İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy102 Physics II
Midterm Examination
November 03, 2019 15:30 – 17:30
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

INSTRUCTOR:

DURATION: 120 minutes

◇ **Answer all the questions.**
◇ **Write the solutions explicitly and clearly. Use the physical terminology.**
◇ **You are allowed to use Formulae Sheet.**
◇ **Calculator is allowed.**
◇ **You are not allowed to use any other electronic equipment in the exam.**

Question	Grade	Out of
1A		15
1B		15
2		20
3		20
4		20
5		20
TOTAL		110

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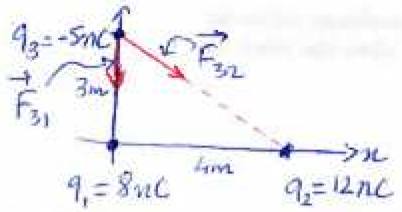
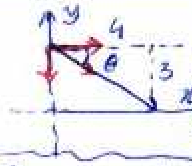
1. A) A point charge $q_1 = 8\ nC$ is at the origin and a second point charge $q_2 = 12\ nC$ is on the x-axis at x=4 m. Find the net electric force they exert on $q_3 = -5\ nC$ located on the y-axis at y=3.0 m in vector notation, magnitude and angle.



$$\vec{F}_{3,net} = \vec{F}_{31} + \vec{F}_{32}$$

$$|\vec{F}_{31}| = k\frac{|q_3||q_1|}{r_{31}^2} = \left(9\times10^{9}\frac{Nm^2}{C^2}\right)\frac{|5\times10^{-9}C||8\times10^{-9}C|}{(3m)^2}$$

$$= \underline{4\times10^{-8}N} \rightsquigarrow \underline{\vec{F}_{31} = 4\times10^{-8}N(-\hat{j})}$$



$\cos\theta = \frac{4}{5} = 0.8$

$\sin\theta = \frac{3}{5} = 0.6$

$$|\vec{F}_{32}| = k\frac{|q_3||q_2|}{r_{32}^2} = \left(9\times10^{9}\frac{Nm^2}{C^2}\right)\frac{|-5\times10^{-9}C||12\times10^{-9}C|}{(4m)^2}$$

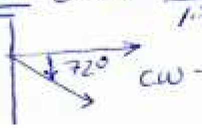
$$= \underline{2.16\times10^{-8}N} \rightsquigarrow \vec{F}_{32} = ?$$

$$F_{32,x} = |\vec{F}_{32}|\cos\theta = 2.16\times10^{-8}N\times0.8 = 1.73\times10^{-8}N$$

$$\underline{\vec{F}_{32} = 1.7\times10^{-8}N\hat{i} + 1.3\times10^{-8}N(-\hat{j})} \quad F_{32,y} = |\vec{F}_{32}|\sin\theta = 2.16\times10^{-8}N\times0.6 = 1.3\times10^{-8}N$$

$$\Rightarrow \vec{F}_{3,net} = (4.0\times10^{-8}(-\hat{j})) + 1.73\times10^{-8}N\hat{i} + 1.3\times10^{-8}(-\hat{j}))N = \underline{\underline{1.73\times10^{-8}N\hat{i} - 5.3\times10^{-8}\hat{j}}}$$

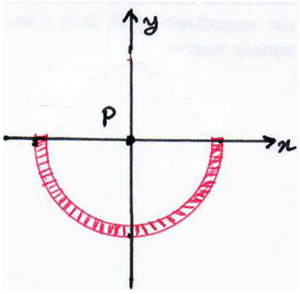
$$|\vec{F}_{3,net}| = \sqrt{(1.73\times10^{-8}N)^2 + (-5.3\times10^{-8}N)^2} = \underline{\underline{5.6\times10^{-8}N}} \quad \theta = \tan^{-1}\frac{-5.3}{1.73} \simeq -72^\circ$$

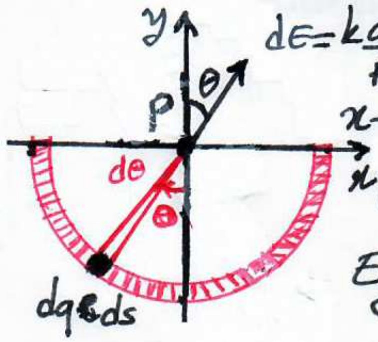


CW

B) Semicircular wire shown in figure below has a non-uniform charge distribution $\lambda(\theta) = \lambda_0 Cos\theta$.

Find the electric field at point P in unit vector notation and in terms of total charge Q.
(Hint: $\int Cos^2 ax dx = x/2 + Sin2ax/4a$)





$$dE = k\frac{dq}{R^2} = k\frac{\lambda R d\theta}{R^2} = \frac{k\lambda d\theta}{R} \Big\} \lambda = \lambda_0 \cos\theta \Big| = \frac{k\lambda_0 \cos\theta}{R} d\theta$$

x-components are cancelling due to symmetry

$$dE_y = |d\vec{E}|\cos\theta = \frac{k\lambda_0 \cos^2\theta \, d\theta}{R} \Big\} E_y = \int dE_y$$

$$E_y = E = \frac{k\lambda_0}{R}\int_{-\pi/2}^{\pi/2}\cos^2\theta \, d\theta = \frac{k\lambda_0}{R}\left(\frac{\theta}{2}+\frac{\sin 2\theta}{4}\right)\Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{k\lambda_0}{R}\frac{\pi}{2} \rightsquigarrow \vec{E} = \frac{k\lambda_0\pi}{2R}(\hat{j})$$

in terms of Q $\Big\}$ $Q = \int\lambda ds = \int_{-\pi/2}^{\pi/2}\lambda_0\cos\theta R d\theta = \lambda_0 R\sin\theta\Big|_{-\pi/2}^{\pi/2} = \underline{2\lambda_0 R}$

$$\Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{2R}\frac{\pi}{2R}\hat{j} = \boxed{\frac{Q}{16\pi\epsilon_0 R^2}\hat{j}}$$

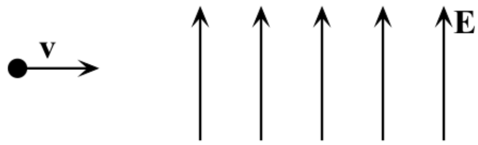
2. A proton moves at $4.5 \times 10^5\ m/s$ in the horizontal direction. It enters a uniform vertical electric field with a magnitude of $9.6 \times 10^3\ N/C$.

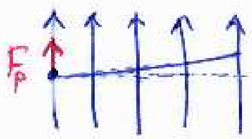
Ignoring any gravitational effects, find

i the time required for the proton to travel 5 cm horizontally,

ii the vertical displacement during that time,

iii the horizontal and vertical components of the velocity after the proton has traveled 5 cm horizontally.





$v = 4.5\times10^{5}\ m/s$ & $E = 9.6\times10^{3}\ N/C$ (uniform)

$v = v_{0x}$ & $v_{0y} = 0$ } Constant $E \sim$ constant

$a = a_y$ & $a_x = 0$ } acceleration $\leftarrow$ force

$qE = ma$

i) $v_x = v_{0x} = v = \frac{\Delta x}{\Delta t} \sim \Delta t = \frac{5\times10^{-2}\ m}{4.5\times10^{5}\ m/s} = 1.11\times10^{-7}\ s = \underline{111\ ns}$

ii) $a_y m_p = q_p E \sim a_y = \frac{q_p E}{m_p} = \frac{(1.6\times10^{-19}\ C)(9.6\times10^{3}\ N/C)}{(1.67\times10^{-27}\ kg)} = 9.21\times10^{11}\ m/s^2$

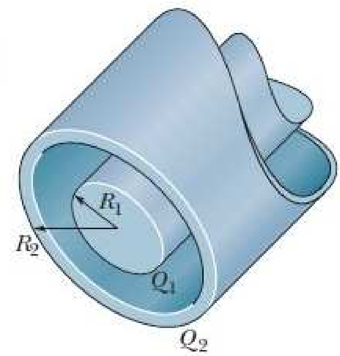
$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \sim y - y_0 = h = \frac{1}{2}a_y t^2 = \frac{1}{2}(9.21\times10^{11}\ m/s^2)(1.11\times10^{-7}\ s)^2$

$= 5.68\times10^{-3}\ m = \underline{5.68\ mm}$

iii) $v_x = v_{0x} = 4.5\times10^{5}\ m/s$

$v_y = v_{0y} + a_y t = a_y t = (9.21\times10^{11}\ m/s^2)(1.11\times10^{-7}\ s) = \underline{1.02\times10^{5}\ m/s}$

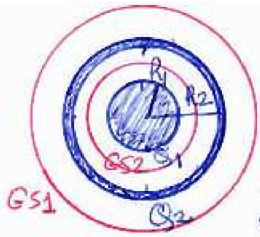
3. Figure below shows a section of a conducting rod of radius $R_1 = 1.30\ mm$ and length $L = 11.00\ m$ inside a thin-walled coaxial conducting cylindrical shell of radius $R_2 = 10.0R_1$ and the (same) length L. The net charge on the rod is $Q_1 = +3.40 \times 10^{-12}\ C$; that on the shell is $Q_2 = -2.00Q_1$



i What are the magnitude E and direction (radially inward or outward) of the electric field at radial distance $r = 2.00R_2$?

ii What are E and the direction at $r = 5.00R_1$?

iii What is the charge on the interior and exterior surface of the shell?



$R_1 = 1.30 \times 10^{-3}\ m$
$R_2 = 10.0\ R_1 = 1.30 \times 10^{-2}\ m$
$L = 11.00\ m$

$Q_1 = +3.40 \times 10^{-12}\ C$ (on rod)
$Q_2 = -2\,Q_1 = -6.80 \times 10^{-12}\ C$ (on shell)
$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$ & cylindrical Gaussian surface

i) GS1: $r = 2R_2 \rightsquigarrow E\, 2\pi r L = \frac{Q_1 + Q_2}{\varepsilon_0} \Rightarrow E = \frac{3.4 \times 10^{-12} C - 6.8 \times 10^{-12} C}{2\pi\, 2 \times 1.30 \times 10^{-2} m\ 11 m\ \varepsilon_0}$

$\vec{E}$ $\hat{r}_1$ $\rightsquigarrow E = -0.214\ N/C \rightsquigarrow |\vec{E}| = 0.214\ N/C$ & inward

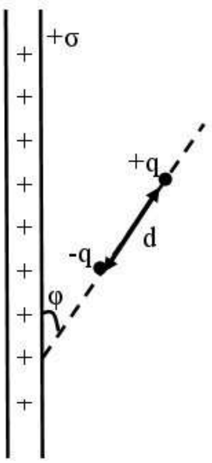
ii) GS2: $r = 5R_1 \rightsquigarrow E 2\pi r L = \frac{Q_1}{\varepsilon_0} \rightsquigarrow E = \frac{3.4 \times 10^{-12} C}{2\pi\, 5 \times 1.30 \times 10^{-3} m\ 11 m\ 8.85 \times 10^{-12}\ C^2/Nm^2}$

$\hat{r}_2$ $\vec{E}$ $\rightsquigarrow E = 0.855\ N/C \rightsquigarrow |\vec{E}| = 0.855\ N/C$ & outward

iii) Q_1 rod (outer) $\quad -Q_1$ shell (inner) $\quad Q_2 - (-Q_1)$ shell (outer) $\rightarrow 3.4 \times 10^{-12} C \quad -3.4 \times 10^{-12} C \quad \underbrace{-6.8 \times 10^{-12} C - (-3.4 \times 10^{-12} C)}_{-3.4 \times 10^{-12} C}$

sum up to $-6.8 \times 10^{-12} C$

4. An electric dipole of two opposite charges of magnitude $q = 1.50\ \mu C$, separated by a distance $d = 1.20\ cm$ is placed near an infinitely large plane of charge of uniform charge density $\sigma = 1.77\ \mu C/m^2$. The axis of the electric dipole makes an angle of $\varphi = 37°$ with the plane, as shown in the figure.

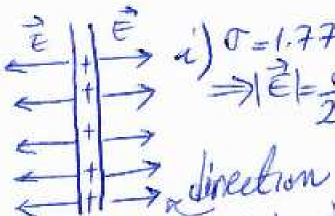


i Find the magnitude of the electric field due to the plane. Show its direction on the figure.

ii Calculate the magnitude of the electric dipole moment. Show its direction on the figure.

iii Calculate the magnitude of the torque acting on the electric dipole. Show its direction on the figure.

iv How much work must be done by an external agent to turn the electric dipole by 90° in clockwise direction?



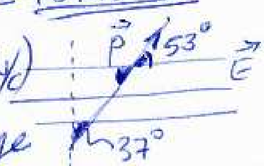
i) $\sigma = 1.77\times10^{-6}\ C/m^2$ & non-conducting plane (uniform charge density)

$\Rightarrow |\vec{E}| = \frac{\sigma}{2\varepsilon_0} = \frac{1.77\times10^{-6}\ C/m^2}{2\times8.85\times10^{-12}\ C^2/Nm^2} = \underline{10^5\ N/C}$ magnitude

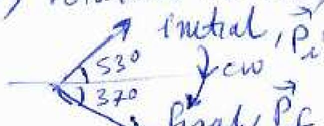
ii) $p = qd$ $\Big\{\ \ominus \xrightarrow{\vec{p}} \oplus$, $|\!-d-\!|$ $\Big\}$ $p = 1.50\times10^{-6}\ C \times 1.20\times10^{-2}\ m = \underline{1.8\times10^{-8}\ Cm}$

iii) $\vec{\tau} = \vec{p}\times\vec{E} \Rightarrow |\vec{\tau}| = |\vec{p}||\vec{E}|\,Sin53 = (1.8\times10^{-8}\ Cm)(10^5\ N/C)\,Sin53$

$= 1.44\times10^{-3}\ Nm$ ⊗ into the page



iv) rotation in cw, 90°



$W_{ext} = -\Delta U = -U_f + U_i = U_i - U_f$ $\{U = \vec{p}\cdot\vec{E}$

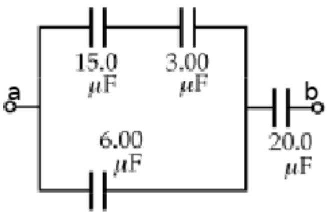
$= \vec{p}_i\cdot\vec{E} - \vec{p}_f\cdot\vec{E}$

$= |\vec{p}||\vec{E}|Cos53 - |p||E|Cos37°$

$= (1.8\times10^{-8}\ Cm)(10^5\ N/C)(Cos53° - Cos37°)$

$= \underline{-3.6\times10^{-4}\ J}$ $J \equiv Nm$

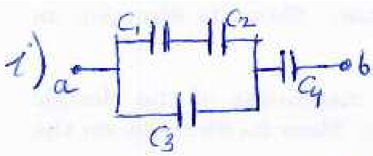
5. Four capacitors are connected as shown in Figure.



i Find the equivalent capacitance between points a and b.

ii Calculate the charge on each capacitor if $\Delta V_{ab} = 15.0\ V$.

1)



$$C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(15\times10^{-6}F)(3\times10^{-6}F)}{18\times10^{-6}F} = 2.5\times10^{-6}F$$

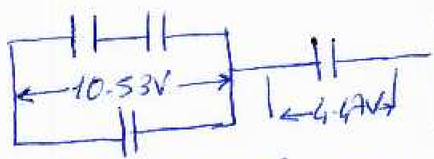
$$C_{123} = C_{12} + C_3 = 2.5\times10^{-6}F + 6\times10^{-6}F = 8.5\times10^{-6}F$$

$$C_{eqv} = C_{1234} = \frac{C_{123}C_4}{C_{123}+C_4} = \frac{(8.5\times10^{-6}F)(20\times10^{-6}F)}{28.5\times10^{-6}F} = 5.97\times10^{-6}F = 5.97\mu F$$

ii) $C = \frac{Q}{V} \rightarrow Q_{eqv} = Q_{1234} = C_{eqv}V = 5.97\times10^{-6}F \times 15V = \boxed{89.47\mu C}$ ← 4

$\rightarrow Q_4 = Q_{123} = Q_{eqv} = 89.47\mu C \rightarrow V_4 = \frac{89.47\mu C}{20\mu F} = 4.47V$ ← 4

→



$10.53V$ ← 3 $\rightarrow Q_3 = C_3V_3 = \boxed{63.18\mu C}$ 3

$Q_{12} = C_{12}V = (2.5\times10^{-6}F)\,10.53V = Q_1 = Q_2$

$\rightarrow Q_1 = Q_2 = \boxed{2.63\mu C}$

$\Rightarrow V_1 = \frac{Q_1}{C_1} = \frac{2.63\mu C}{15\mu F} = 1.75V$ ← 1

$V_2 = \frac{Q_1}{C_2} = 8.78V$ ← 2