \phi$ (shell theorem)

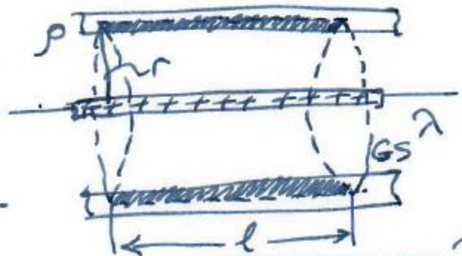
$\rightsquigarrow Q_{enc} = \lambda l + \phi$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

ii) $a<r<b$

$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$

$E(2\pi r l) = \frac{Q_{enc}}{\varepsilon_0}$



$Q_{line} = \lambda l$

$Q_{cylinder} = \rho * \text{Volume}$

$= \rho * (\pi r^2 l - \pi a^2 l)$

$= \pi l \rho (r^2 - a^2)$

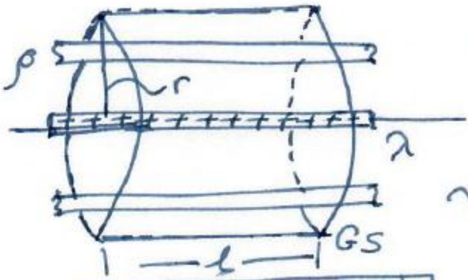
$\rightsquigarrow Q_{enc} = \lambda l + \pi l \rho (r^2 - a^2)$

$$\boxed{E = \frac{\lambda + \rho\pi(r^2 - a^2)}{2\pi r}}$$

iii) $r>b$

$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$

$E(2\pi r l) = \frac{Q_{enc}}{\varepsilon_0}$



$Q_{line} = \lambda l$

$Q_{cylinder} = \rho(\pi b^2 l - \pi a^2 l)$

$\rightsquigarrow Q_{enc} = \lambda l + \rho l \pi (b^2 - a^2)$

$$\boxed{E = \frac{\lambda + \rho\pi(b^2 - a^2)}{2\pi r}}$$

Open Book Quiz - Ch24 Electric Potential
(Duration: 30 minutes)

1. What is the **magnitude** of the electric field at the point (3i-2j+4k) m if the electric potential in the region is given by $V=5xyz^2$, where V is in volts and coordinates x, y, and z are in meters?

$V(x,y,z) = 5xyz^2$ $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, $E_z = -\frac{\partial V}{\partial z}$

$\rightarrow E_x = -5yz^2$
$E_y = -5xz^2$
$E_z = -10xyz$

point $(3,-2,4)$

$E_x = 160 N/C$
$E_y = -240 N/C$
$E_z = 240 N/C$

$|\vec{E}| = \sqrt{E_x^2 + E_y^2 + E_z^2} = 375 N/C$

Open Book Quiz - Ch26 Current and Resistance
(Duration: 30 minutes)

1. The magnitude J of the current density in a certain lab wire with a circular cross section of radius R=2.00 mm is given by J=(3.00x10^8)r^2, with J in amperes per square meter and radial distance r in meters. What is the current through the outer section bounded by r=0.900R and r=R?

$R = 2\times10^{-3}\ m$

$J(r) = 3\times10^{8} r^{2}\ A/m^{2}$

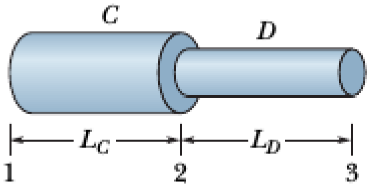
$i = ?$ from $r = 0.9R$ to $r = R$

$$i = \int \vec{J}\cdot d\vec{A} = \int_{0.9R}^{R} 3\times10^{8} r^{2}\, 2\pi r\, dr = 6\pi\times10^{8}\int_{0.9R}^{R} r^{3} dr$$

$$= 6\pi\times10^{8}\left.\frac{r^{4}}{4}\right|_{0.9R}^{R} = \frac{6\pi\times10^{8}}{4}\left(R^{4} - (0.9R)^{4}\right)$$

$$= \frac{6\pi\times10^{8}}{4}\, 0.344 R^{4} = \underline{2.59\times10^{-3} A}$$

2. Wire C and wire D are made from different materials and have length L_C=L_D=1.0 m. The resistivity and diameter of wire C are 2.0x10^{-6} Ω.m and 1.00 mm, and those of wire D are 1.0x10^{-6} Ω.m and 0.50 mm. The wires are joined as shown in Figure, and a current of 2.0 A is set up in them. What is the electric potential difference between (a) points 1 and 2 and (b) points 2 and 3? What is the rate at which energy is dissipated between (c) points 1 and 2 and (d) points 2 and 3?



$L_C = L_D = 1 m$

$\rho_C = 2\times10^{-6}\ \Omega m$

$\rho_D = 1\times10^{-6}\ \Omega m$

$d_C = 1\times10^{-3}\ m$

$d_D = 0.5\times10^{-3}\ m$

$i = 2\ A$

same current at both segments. $R = \rho\, l/A = V/I$

$R_C = \rho_C\, L_C/\pi(d_C/2)^2 = 2\times10^{-6}\ \Omega m\ 1m/\pi(0.50\times10^{-3} m)^2 = \underline{2.55\ \Omega}$

$R_D = \rho_D\, L_D/\pi(d_D/2)^2 = 1\times10^{-6}\ \Omega m\ 1m/\pi(0.25\times10^{-3} m)^2 = \underline{5.09\ \Omega}$

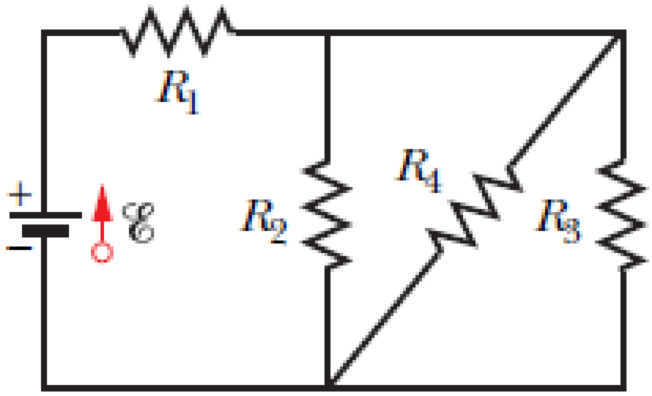
$V_C = R_C\, i = 2.55\ \Omega\ 2A = \underline{5.1 V}$ & $V_D = \underline{10.2 V}$

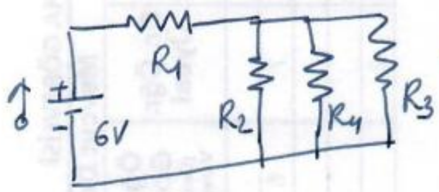
$(V_2 - V_1)$ $(V_3 - V_2)$

$P_C = i^2 R_C = \underline{10 W}$ & $P_D = i^2 R_D = \underline{20 W}$

Open Book Quiz - Ch27 Circuits
(Duration: 30 minutes)

1. In Figure, R_1=100 Ω, R_2=R_3=50.0 Ω, R_4=75.0 Ω, and the ideal battery has emf ξ=6.00 V. (a) What is the equivalent resistance? What is *i* in (b) resistance 1, (c) resistance 2, (d) resistance 3, and (e) resistance 4?

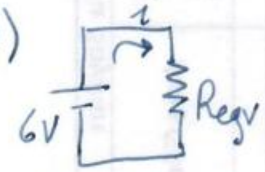




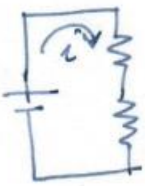
i) $R_{234} = \left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)^{-1} = \left(\frac{1}{50}+\frac{1}{50}+\frac{1}{75}\right)^{-1}$

$= \left(\frac{150+50}{3750}\right)^{-1} = 18.75\,\Omega \Rightarrow R_{1234} = R_1 + R_{234}$

(40) $= 118.75\,\Omega$

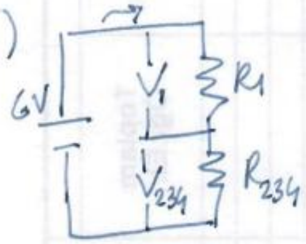


ii) $i = \frac{V}{R_{eqv}} = \frac{6V}{118.75} = 0.051\,A$ (10)



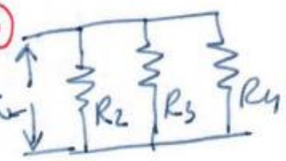
$R_1 \sim i$

$R_{234} \sim i$ $i = 0.051A$ (10)



iii) $V_1 = R_1 i = 5.05\,V$ (5)

$V_{234} = R_{234}\, i = 0.95\,V = V_{R_2}$ (5)



$i_2 = \frac{V}{R_2} = \frac{0.95V}{50\Omega} = 0.019A$ (10)

iv) $i_3 = \frac{V}{R_3} = \frac{0.95V}{50\Omega} = 0.019A$ (10)

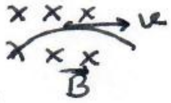
v) $i_4 = \frac{V}{R_4} = \frac{0.95V}{75\Omega} = 0.013\,A$ (10)

Open Book Quiz - Ch28 Magnetic Fields
(Duration: 30 minutes)

1. An electron of kinetic energy 1.20 keV circles in a plane perpendicular to a uniform magnetic field.The orbit radius is 25.0 cm. Find (a) the electron's speed, (b) the magnetic field magnitude, (c) the circling frequency, and (d) the period of the motion.

electron
$KE = 1.2\times10^{3}\,eV$
$R = 25\times10^{-2}\,m$

$\frac{mv^2}{R} = qvB\sin 90^\circ \rightarrow v = \frac{qRB}{m}$



$R = \frac{mv}{qB}$

i) $\frac{1}{2}m_e v^2 = 1.2\times10^{3}\,eV \rightarrow v = \frac{2(1.2\times10^{3}\,eV)(1.6\times10^{-19}\,J/eV)}{(9.1\times10^{-31}\,kg)} = 2.05\times10^{7}\,m/s$

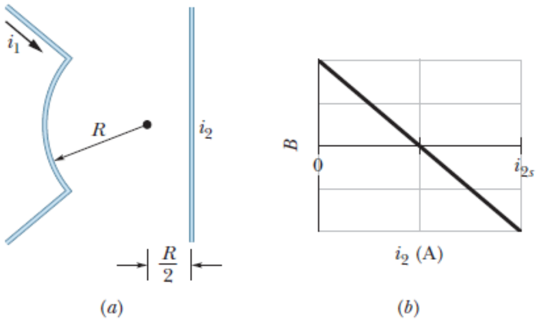
ii) $B = \frac{mv}{eR} = \frac{(9.1\times10^{-31}\,kg)(2.05\times10^{7}\,m/s)}{(1.6\times10^{-19}\,C)(25\times10^{-2}\,m)} = 4.67\times10^{-4}\,T$

iii) $T = \frac{1}{f} = \frac{2\pi R}{v} \rightarrow f = \frac{v}{2\pi R} = \frac{(2.05\times10^{7}\,m/s)}{2\pi(25\times10^{-2}\,m)} = 1.31\times10^{7}\,Hz$

iv) $T = \frac{1}{f} = 7.63\times10^{-8}\,s$

Open Book Quiz - Ch29 Magnetic Fields due to Currents (Duration: 30 minutes)

1. Figure(a) shows two wires, each carrying a current. Wire 1 consists of a circular arc of radius R and two radial lengths; it carries current i_1= 2.0 A in the direction indicated. Wire 2 is long and straight; it carries a current i_2 that can be varied; and it is at distance R/2 from the center of the arc. The net magnetic field B due to the two currents is measured at the center of curvature of the arc. Figure(b) is a plot of the component of B in the direction perpendicular to the figure as a function of current i_2. The horizontal scale is set by i_{2s} = 1.00 A. What is the angle subtended by the arc?



$i_1 = 2A$, R ; i_2: variable, R/2

net magnetic field at point P

$$B_P = \frac{\mu_0 i_1}{4\pi R}\phi - \frac{\mu_0 i_2}{2\pi (R/2)}$$

at $i_2 = 0.5A \rightsquigarrow B_P = 0$

$$\Rightarrow 0 = \frac{\mu_0 (2A)\phi}{4\pi R} - \frac{\mu_0 (0.5A)}{2\pi R/2} \rightsquigarrow \phi = 4\left(\frac{0.5A}{2A}\right) = 1 \text{ rad} = 57.3^\circ$$