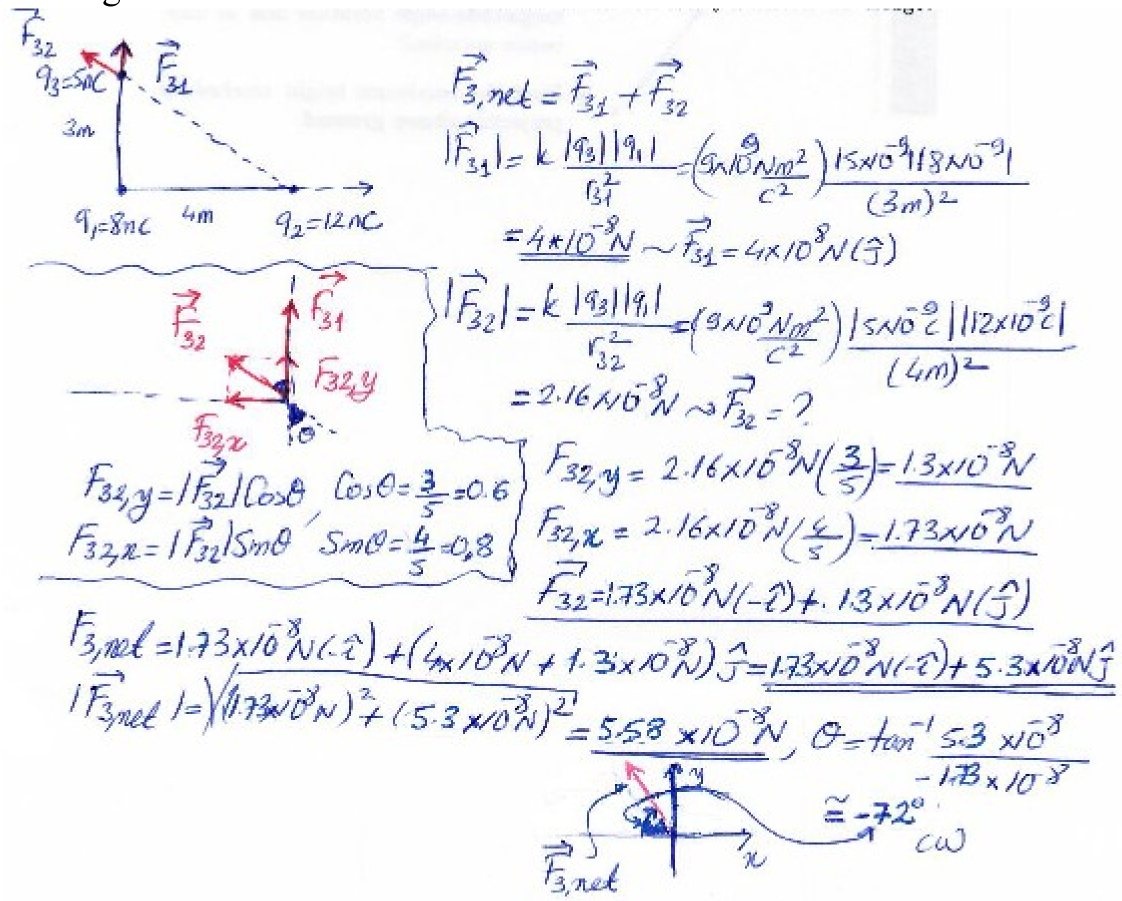
Open Book Quiz - Ch21 Electric Charge
(Duration: 30 minutes)

1. (60 pts) A point charge q_1=8 nC is at the origin and a second point charge q_2=12 nC is on the x-axis at x=4 m. Find the net electric force they exert on q_3= 5 nC located on the y-axis at y=3.0 m in vector notation, magnitude and angle.



$\vec{F}_{3,net} = \vec{F}_{31} + \vec{F}_{32}$

$|\vec{F}_{31}| = k\frac{|q_3||q_1|}{r_{31}^2} = (9\times10^9 \frac{Nm^2}{C^2})\frac{|5\times10^{-9}||8\times10^{-9}|}{(3m)^2} = 4\times10^{-8}N \sim \vec{F}_{31} = 4\times10^{-8}N(\hat{j})$

$|\vec{F}_{32}| = k\frac{|q_3||q_1|}{r_{32}^2} = (9\times10^9 \frac{Nm^2}{C^2})\frac{|5\times10^{-9}C||12\times10^{-9}C|}{(4m)^2} = 2.16\times10^{-8}N \sim \vec{F}_{32} = ?$

$F_{32,y} = |\vec{F}_{32}|\cos\theta$, $\cos\theta = \frac{3}{5} = 0.6$

$F_{32,x} = |\vec{F}_{32}|\sin\theta$, $\sin\theta = \frac{4}{5} = 0.8$

$F_{32,y} = 2.16\times10^{-8}N(\frac{3}{5}) = 1.3\times10^{-8}N$

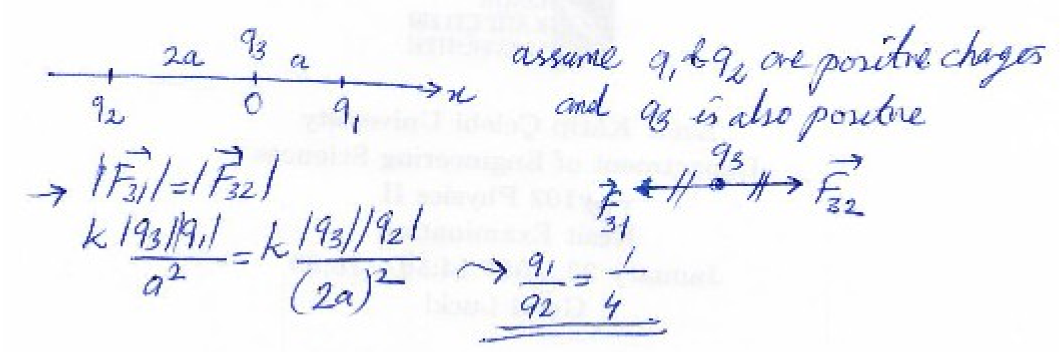
$F_{32,x} = 2.16\times10^{-8}N(\frac{4}{5}) = 1.73\times10^{-8}N$

$\vec{F}_{32} = 1.73\times10^{-8}N(-\hat{i}) + 1.3\times10^{-8}N(\hat{j})$

$\vec{F}_{3,net} = 1.73\times10^{-8}N(-\hat{i}) + (4\times10^{-8}N + 1.3\times10^{-8}N)\hat{j} = 1.73\times10^{-8}N(-\hat{i}) + 5.3\times10^{-8}N\hat{j}$

$|\vec{F}_{3,net}| = \sqrt{(1.73\times10^{-8}N)^2 + (5.3\times10^{-8}N)^2} = 5.58\times10^{-8}N$, $\theta = \tan^{-1}\frac{5.3\times10^{-8}}{-1.73\times10^{-8}} \cong -72°$ cw

2. (40 pts) Particle 1 with charge q_1, and particle 2, with a charge q_2 are on the x-axis, with particle 1 at x=a and particle 2 at x=-2a. For the net force on a third charged particle, at the origin to be zero what must be the ratio q_1/q_2.



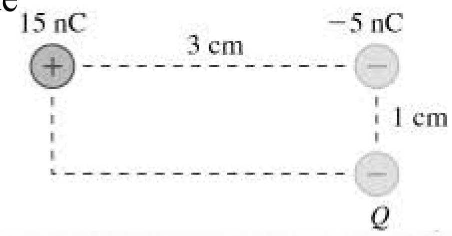
assume q_1 & q_2 are positive charges and q_3 is also positive

$\rightarrow |\vec{F}_{31}| = |\vec{F}_{32}|$

$k\frac{|q_3||q_1|}{a^2} = k\frac{|q_3||q_2|}{(2a)^2} \rightarrow \frac{q_1}{q_2} = \frac{1}{4}$

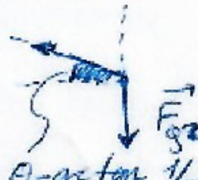
Open Book Quiz - Ch21 Electric Charge
(Duration: 30 minutes)

1. (60 pts) What is in vector notation, magnitude and angle of the net force on Q=-45nC?



15nC ① ⊕ --- 3cm --- ⊖ -5nC ②, 1cm, ⊖ Q

$\vec{F}_{Q,net} = \vec{F}_{Q1} + \vec{F}_{Q2}$ | when Q = -45nC



$\theta = \arctan 1/3 = 18.4°$

$\to F_{Q2} = k\frac{|Q||-5nC|}{(1\times10^{-2}m)^2} = 9\times10^{9}\frac{Nm^2}{C^2}\frac{|-45\times10^{-9}C||-5\times10^{-9}C|}{(1\times10^{-2}m)^2}$
$= 0.0203N$

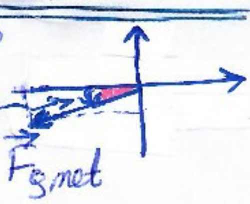
$F_{Q1,x} = |\vec{F}_{Q1}|\cos 18.4 = \frac{k|Q||15nC|\cos 18.4}{(1\times10^{-2}m)^2+(3\times10^{-2}m)^2} = 9\times10^{9}\frac{Nm^2}{C^2}\frac{45\times10^{-9}C\ 15\times10^{-9}C}{1\times10^{-3}m}\cos 18.4$
$= (6.08\times10^{-3}N)\cos 18.4 = 5.76\times10^{-3}N$

$F_{Q1,y} = |\vec{F}_{Q1}|\sin 18.4 = 1.92\times10^{-3}N$

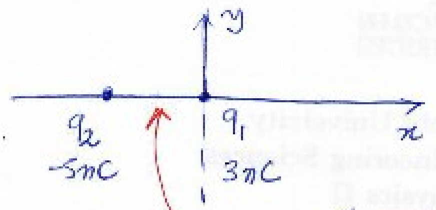
$\vec{F}_{Q,net} = 5.76\times10^{-3}N(-\hat{i}) + (1.92\times10^{-3}N - 0.0203N)\hat{j}$
$= 5.76\times10^{-3}N(-\hat{i}) + 1.83\times10^{-2}N(-\hat{j})$

$|\vec{F}_{Q,net}| = [(-5.76\times10^{-3}N)^2 + (-1.83\times10^{-2}N)^2]^{1/2}$
$= 1.92\times10^{-2}N$

$\theta = \tan^{-1}\frac{-1.83\times10^{-2}}{-5.76\times10^{-3}} \cong 17.6°$ (ccw)



2. (40 pts)Three point charges are arranged along the x-axis. Charge q_1=3.00 nC is at the origin, and charge q_2=-5.00 nC is at x=0.200 m. Charge q_3=8.00 nC. Where is q_3 located if the net force on q_1 is 7.00 N in the -x direction?



$\vec{F}_{net,1} = \vec{F}_{12} + \vec{F}_{13}$ & $q_3 = 8nC$

$7N = k\frac{|q_1||q_2|}{(0.2m)^2} + k\frac{|q_1||q_3|}{x^2}$

$\leftarrow F_{1,net}$, $\leftarrow F_{12}$ $\Big\{ |F_{12}| = 9\times10^{9}\frac{Nm^2}{C^2}\frac{|3\times10^{-9}C||-5\times10^{-9}|}{(0.2m)^2}$
$= 3.38\times10^{-6}N$ very small

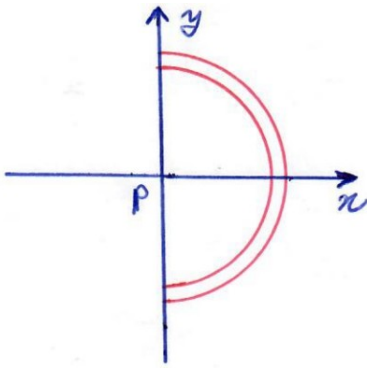
$\rightsquigarrow$ so q_3 should be in between

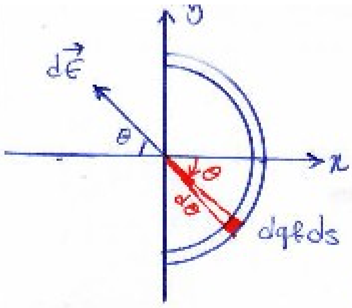
$7N = 3.38\times10^{-6}N + \left(9\times10^{9}\frac{Nm^2}{C^2}\right)\frac{|3\times10^{-9}C||8\times10^{-9}C|}{x^2}$

$\rightsquigarrow x = 1.76\times10^{-4}m$ (in $-\hat{i}$)

Open Book Quiz - Ch22 Electric Fields
(Duration: 30 minutes)

1. (100 pts) Semicircular wire shown in figure below has a non-uniform charge distribution $\lambda(\theta)=\lambda_0 \cos\theta$. Find the electric field at point P in unit vector notation and in terms of total charge Q.
   Hint: $\int \cos^2 ax\, dx = x/2 + \sin 2ax/4a$





$dE = k\frac{dq}{R^2} = k\frac{\lambda R\, d\theta}{R^2} = k\frac{\lambda\, d\theta}{R}$ $\{\lambda = \lambda_0 \cos\theta\}$ $= k\frac{\lambda_0 \cos\theta\, d\theta}{R}$

y-components are cancelling out due to symmetry

$dE_x = |d\vec{E}|\cos\theta = k\frac{\lambda_0 \cos^2\theta\, d\theta}{R}$ $\{ E_x = \int dE_x$

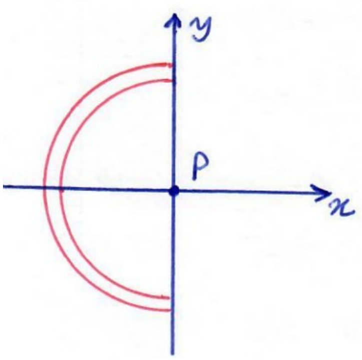
$E_x = E = \frac{k\lambda_0}{R}\int_{-\pi/2}^{\pi/2}\cos^2\theta\, d\theta = \frac{k\lambda_0}{R}\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right)\Big|_{-\pi/2}^{\pi/2} = \frac{k\lambda_0}{R}\frac{\pi}{2}$

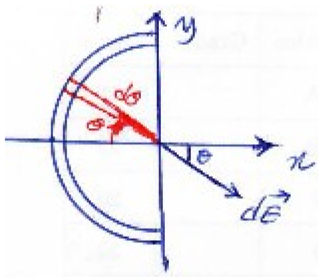
$\rightarrow \vec{E} = \frac{k\lambda_0 \pi}{2R}(-\hat{i})$ | in terms of Q: $Q = \int \lambda\, ds = \int_{-\pi/2}^{\pi/2}\lambda_0 \cos\theta\, d\theta R = \lambda_0 R \sin\theta\Big|_{-\pi/2}^{\pi/2} = 2\lambda_0 R$

$= \frac{1}{4\pi\epsilon_0}\frac{Q}{2R}\frac{\pi}{2R}(-\hat{i}) = \boxed{\frac{Q}{16\epsilon_0 R^2}(-\hat{i})}$

Open Book Quiz - Ch22 Electric Fields
(Duration: 30 minutes)

1. (100 pts) Semicircular wire shown in figure below has a non-uniform charge distribution $\lambda(\theta)=\lambda_0 \text{Cos}\theta$. Find the electric field at point P in unit vector notation and in terms of total charge Q.
   Hint: $\int \text{Cos}^2 ax\, dx = x/2 + \text{Sin}2ax/4a$





$$dE = k\frac{dq}{R^2} = k\frac{\lambda R\,d\theta}{R^2} = k\frac{\lambda\,d\theta}{R} \;\{\lambda=\lambda_0 \text{Cos}\theta\} = \frac{k\lambda_0 \text{Cos}\theta\,d\theta}{R}$$

y-components are cancelling out due to symmetry

$$dE_x = |d\vec{E}|\text{Cos}\theta = \frac{k\lambda_0 \text{Cos}^2\theta\,d\theta}{R} \;\Big\{ E_x = \int dE_x$$

$$E_x = E = \frac{k\lambda_0}{R}\int_{-\pi/2}^{\pi/2} \text{Cos}^2\theta\,d\theta = \frac{k\lambda_0}{R}\left(\frac{\theta}{2} + \frac{\text{Sin}2\theta}{4}\right)\Big|_{-\pi/2}^{\pi/2} = \frac{k\lambda_0}{R}\frac{\pi}{2}$$

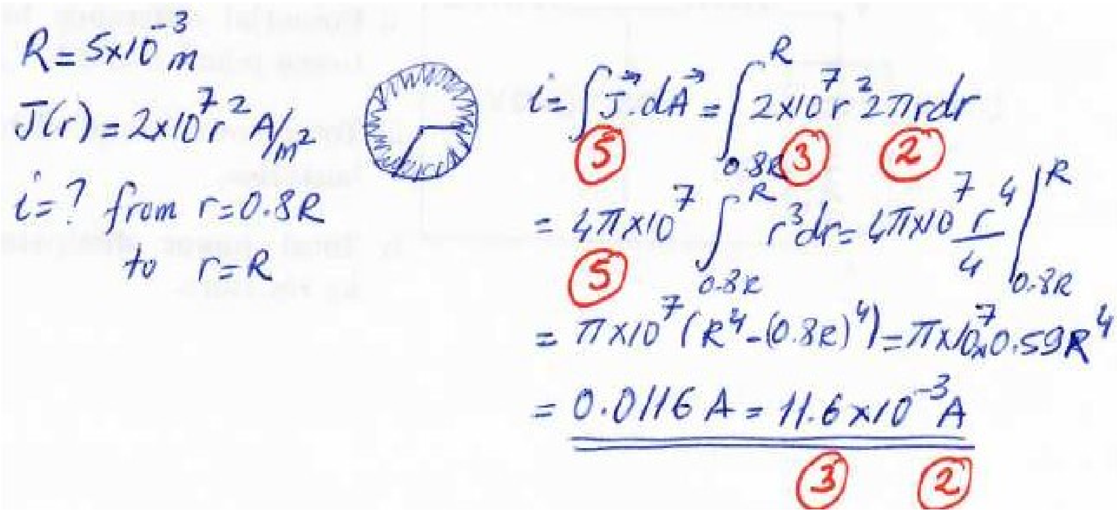
$$\vec{E} = \frac{k\lambda_0\pi}{2R}(+\hat{i}) \;\Big|\; \text{in terms of } Q: \quad Q = \int \lambda\,ds = \int_{-\pi/2}^{\pi/2} \lambda_0 R \text{Cos}\theta\,d\theta = \lambda_0 R\,\text{Sin}\theta\Big|_{-\pi/2}^{\pi/2} = 2\lambda_0 R$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R\,2R}\pi(+\hat{i}) = \boxed{\frac{Q}{16\varepsilon_0 R^2}\hat{i}}$$

Open Book Quiz - Ch26 Current and Resistance

(Duration: 30 minutes)

1. (100 pts) The magnitude J of the current density in a certain lab wire with a circular cross section of radius R=5.00 mm is given by $J = (2.00 \times 10^7) r^2$, with J in amperes per square meter and radial distance r in meters. What is the current through the outer section bounded by r=0.800R and r=R?



$R = 5 \times 10^{-3}$ m

$J(r) = 2 \times 10^7 r^2$ A/m^2

$i = ?$ from $r = 0.8R$ to $r = R$

$i = \int \vec{J} \cdot d\vec{A} = \int_{0.8R}^{R} 2 \times 10^7 r^2 \, 2\pi r dr$ (5) (3) (2)

$= 4\pi \times 10^7 \int_{0.8R}^{R} r^3 dr = 4\pi \times 10^7 \frac{r^4}{4} \Big|_{0.8R}^{R}$ (5)

$= \pi \times 10^7 (R^4 - (0.8R)^4) = \pi \times 10^7 \times 0.59 R^4$

$= 0.0116$ A $= 11.6 \times 10^{-3}$ A (3) (2)

Open Book Quiz - Ch26 Current and Resistance
(Duration: 30 minutes)

1. (100 pts) The magnitude J of the current density in a certain lab wire with a circular cross section of radius R=10.00 mm is given by $J = (4.00 \times 10^7) r^2$, with J in amperes per square meter and radial distance r in meters. What is the current through the section bounded by r=0.400R and r=0.800R?

$R = 10 \times 10^{-3}\, m$

$J(r) = 4 \times 10^7 r^2\, A/m^2$

$i = ?$ from $r = 0.4R$ to $r = 0.8R$

$i = \int \vec{J} \cdot d\vec{A} = \int_{0.4R}^{0.8R} 4 \times 10^7 r^2 \, 2\pi r\, dr$ (5) (3) (2)

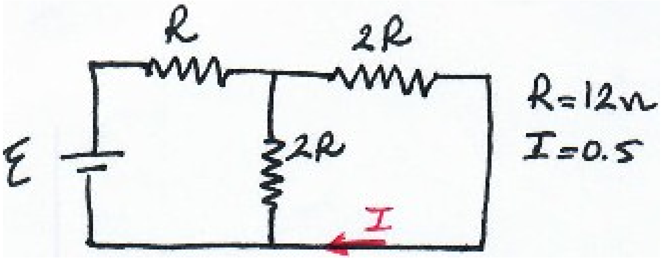
$= 8\pi \times 10^7 \int_{0.4R}^{0.8R} r^3 dr = 8\pi \times 10^7 \frac{r^4}{4} \Big|_{0.4R}^{0.8R}$ (5)

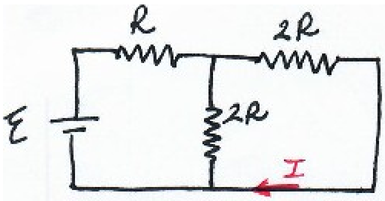
$= 2\pi \times 10^7 \left((0.8R)^4 - (0.4R)^4 \right) = 0.24 A$ (3) (2)

Open Book Quiz - Ch27 Circuits

(Duration: 30 minutes)

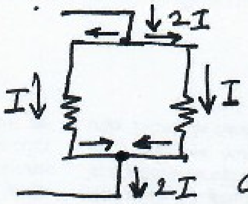
2. (100 pts) Find
   i) the current in each resistors
   ii) R_{eqv}
   iii) power dissipated at R.





QUIZ 4

i) the current in each resistor



same resistors ⇒ same current

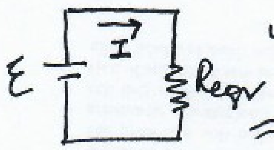
at R ⇝ $2I = 1A$

at 2R ⇝ $I = 0.5A$

ii) R_{eqv} ~ ?

⇝ $R + \left(\frac{1}{2R} + \frac{1}{2R}\right)^{-1}$

$12\Omega + 12\Omega = 24\Omega = R_{eqv}$



where $I = 1A$

$R_{eqv} = 24\Omega$

⇒ $\mathcal{E} = I R_{eqv} = 24V$

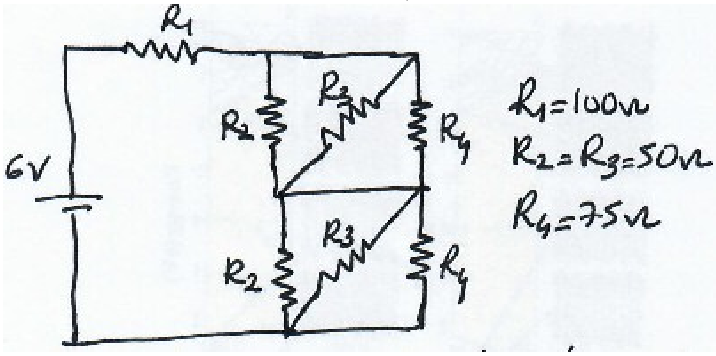
iii) Power dissipated at R

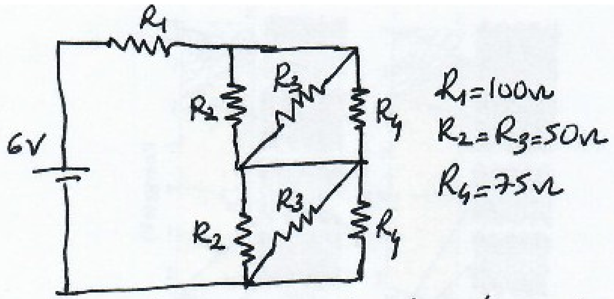
$P = I^2 R$ (or V^2/R)

$= 12\ W$

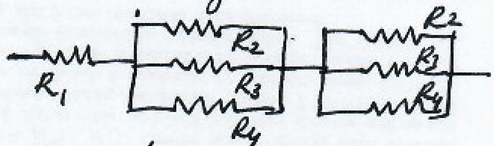
Open Book Quiz - Ch27 Circuits
(Duration: 30 minutes)

3. (100 pts) Find
   i) R_{eqv}
   ii) the current in each resistors





i) Find the equivalent resistance

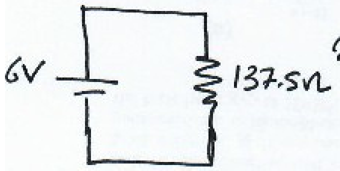


$R_{234} = \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)^{-1} = \left(\frac{1}{50} + \frac{1}{50} + \frac{1}{75}\right)^{-1}$

$= 18.75\Omega$

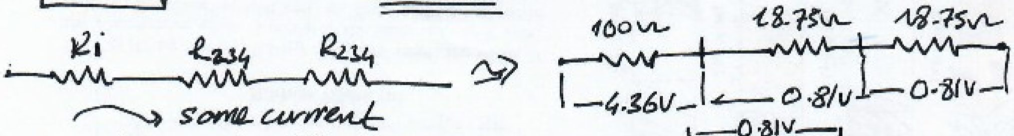
$\Rightarrow R_{eqv} = R_1 + R_{234} + R_{234} = 137.5\Omega$

ii) Find the current at each resistor

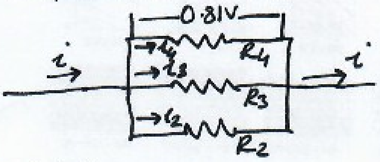


$\Rightarrow i = \frac{V}{R_{eqv}} = \frac{6V}{137.5\Omega}$

$= 0.0436A$



$V = iR$ — $V_1 = iR_1 = 4.36V$; $V_{234} = iR_{234} = 0.81V$



$i_2 = \frac{0.81V}{50\Omega} = 0.0162A = i_3$

$i_4 = \frac{0.81V}{75\Omega} = 0.0108A$