İZMİR KATİP ÇELEBİ UNIVERSITY	FACULTY OF ENG. & ARCH. PHY102, MIDTERM EXAM 6 April 2019, 10:30, DURATION: 120 MIN			
Student Name	**ID Number**	**Instructor Name**	**Department**	**Signature**

Please read the following directions carefully.

- You must show all your work to get credit; you will not be given any points unless you show the details of your work (this applies even if your final answer is correct).
- Write neatly and clearly; unreadable answers will not be given any credit. If you need more writing space, use the backs of the question pages and put down the appropriate pointer marks.
- Make sure that you include units in your results. Incomplete calculations will not be graded.
- Turn off your mobile phones, and put away. No notebooks or textbooks are allowed to use during the exam.
- You are not allowed to leave the class during the first 15 minutes, and last 15 minutes.
- Calculator is allowed to use. Calculator is assumed to be used only for simple arithmetics, other intentions will be considered as cheating. Everybody must use his/her own calculator. Do not exchange calculators during the exam!
- There are 8 questions. Grade point values are under question numbers.
- Before you begin, please check all pages.
- At the end of the exam make sure that you turn in your exam paper to your proctor by yourself! Do not give your exam paper to others!

Constants

$e = 1.602 \times 10^{-19}$ C (charge on e^- or p^+)
$1\text{eV} = 1.602 \times 10^{-19}$ J
$m_e = 9.11 \times 10^{-31}$ kg
$m_p = 1.67 \times 10^{-27}$ kg
$k = 1/(4\pi\varepsilon_o) = 8.99 \times 10^9$ N m^2/C^2
$\varepsilon_o = 8.85 \times 10^{-12}$ F/m (or C^2/N m^2)
$\mu_o = 4\pi \times 10^{-7}$ T m/A
1 T $= 10^4$ G
$g = 9.8$ m/s^2

1 (15pts)	2 (10pts)	3 (10pts)	4 (15pts)	5 (10pts)	6 (15pts)	7 (15pts)	8 (10pts)	Total grade

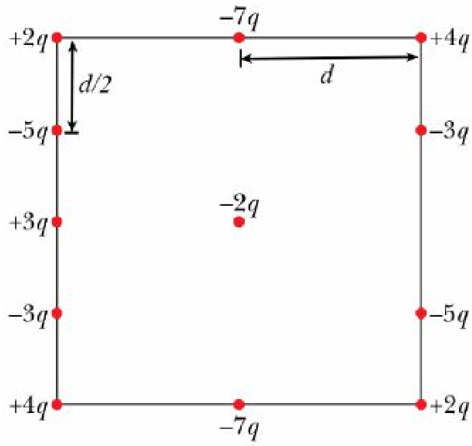
This paper is not to be removed from the Examination Halls

This page can be used if necessary.

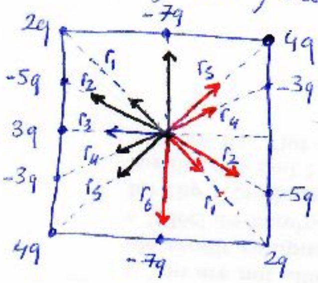
QUESTIONS

(Put your solutions under each question!)

1. In the figure, a central particle of charge -2q is surrounded by a square array of charged particles, separated by either distance d or d/2 along the perimeter of the square.
    a. What are the magnitude and direction of the net electrostatic force on the central particle due to the other particles? (Hint: Some forces on the central particle cancel each other!)
    b. What is the work you need to apply to bring central particle to its place from infinity?



i) Some of the forces cancel each other!



$\vec F = k\frac{|q_1||q_2|}{r^2}\hat r$, $\vec F_{net} = \sum_{i=1}^{11} k\frac{|-2q||q_i|}{r_i^2}\hat r_i$ (1)

$\vec F_{net} = k|-2q|\Big(\frac{|2q|}{r_1^2}\hat r_1 + \frac{|-5q|}{r_2^2}\hat r_2 + \frac{|3q|}{r_3^2}\hat r_3 + \frac{|-3q|}{r_4^2}\hat r_4 + \frac{|4q|}{r_5^2}\hat r_5 + \frac{|-7q|}{r_6^2}\hat r_6 + \frac{|2q|}{r_1^2}(-\hat r_1) + \frac{|-5q|}{r_2^2}(-\hat r_2) + \frac{|-3q|}{r_4^2}(-\hat r_4) + \frac{|4q|}{r_5^2}(-\hat r_5) + \frac{|-7q|}{r_6^2}(-\hat r_6)\Big)$

(5.5) each 0.5

only survival force is due to 3q (1) (0.5)

$\rightarrow \vec F_{net} = \frac{k|-2q||3q|}{r_3^2}\hat r_3 = k\frac{6q}{d^2}(-\hat x) = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{6q^2}{d^2}(-\hat x)}$

ii) To bring the central particle. First find the potential present at that central point. Potential is a scalar quantity. No cancellation as force vectors. $V = k\frac{q}{r}$

$V_{net} = \sum_{i=1}^{11} V_i = k\left(\frac{-7q-7q+3q}{d} + \frac{+4q+4q+2q+2q}{d\sqrt2} + \frac{-5q-5q-3q-3q}{[d^2+(d/2)^2]^{1/2}}\right)$ (2.75) each 0.25

$= \frac{q}{4\pi\varepsilon_0}\left(\frac{-11}{d} + \frac{12}{d\sqrt2} + \frac{-16}{d(\sqrt5/2)}\right) = \frac{q}{4\pi\varepsilon_0 d}\left(-11+6\sqrt2-32/\sqrt5\right) = \frac{q}{4\pi\varepsilon_0 d}(-16.8)$

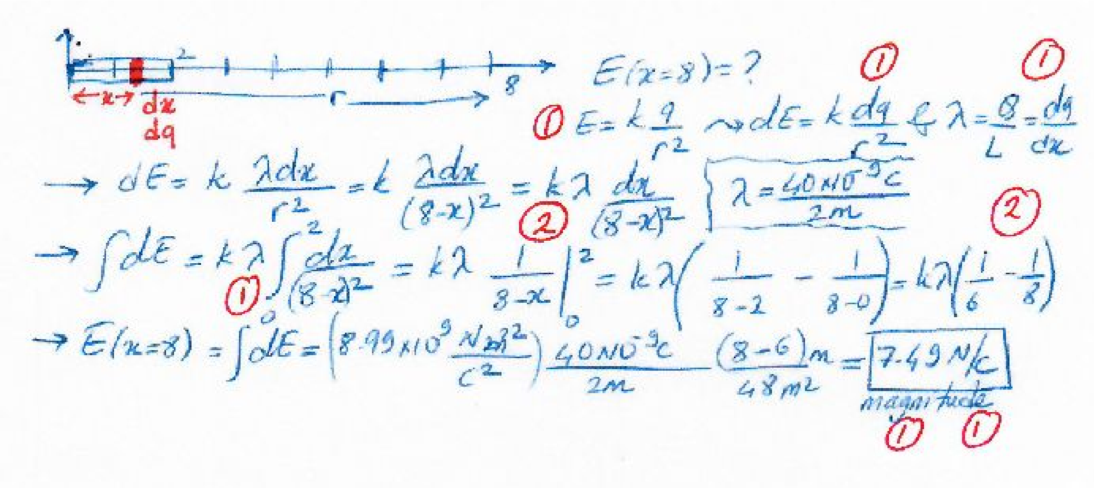
$= -\frac{4.2q}{\pi\varepsilon_0 d}$ (1.25)

Now find the required potential energy (or work)

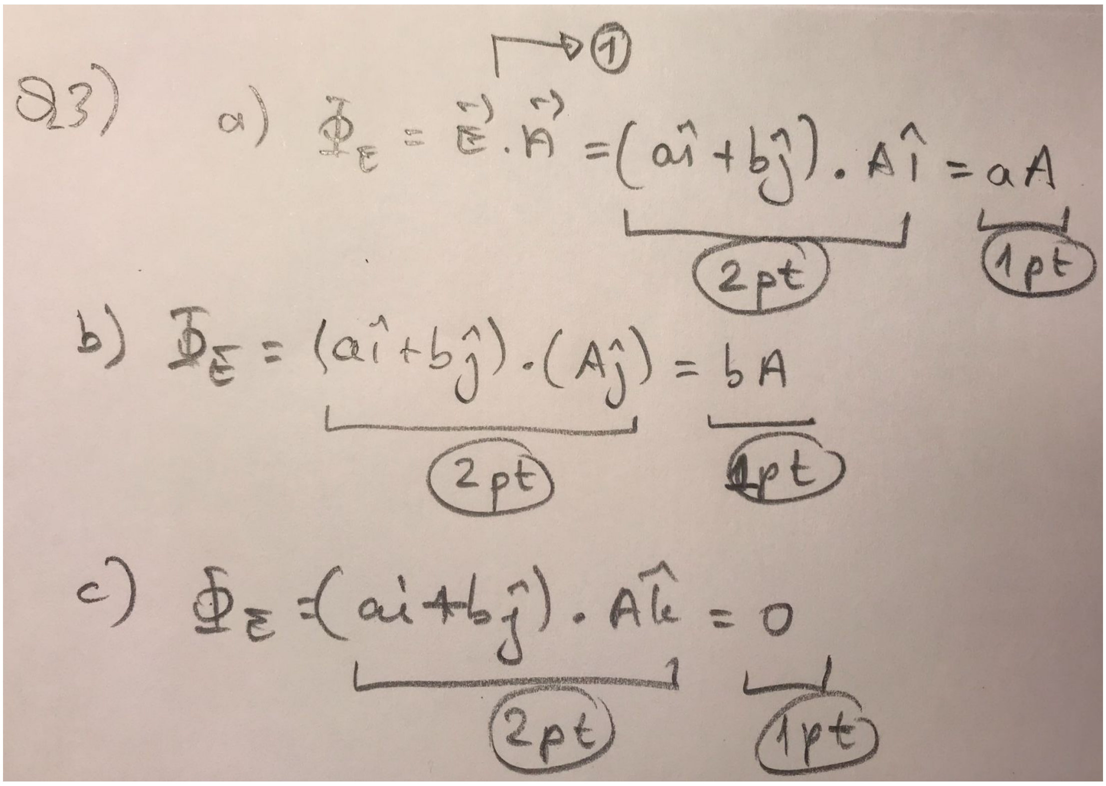
$W = \Delta U = (U_f - U_i) = U_f = (-2q)\left(\frac{-4.2q}{\pi\varepsilon_0 d}\right) = \boxed{\frac{8.4q}{\pi\varepsilon_0 d}}$ (1)

(qV) (1) 0 at infinity (1)

2. A charge of 40 nC is uniformly distributed along the axis from x = 0 to x=2 m. Determine the magnitude of the electric field at a point on the x-axis with x =8 m. (Take $k=8.99 \times 10^{9}$ N m^2 /C^2,1 nC =10^{-9} C)



3. A uniform electric field $a\hat{i} + b\hat{j}$ intersects a surface of area A. What is the flux through this area if the surface lies (a) in the yz plane? (b) in the xz plane? (c) in the xy plane? Your answers should be in terms of a, b and A.

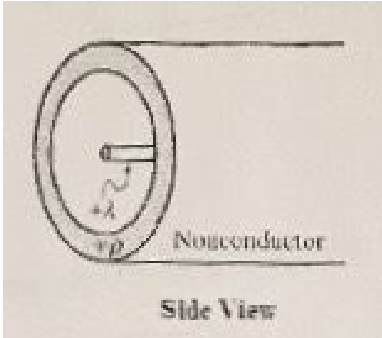


4. An infinitely long cylindrical insulating shell of inner radius ***a*** and outer radius ***b*** has a uniform volume charge density ρ. A line of uniform linear charge density λ is placed along the axis of the shell (the figure below). Determine the electric field in terms of a,b, k, λ. (Hint: Choose a Gaussian cylinder of radius r and length L. Then apply Gauss' law.)

   i) r < a

   ii) a < r < b

   iii) b < r



Q4) Gauss' law: $\vec{E} \cdot \left(\text{Gaussian Surface Area}\right) = \dfrac{\left(\text{Total enclosed charge}\right)}{\epsilon_0}$

So, if $r < a$:

$$\underbrace{E(2\pi r L) = \frac{\lambda L}{\epsilon_0}}_{4pt} \Rightarrow \boxed{E = \frac{\lambda}{2\pi r \epsilon_0}\hat{r}} \quad 1pt$$

If $a < r < b$:

$$\underbrace{E(2\pi r L) = \frac{\lambda L + \rho\pi(r^2 - a^2)L}{\epsilon_0}}_{4pt}$$

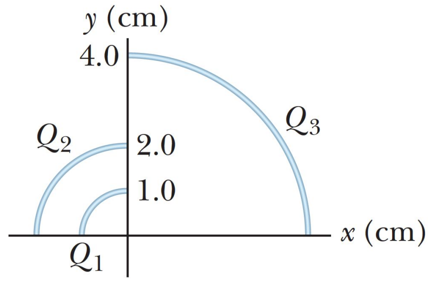
$$\boxed{E = \frac{\lambda + \rho\pi(r^2 - a^2)}{2\pi r \epsilon_0}\hat{r}} \rightarrow 1pt$$

If $r > b$:

$$E(2\pi r L) = \frac{\lambda L + \rho\pi(b^2 - a^2)L}{\epsilon_0} \rightarrow 4pt$$

$$\boxed{E = \frac{\lambda + \rho\pi(b^2 - a^2)}{2\pi r \epsilon_0}\hat{r}} \rightarrow 1pt$$

5. In the Figure, three thin plastic rods form quarter-circles with a common center of curvature at the origin. The uniform charges on the rods are $Q_1 = +30$ nC, $Q_2 = +3.0\,Q_1$, and $Q_3 = -8.0\,Q_1$. What is the net electric potential at the origin due to the rods? (Take k=9x10^9 N m^2 /C^2). 1 nC =10^{-9} C.



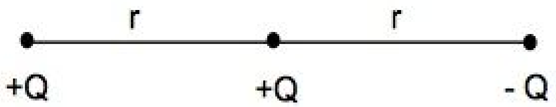
$$V_i = k\frac{Q_i}{r_i} \quad (4)$$

$$V = V_1 + V_2 + V_3 = k\left\{\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3}\right\} \quad (5)$$

$$= \frac{9\times10^{9} \times 30\times10^{-9}}{10^{-2}}\left\{\frac{1}{1} + \frac{3}{2} - \frac{8}{4}\right\}$$

$$= \boxed{1{,}35\times10^{5}\ \text{volt}} \quad (1)$$

6. Three charges are placed in a line as shown. What is the total potential energy of these charges ? Your answer should be in terms of kQ^2/r.



Use $U_{ij} = k\frac{q_i q_j}{r_{ij}}$ (5) $U_{total} = \sum_i \sum_j k\frac{q_i q_j}{r_{ij}}$

$$\Rightarrow U = k\left\{\frac{Q^2}{r} - \frac{Q^2}{2r} - \frac{Q^2}{r}\right\} = \boxed{-k\frac{Q^2}{2r}} \quad (5)\ (5)$$

7. A parallel-plate capacitor with square plates 14 cm on a side and separated by 2.0 mm is connected to a battery and charged to 12V. (a) What is the charge on the capacitor? (b) How much energy is stored in the capacitor? (c) The battery is then disconnected from the capacitor and the plate separation is then increased to 3.5 mm. By how much is the energy increased when the plate separation is changed?

a) $Q_0 = C_0 V_0 = \frac{\epsilon_0 A}{d_0} V_0 = \frac{8{,}85\times10^{-12} \times (0{,}14)^2}{0{,}002} \times 12 = 1{,}04\,nC$ (2) (2) (1)

b) $U_0 = \frac{1}{2} Q_0 V_0 = \frac{1}{2}\, 1{,}04\times10^{-9} \times 12 = 6{,}24\,nJ$ (4) (1)

c) $Q' = Q_0$ (1), $E' = E_0$ because $E = \frac{Q}{A\epsilon_0}$

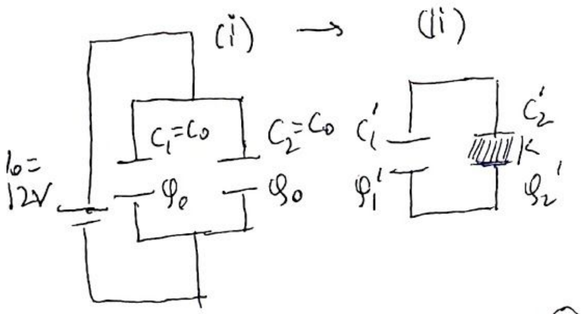
$V = Ed$ (1) $\Rightarrow \frac{V'}{d'} = \frac{V_0}{d_0} \Rightarrow V' = \frac{V_0}{d_0} \cdot d'$

$U' = \frac{1}{2} Q' V' = \frac{1}{2} Q_0 V_0 \frac{d'}{d_0} = U_0 \cdot \frac{d'}{d_0}$ (3)

$\Delta U = U' - U_0 = \frac{d'}{d_0} U_0 - U_0 = \left(\frac{d'}{d_0} - 1\right) U_0 = \left(\frac{3{,}5}{2} - 1\right) U_0$ (1) (4)

$= \left(\frac{3{,}5}{2} - 1\right) \cdot 6{,}24\,nJ = 4{,}68\,nJ$ (1)

8. Two parallel-plate capacitors, each having a capacitance of $C_1 = C_2 = 2\ \mu F$, are connected in parallel across a 12-V battery. The parallel combination is then disconnected from the battery and a dielectric slab of constant $K = 2.5$ is inserted between the plates of the capacitor C_2, completely filling the gap. After the dielectric is inserted, find (a) the charge on each capacitor, and (b) the total energy stored in the capacitors.



a) $Q_0 = C_0 V_0 = 2\mu F \times 12\ V$

$= 24 \mu C$ (1)

$Q_{total} = 2 \times 24 = 48 \mu C$ (1)

in (ii) $C_{eq} = C_1' + C_2' = C_0 + K C_0 = 2\mu F + 2{,}5 \times 2\mu F = 7\mu F$ (2)

$V' = \frac{Q_{total}}{C_{eq}} = \frac{48\ \mu C}{7\mu F} = 6.86\ V$ (1) $\Rightarrow$ $Q_1' = C_1' \cdot V' = 2\mu F \times 6{,}86 V = 13{,}7 \mu C$

$Q_2' = C_2' \cdot V' = 5\mu F \times 6{,}86 V = 34.3 \mu C$ (1)

b) $U = U_1 + U_2 = \frac{1}{2} Q_1' \cdot V' + \frac{1}{2} Q_2' V' = 165 \mu J$ (1) (3) (1)