İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy102 Physics II
Midterm Examination
November 06, 2018 16:30 – 18:30
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

DURATION: 120 minutes

◇ **Answer all the questions.**
◇ **Write the solutions explicitly and clearly. Use the physical terminology.**
◇ **You are allowed to use Formulae Sheet.**
◇ **Calculator is allowed.**
◇ **You are not allowed to use any other electronic equipment in the exam.**

Question	Grade	Out of
1A		15
1B		15
2		20
3		20
4		20
5		20
TOTAL		110

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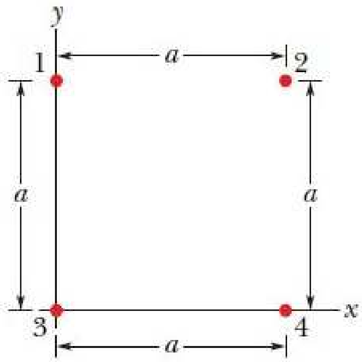
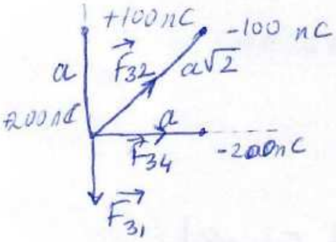
1. A) In Figure, four particles form a square.



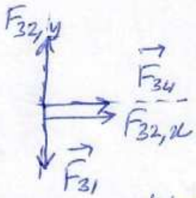
The particles have charges $q_1 = -q_2 = 100\ nC$ and $q_3 = -q_4 = 200\ nC$, and distance $a = 5.0\ cm$. What are the x and y components of the net electrostatic force on particle 3?



$$\vec{F}_{3,net} = \sum_{i=1}^{3} \vec{F}_{3i} = \vec{F}_{31} + \vec{F}_{32} + \vec{F}_{34}$$

$$F_{3,net,x} = F_{31,x} + F_{32,x} + F_{34,x}$$

$$F_{3,net,y} = F_{31,y} + F_{32,y} + F_{34,y}$$



$$F_{3,net,x} = |F_{32}| \cos 45 + |F_{34}| = k\frac{|q_3||q_2|}{a^2 2}\cos 45 + k\frac{|q_3||q_4|}{a^2}$$

$$= 8.99\times10^{9}\frac{Nm^2}{C^2}\frac{(200\times10^{-9}C)}{(5\times10^{-2}m)^2}\left(\frac{100\times10^{-9}C}{2}\frac{\sqrt{2}}{2} + 200\times10^{-9}C\right)$$

$$\boxed{= 0.17\ N}$$

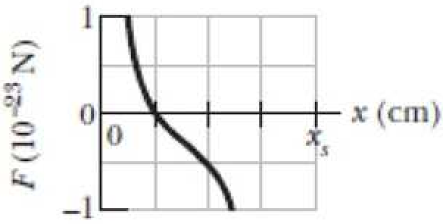
$$F_{3,net,y} = |F_{32}| \sin 45 - |F_{31}| = k\frac{|q_3||q_2|}{a^2 2}\sin 45 - k\frac{|q_3||q_1|}{a^2}$$

$$= 8.99\times10^{9}\ Nm^2/C^2\frac{(200\times10^{-9}C)}{(5\times10^{-2}m)^2}\left(\frac{100\times10^{-9}C}{2}\frac{\sqrt{2}}{2} - 100\times10^{-9}C\right)$$

$$\boxed{= -0.046\ N}$$

B) In Figure (a), particle 1 (of charge q_1) and particle 2 (of charge q_2) are fixed in place on an x-axis, 8.00 cm apart. Particle 3 (of charge $q_3 = +8.00 \times 10^{-19}\ C$) is to be placed on the line between particles 1 and 2 so that they produce a net electrostatic force $F_{3,net}$ on it.

(a)



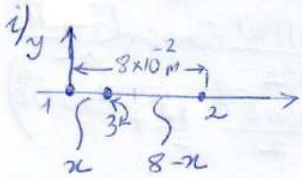
(b)

Figure (b) gives the x component of that force versus the coordinate x at which particle 3 is placed. The scale of the x axis is set by $x_s = 8.0\ cm$.

i What is the sign of charge q_1?

ii What is the ratio q_2/q_1?

i)



if ⊖ ⊕ ⊖ F_{31} F_{32} ✓ but Figure(b)

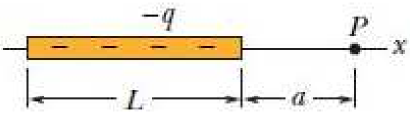
if ⊕ ⊕ ⊕ F_{32} F_{31} ✓ when $x>2$ repulsive force (positive value) → q_1 should be (+)

ii) $F_{3,net}(x=2)=0 \rightarrow |F_{32}(x=2)| = |F_{31}(x=2)|$

$$k\frac{|q_3||q_2|}{(8-x)^2} = k\frac{|q_3||q_1|}{x^2} \quad \text{when } x = 2\times10^{-2}\ m$$

$$\frac{q_2}{(6\times10^{-2}\ m)^2} = \frac{q_1}{(2\times10^{-2}\ m)^2} \rightarrow \boxed{\frac{q_2}{q_1} = 9}$$

2. In the figure below, a nonconducting rod of length $L = 8.15\ cm$ has a charge $q = -4.23\ fC$ uniformly distributed along its length.



i What is the linear charge density of the rod?

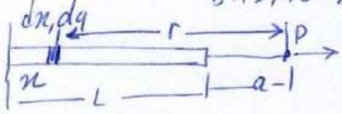
ii What are the magnitude and direction (relative to the $+x$-axis) of the electric field produced at point P, at distance $a = 12.0\ cm$ from the rod?

iii What is the electric field magnitude produced at distance $a = 50.0\ cm$ by the rod?

iv What is the electric field magnitude produced at distance $a = 50.0\ cm$ by <u>a particle of charge</u> $q = -4.23\ fC$ that replaces the rod?

i) $\lambda = \frac{Q}{L} = \frac{-4.23\times10^{-15}C}{8.15\times10^{-2}m} = -5.19\times10^{-14}C/m$

ii)



$\rightsquigarrow r = L + a - x$

$dE = k\frac{dq}{r^2} = k\frac{\lambda dx}{(L+a-x)^2}$ $\Big\}$ $E = \int_0^L dE$

$E_p = k\lambda \int_0^L \frac{dx}{(L+a-x)^2} = k\lambda \frac{1}{(L+a-x)}\Big|_0^L = k\lambda\left(\frac{1}{a} - \frac{1}{L+a}\right)$

$= 8.99\times10^9 \frac{Nm^2}{C^2}\left(5.19\times10^{-14}\frac{C}{m}\right)\left(\frac{L}{a(L+a)}\right) = 4.67\times10^{-4}\frac{Nm}{C}$ $(-\hat{\imath})$

$\Rightarrow$ $L = 8.15\times10^{-2}m$, $a = 12\times10^{-2}m$ $\Big\}$ $E_p = 4.67\times10^{-4}Nm/C\left(\frac{8.15\times10^{-2}m}{(12\times10^{-2}m)20.15\times10^{-2}m}\right) = \boxed{1.57\times10^{-3}\frac{N}{C}}$

iii) $L = 8.15\times10^{-2}m$, $a = 50\times10^{-2}m$ $\Big\}$ $E_p = 4.67\times10^{-4}Nm/C\left(\frac{8.15\times10^{-2}m}{(50\times10^{-2}m)58.15\times10^{-2}m}\right) = \boxed{1.31\times10^{-4}N/C}$

iv) Point charge: $E_p = k\frac{|q|}{r^2} = 8.99\times10^9\frac{Nm^2}{C^2}\frac{4.23\times10\times10^{-15}C}{(50\times10^{-2}m)^2} = \boxed{1.54\times10^{-4}N/C}$

3. An infinitely long cylindrical insulating shell of inner radius a and outer radius b has a uniform volume charge density ρ. A line of uniform linear charge density λ, is placed along the axis of the shell. Determine the electric field in the following regions:

   i $r < a$

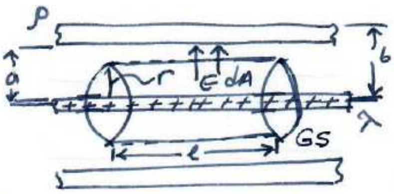
   ii $a < r < b$

   iii $r > b$

i) $r < a$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E(2\pi r l) = \frac{q_{enc}}{\varepsilon_0}$$



$q_{line} = \lambda l$

$q_{cylinder} = \phi$ (shell theorem)

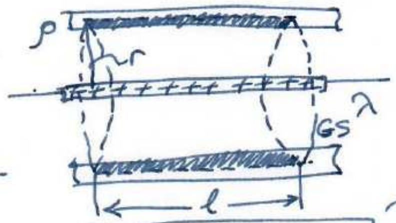
$\rightsquigarrow q_{enc} = \lambda l + \phi$

$$\boxed{E = \frac{\lambda}{2\pi \varepsilon_0 r}}$$

ii) $a < r < b$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E(2\pi r l) = \frac{q_{enc}}{\varepsilon_0}$$



$q_{line} = \lambda l$

$q_{cylinder} = \rho * \text{Volume}$

$= \rho * (\pi r^2 l - \pi a^2 l)$

$= \pi l \rho (r^2 - a^2)$

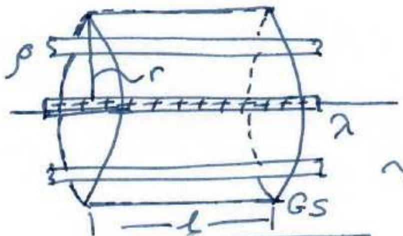
$\rightsquigarrow q_{enc} = \lambda l + \pi l \rho (r^2 - a^2)$

$$\boxed{E = \frac{\lambda + \rho \pi (r^2 - a^2)}{2\pi r}}$$

iii) $r > b$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E(2\pi r l) = \frac{q_{enc}}{\varepsilon_0}$$



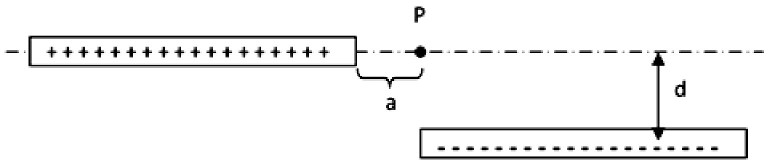
$q_{line} = \lambda l$

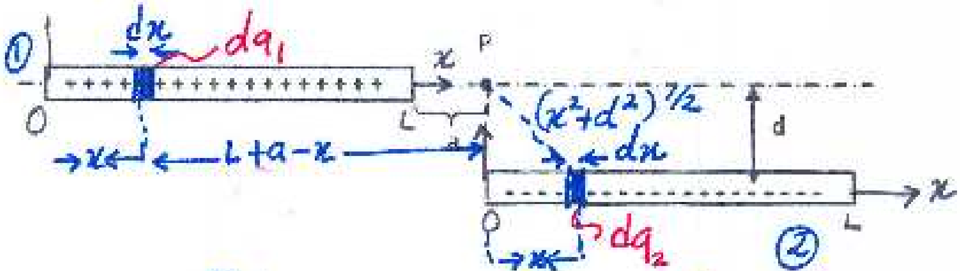
$q_{cylinder} = \rho(\pi b^2 l - \pi a^2 l)$

$\rightsquigarrow q_{enc} = \lambda l + \rho l \pi (b^2 - a^2)$

$$\boxed{E = \frac{\lambda + \rho \pi (b^2 - a^2)}{2\pi r}}$$

4. Two very thin non-conducting rods are placed together as shown. Both rods have lengths of L and they carry uniform charges of $+q$ and $-q$ over their lengths. Find the potential at point P at a distance a and d from the positively and negatively charged rods as shown. Don't perform integration.





$$V_{1\,at\,P} = \int dV = \int \frac{1}{4\pi\varepsilon_0}\frac{dq_1}{r_1}$$

$dq_1 = \lambda\, dx$

$r_1 = L+a-x$

$$V_{2\,at\,P} = \int dV = \int \frac{1}{4\pi\varepsilon_0}\frac{dq_2}{r_2}$$

$dq_2 = -\lambda\, dx$

$r_2 = \sqrt{x^2+d^2}$

$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

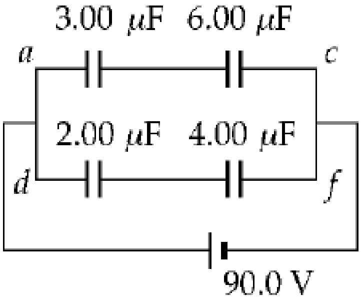
$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$

$dq = ?$

$dq = \lambda\, dx$

$$V_{tot} = V_{1\,at\,P} + V_{2\,at\,P} = \boxed{\frac{\lambda}{4\pi\varepsilon_0}\left(\int_0^L \frac{dx}{(L+a-x)} - \int_0^L \frac{dx}{\sqrt{x^2+d^2}}\right)}$$

5. For the system of capacitors shown in Figure,



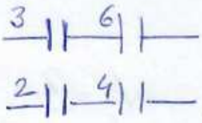
find

i the equivalent capacitance of the system,

ii the potential across each capacitor,

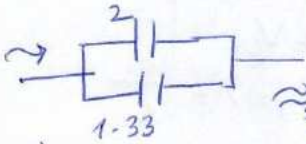
iii the charge on each capacitor.

i) $C_{eqv} = ?$



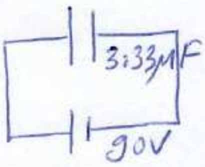
$\frac{1}{C_{ac}} = \frac{1}{3\mu F} + \frac{1}{6\mu F} \Rightarrow C_{ac} = 2\mu F$

$\frac{1}{C_{df}} = \frac{1}{2\mu F} + \frac{1}{4\mu F} \Rightarrow C_{df} = 1.33\mu F$

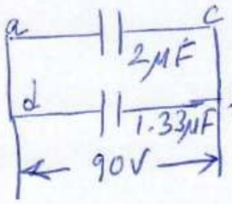


$\Rightarrow \underline{\underline{C_{eqv} = 3.33\mu F}}$

ii)



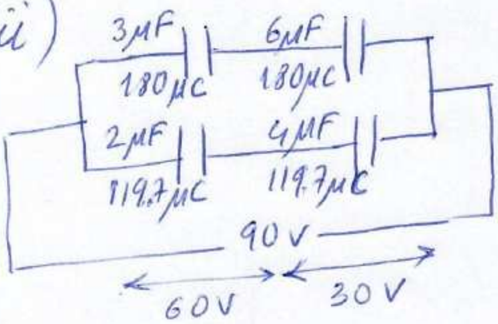
$C = \frac{Q}{V} \sim Q = C_{eqv} * V = (3.33 \times 10^{-6} F) 90V = \underline{\underline{299.7\mu C}}$ (total charge)



$Q_{ac} = (2\mu F) 90V = 180\mu C = q_a = q_c$

$Q_{df} = (1.33\mu F) 90V = \underline{\underline{119.7\mu C = q_d = q_f}}$

iii)



$V_a = \frac{q_a}{C_a} = \frac{180\mu C}{3\mu F} = \boxed{60V}$

$V_b = \frac{180\mu C}{6\mu F} = \boxed{30V}$

$V_c = \frac{119.7\mu C}{2\mu F} \sim \boxed{60V}$

$V_d = \frac{119.7\mu C}{4\mu F} \cong \boxed{30V}$