İZMİR KATİP ÇELEBİ UNIVERSITY	FACULTY OF ENG. & ARCH. PHY102, TAKE HOME EXAM DUE 18 April 2020, 12:30			
Student Name	**ID Number**	**Instructor Name**	**Department**	**Signature**

Please read the following directions carefully.

- This document is provided to you both as a docx and a pdf file through UBYS and Canvas LMS systems.
- This document must be returned with solutions pasted under every question. You can solve the question on a paper, take a picture of it and paste that picture under the relevant question on the docx file. Then submit your document to UBYS and Canvas LMS systems as a single docx or a pdf document.
- The other alternative way is to print out the entire document, write out your solutions under every question. Scan or take a picture of every page with solutions, make a single word file out of it and submit the entire document to both UBYS and Canvas LMS systems.
- Submissions through email **will not be accepted**.
- Submissions containing many different picture files or zipped files **will not be accepted.**
- You must show all your work to get credit; you will not be given any points unless you show the details of your work (this applies even if your final answer is correct).
- Write neatly and clearly; **unreadable answers will not be given any credit.**
- Make sure that you include units in your results. Incomplete calculations will not be graded.
- There are 20 questions. Every question is worth 5 points.
- Do not forget to put your name, instructor's name, id number and your signature. 1

Constants

$e = 1.602 \times 10^{-19}$ C (charge on e^- or p^+)

$1 eV = 1.602 \times 10^{-19}$ J

$m_e = 9.11 \times 10^{-31}$ kg

$m_p = 1.67 \times 10^{-27}$ kg

$k = 1/(4\pi\varepsilon_o) = 8.99 \times 10^9$ N m^2/C^2

$\varepsilon_o = 8.85 \times 10^{-12}$ F/m (or C^2/N m^2)

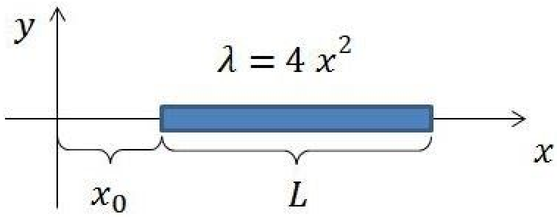
$\mu_o = 4\pi \times 10^{-7}$ T m/A

1 T $= 10^4$ G

$g = 9.8$ m/s^2

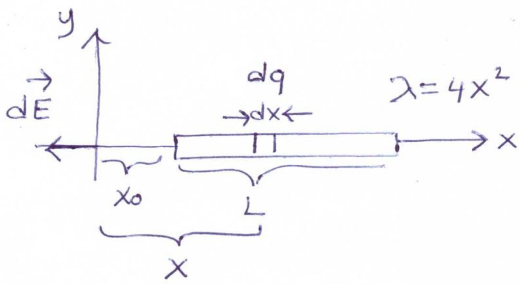
QUESTIONS

1. A non-uniform positive line charge of length 2 m is put along the x-axis as shown in the figure, where x_0=1.5 m. The linear charge density is given by $\lambda(x)=4x^2$ C/m^3. Find the magnitude of the total electric field, E, created by the line charge at the origin using integration.
   (Take $k=9\times10^9$ N m^2 /C^2)



$x_0 = 0.5$ m , $L = 2$m

$E = ?$



$$dE = k\frac{dq}{x^2} \quad , \quad dq = \lambda\, dx = 4x^2 dx$$

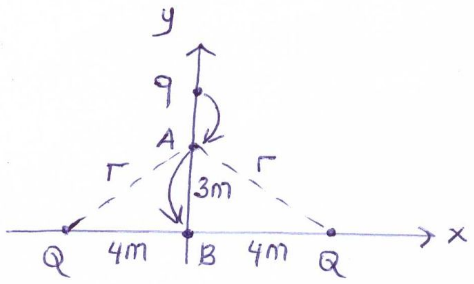
$$\int dE = \int_{x_0}^{x_0+L} k\,\frac{4x^2 dx}{x^2}$$

$$E = 4k \underbrace{\int_{x_0}^{x_0+L} dx}$$

$$x\Big|_{x_0}^{x_0+L} = \cancel{x_0} + L - \cancel{x_0} = L$$

$$E = 4kL = 4(9\times10^9)(2) = 72\times10^9 \ N/C$$

2. Identical 45 μC charges are fixed on an x-axis at x = ±4 m. A particle of charge q =-16 μC is then released from rest at a point on the positive part of the y-axis. Due to the symmetry of the situation, the particle moves along the y-axis and has kinetic energy 2 J as it passes through the point x =0, y=3 m. What is the kinetic energy of the particle as it passes through the origin?
(Take $k=9\text{x}10^9$ N m^2 /C^2)



$Q = 45\ \mu C$

$q = -16\ \mu C$

$K_A = 2J$

$K_B = ?$

Energy is conserved $\rightarrow$ $E_A = E_B$

$$K_A + U_A = K_B + U_B$$

$$U = qV, \quad V = \sum k \frac{q_i}{r_i}$$

$$U_A = q\left(k\frac{Q}{r} + k\frac{Q}{r}\right)$$

$$= (-16\times10^{-6})\left[(9\times10^{9})\frac{45\times10^{-6}}{5} + (9\times10^{9})\frac{45\times10^{-6}}{5}\right]$$

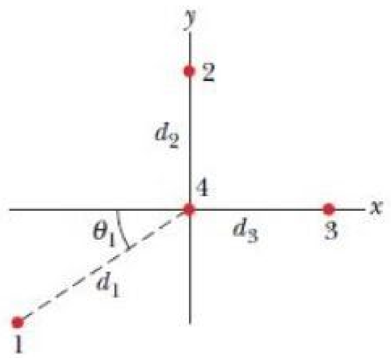
$$= -2.592\ J \cong -2.60\ J$$

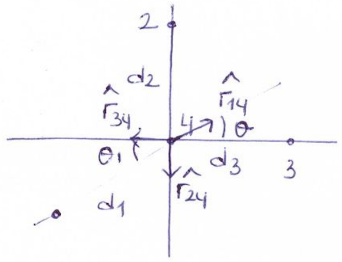
$$U_B = (-16\times10^{-6})\left[(9\times10^{9})\frac{45\times10^{-6}}{4} + (9\times10^{9})\frac{45\times10^{-6}}{4}\right]$$

$$= -3.24\ J$$

$$2 - 2.60 = K_B - 3.24 \rightarrow K_B = 2.64\ J$$

3. In figure, all four particles are fixed in the xy-plane, and q_1 = -q , q_2 =q_3 =+4q, q_4 =+5q, where q is 1.6x10^{-19} C, θ_1 = 40°, d_1 =5 cm and d2 =d3 = 3 cm. What is the magnitude of the net electrostatic force on particle 4 due to the other three particles. (Take k=9x10^9 Nm^2/C^2)





$q_1 = -q \quad , \quad q = 1.6\times10^{-19}C$

$q_2 = q_3 = 4q$

$q_4 = 5q$

$\theta_1 = 40°$

$d_1 = 5\,cm$

$d_2 = d_3 = 3cm$

$$\vec{F}_4 = \vec{F}_{14} + \vec{F}_{24} + \vec{F}_{34}$$

$$\vec{F}_{ij} = k\,\frac{q_i q_j}{r_{ij}}\,\hat{r}_{ij}$$

$$\vec{F}_{14} = k\,\frac{(-q)(5q)}{d_1^2}\,(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$$

$$= -(8.99\times10^9)\,\frac{5\,(1.6\times10^{-19})^2}{0.05^2}\,(\cos 40\,\hat{\imath} + \sin 40\,\hat{\jmath})$$

$$= -(3.53\,\hat{\imath} - 2.96\,\hat{\jmath})\times10^{-25}\,N$$

$$\vec{F}_{24} = \frac{k(4q)(5q)}{d_2^2}\,(\cos(-90)\,\hat{\imath} + \sin(-90)\,\hat{\jmath})$$

$$= -(8.99.10^9).20\,\frac{(1.6\times10^{-19})^2}{0.03^2}\,\hat{\jmath} = -5.11\times10^{-24}\,\hat{\jmath}\,N$$

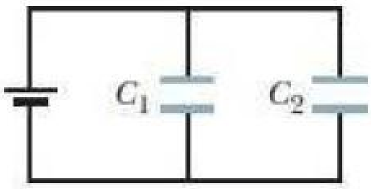
$$\vec{F}_{34} = \frac{k(4q)(5q)}{d_3^2}\,(\cos180\,\hat{\imath} + \sin180\,\hat{\jmath}) = -5.11\times10^{-24}\,\hat{\imath}\,N$$

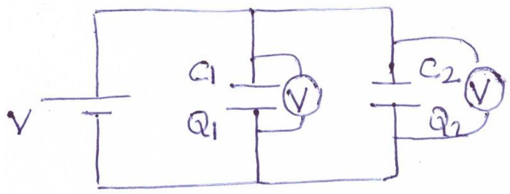
$$\vec{F}_4 = -[\,(0.35 + 5.11)\,\hat{\imath} + (0.30 + 5.11)\,\hat{\jmath}\,]\times10^{-24}$$

$$= -(5.46\,\hat{\imath} + 5.41\,\hat{\jmath})\times10^{-24}\,N$$

$$F_4 = \sqrt{(5.46\times10^{-24})^2 + (5.41\times10^{-24})^2} = 7.69\times10^{-24}\,N$$

4. Two parallel-plate capacitors (with air between the plates) are connected to a battery as shown in the Figure. Capacitor 1 has a plate area of 1.4 cm^2 and an electric field (between its plates) of magnitude $2x10^3$ V /m. Capacitor 2 has a plate area of 0.5 cm^2 and an electric field of magnitude $1.2x10^3$ V /m. What is the total charge on the two capacitors?
(Take ε_0 =$8.85x10^{-12}$ $F.m^{-1}$)





$Q_1 + Q_2 = ?$

$A_1 = 1{,}4 \ cm^2$ $A_2 = 0{,}5 \ cm^2$

$E_1 = 2 \times 10^3 \ V/m$ $E_2 = 1{,}2 \times 10^3 \ V/m$

$Q_1 = C_1 V_1$

$$Q_1 = \left(\varepsilon_0 \frac{A_1}{d_1} \right) \underbrace{V_1}_{E_1 d_1}$$

$$Q_1 = \left(\varepsilon_0 \frac{A_1}{\cancel{d_1}} \right) (E_1 \cancel{d_1})$$

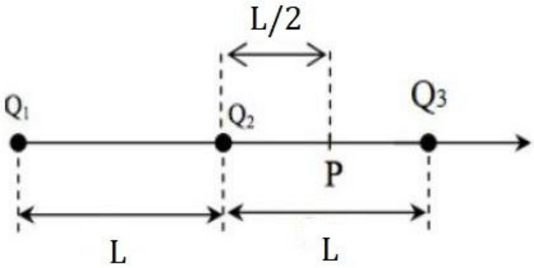
$Q_1 = \varepsilon_0 A_1 E_1$ $Q_2 = \varepsilon_0 A_2 E_2$

$$Q_1 + Q_2 = \varepsilon_0 (A_1 E_1 + A_2 E_2)$$

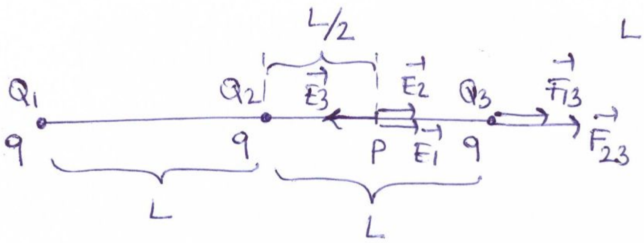
$$= (8.85 \times 10^{-12}) \left((1.4 \times 10^{-4})(2 \times 10^3) + (0.5 \times 10^{-4})(1.2 \times 10^3) \right)$$

$$= 3.01 \times 10^{-12} C \cong 3.0 \ pC$$

5. Three equal charges Q_1 , Q_2 , and Q_3 are placed along a straight line as shown in figure below. L=2.8 m, Q_1 = Q_2 = Q_3 and the net force acting on charge Q_3 is 1.55 N . What is the magnitude of the electric field at point P?
(Take $k=9x10^9$ Nm^2/C^2)



$L = 2.8m$, $F_3 = 1.55 N$



$$\vec{F}_{ij} = k \frac{q_i q_j}{r_{ij}^2} \hat{r}_{ij}$$

$$\vec{F}_{13} = k \frac{q q}{(2L)^2} \hat{\imath} , \qquad \vec{F}_{23} = k \frac{q^2}{L^2} \hat{\imath}$$

$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23} = \underbrace{k \frac{q^2}{L^2} \left(\frac{1}{4} + 1 \right)}_{F_3} \hat{\imath}$$

$$F_3 \rightarrow 1.55 = (9 \times 10^9) \frac{q^2}{2.8^2} \cdot \left(\frac{5}{4} \right)$$

$$q = 32.9 \times 10^{-6}\ C$$

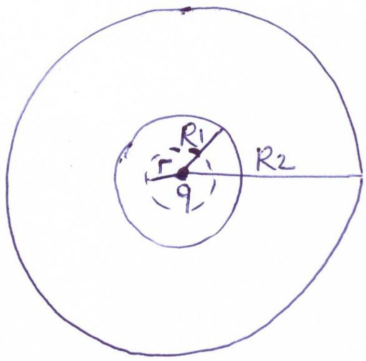
$$\vec{E}_i = k \frac{q_i}{r_i^2} \hat{r}_i$$

$$\vec{E}_1 = k \frac{q}{(3L/2)^2} \hat{\imath} , \qquad \vec{E}_2 = k \frac{q}{(L/2)^2} \hat{\imath} , \qquad \vec{E}_3 = - k \frac{q}{(L/2)^2} \hat{\imath}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = k \frac{q}{(3L/2)^2} \hat{\imath}$$

$$E = (9 \times 10^9) \frac{(32.8 \times 10^{-6})}{9 .\ 2.8^2} \ 4 = 16.8 \times 10^3\ N/C$$

6. Positive charge Q=5 nC is placed on a conducting spherical shell with inner radius R_1=20 cm and outer radius R_2=30 cm . A particle with charge q=4 nC is placed at the center of the cavity. What is the magnitude of the electric field at a point in the cavity, a distance r=12 cm from the center?
(Take Coulomb's constant $k=9\times10^{9}$ N m^2 /C^2)



$Q = 5\ nC$

$R_1 = 20\,cm, \quad R_2 = 30\,cm$

$q = 4\ nC$

$E(r = 12cm) = ?$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

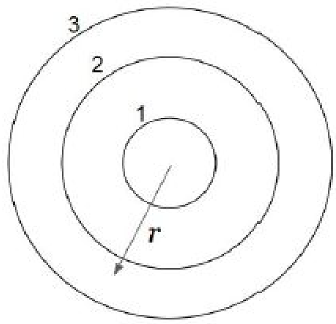
$$\oint \vec{E}\cdot d\vec{A} = \int E\, dA \cos 0 = E\int dA = E\, 4\pi r^2$$

$q_{enc} = q$

$$\Rightarrow \quad E\, 4\pi r^2 = \frac{q}{\varepsilon_0} \quad \rightarrow \quad E = k\frac{q}{r^2} = (8.99\times10^{9})\,\frac{(4\times10^{-9})}{(0.12)^2}$$

$$E = 2.5\times10^{3}\ N/C$$

7. The figure shows three concentric spherical conducting shells of radii a, 3a and 4a respectively. The shells are very thin. r shows the radial distance from the common center. The shell 1, 2 and 3 are initially charged as 3q, − q and − 2q respectively. What is the electric potential difference between any two points on the surfaces of the shell 2 and 3, that is $\Delta V = V_2 - V_3$? Your answer should be in units of kq/a .



$$r_2 < r < r_3 \rightarrow E(r) = k\frac{(q_1+q_2)}{r^2}\hat{r}$$

$$\Delta V = -\int \vec{E}\cdot d\vec{l}$$

$$V_2 - V_3 = -\int_{r_3}^{r_2} k\frac{(q_1+q_2)}{r^2}\hat{r}\cdot d\vec{l}\ , \quad \hat{r}\cdot d\vec{l} = dr$$

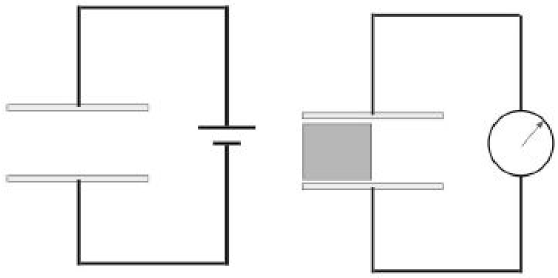
$$= -k(q_1+q_2)\underbrace{\int_{r_3}^{r_2}\frac{dr}{r^2}}_{-\frac{1}{r}\Big|_{r_3}^{r_2}}$$

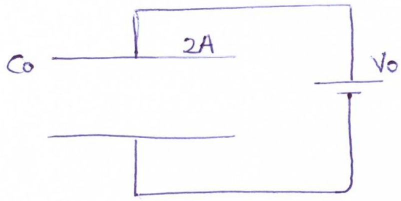
$$= k(q_1+q_2)\left[\frac{1}{r_2} - \frac{1}{r_3}\right]$$

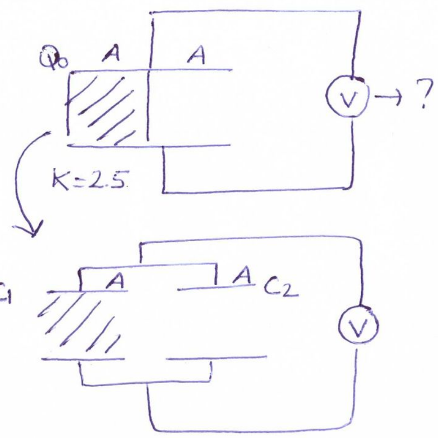
$$= k(3q+(-q))\left[\frac{1}{3a} - \frac{1}{4a}\right] = \frac{1}{6}\frac{kq}{a}$$

$$= 0.17\frac{kq}{a}$$

8. A capacitor with capacitance C_0 is connected to a battery of voltage V_0 and charged. Then it is disconnected from the battery and a dielectric is inserted filling the half of the space as shown in the figure. If the dielectric has $\kappa = 2.5$ what would be the potential difference between the plates in units of V_0 ?







$$C_0 = \frac{\varepsilon_0 2A}{d}$$

$$Q_0 = C_0 V_0$$

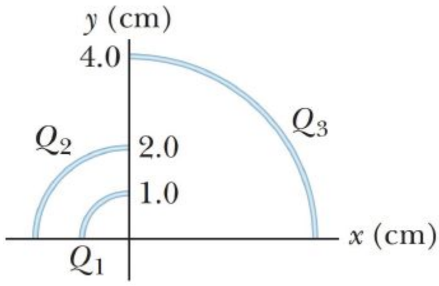
$$C_1 = K\frac{\varepsilon_0 A}{d} = K\frac{C_0}{2}, \qquad C_2 = \frac{\varepsilon_0 A}{d} = \frac{C_0}{2}$$

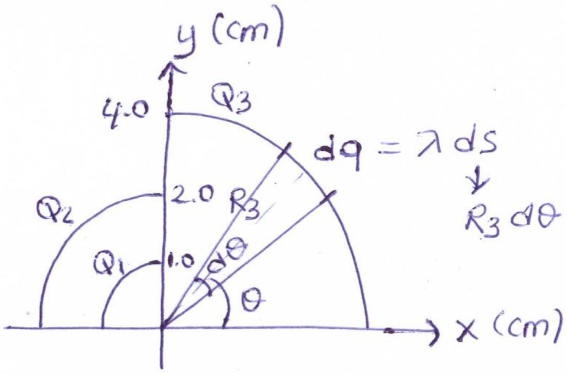
$$C_{eq} = C_1 + C_2 = \frac{C_0}{2}(K+1)$$

$$V = \frac{Q}{C} = \frac{Q_0}{\frac{C_0}{2}(K+1)} = \frac{2V_0}{K+1} = \frac{2}{2.5+1}V_0$$

$$V = 0.571\, V_0$$

9. In the Figure, three thin plastic rods form quarter-circles with a common center of curvature at the origin. The uniform charges on the rods are $Q_1 = +28$ nC, $Q_2 = +4.0\,Q_1$, and $Q_3 = -6.0\,Q_1$. What is the net electric potential at the origin due to the rods? (Take k=9x10^9 N m^2 /C^2).





$dq = \lambda\, ds$

$\downarrow$

$R_3\, d\theta$

$V(r=0) = ?$

$V(0,0) = ?$

$$dV_3 = k\frac{dq}{R_3} = k\frac{\lambda \cancel{R_3}\, d\theta}{\cancel{R_3}}$$

$$dV_3 = k\lambda \underbrace{\int_0^{\pi/2} d\theta}_{\theta\Big|_0^{\pi/2}} = k\lambda\frac{\pi}{2} \quad , \quad \lambda = \frac{Q_3}{\frac{\pi}{2}R_3}$$

$$V_3 = k\frac{Q_3}{R_3} \rightarrow V_2 = k\frac{Q_2}{R_2} \quad , \quad V_1 = k\frac{Q_1}{R_1}$$

$$V = V_1 + V_2 + V_3 = k\left(\frac{Q_3}{R_3} + \frac{Q_2}{R_2} + \frac{Q_1}{R_1}\right)$$

$$V = (8.99\times10^{9})\left(-\frac{6.}{0.04} + \frac{4}{0.02} + \frac{1}{0.01}\right).(28\times10^{-9})$$

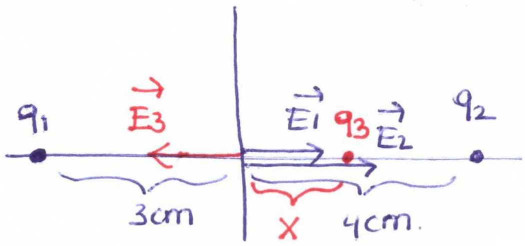
$$V = 3.78\times10^{4}\ V$$

10. A point charge of +5.0 μC is located at x = -3.0 cm, and a second point charge of -8.0 μC is located at x = +4.0 cm. Where should a third charge of +6.0 μC be placed so that the electric field at x = 0 is zero?

$q_1 = +5\ \mu C$

$q_2 = -8\ \mu C$

$q_3 = +6\ \mu C$



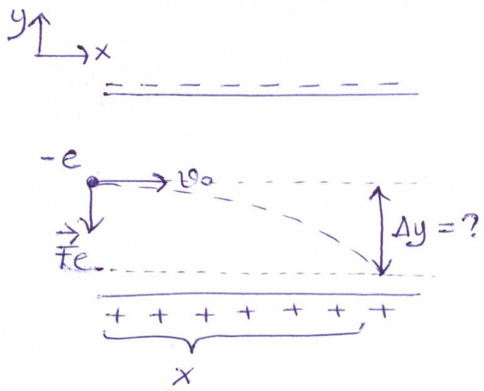
$$\vec{E}(0) = \vec{E_1} + \vec{E_2} + \vec{E_3} = 0$$

$$k\frac{q_1}{r_1^2}\hat{r}_1 + k\frac{q_2}{r_2^2}\hat{r}_2 + k\frac{q_3}{r_3^2}\hat{r}_3 = 0$$

$$9.10^9\left(\frac{5\times10^{-6}}{0.03^2} + \frac{8\times10^{-6}}{0.04^2} - \frac{6\times10^{-6}}{x^2}\right)\hat{i} = 0$$

$$x = 0.0238\ m = 2.38\ cm$$

11. An electron has an initial velocity of 2×10^6 m/s in the +x direction. It enters a uniform electric field E = 410 N/C which is in the +y direction. By how much, and in what direction, is the electron deflected after traveling 9 cm in the +x direction in the field? (Take elementary charge $1.6\text{x}10^{-19}$ C and take mass of electron $9.1\text{x}10^{-31}$ kg)



$q = -e$, $e = 1.6\times10^{-19}$ C

$\vec{v}_0 = 2\times10^{6}\,\hat{i}$ m/s

$\vec{E} = 410\,\hat{j}$ N/C

$x = 9$ cm

$x = v_x \cdot t$

$0.09 = (2\times10^6)\,t$

$t = 4.5\times10^{-8}$ s

$\Delta y = v_{0y} t - \frac{1}{2} a t^2$

$F_e = ma$

$qE = ma \rightarrow a = \frac{qE}{m}$

$\Delta y = 0 - \frac{1}{2}\left(\frac{qE}{m}\right)t^2$

$\Delta y = -\frac{1}{2}\left(\frac{(1.6\times10^{-19})(410)}{9.1\times10^{-31}}\right)(4.5\times10^{-8})^2$

$\Delta y = -0.0730\text{ m} = -7.30\text{ cm}$

12. Three identical capacitors are connected so that their maximum equivalent capacitance is 15 μF. There are three other ways to combine all three capacitors in a circuit. What is the total of the equivalent capacitances for each arrangement?

If their capacitance is to be maximum, they must be connected in parallel.

$$C_{eq} = 3C = 15$$
$$C = 5\,\mu F$$

1 – Connected the three capacitors in series:

$$\frac{1}{C_{eq}} = \frac{3}{5} \rightarrow C_{eq} = \boxed{1.67\,\mu F}$$

2 – Connected two in parallel, with the third in series with that combination:

$$C_{eq,\,\text{two in parallel}} = 2(5) = 10\,\mu F$$

and

$$\frac{1}{C_{eq}} = \frac{1}{10} + \frac{1}{5} \rightarrow C_{eq} = \boxed{3.33\,\mu F}$$

3 – Connect two in series, with the third in parallel with that combination

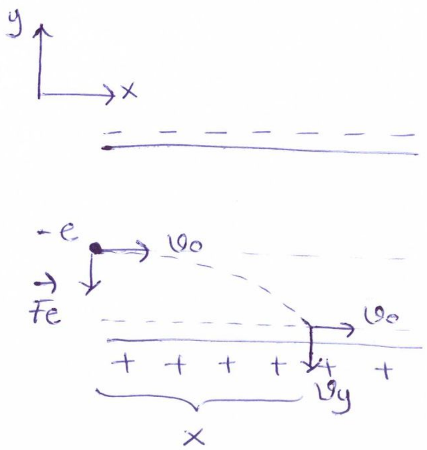
$$\frac{1}{C_{eq,\,\text{two in series}}} = \frac{2}{5} \rightarrow C_{eq,\,\text{two in series}} = 2.5\,\mu F$$

and

$$C_{eq} = 2.5 + 5 = \boxed{7.5\,\mu F}$$

$$\text{Answer} = 1.67 + 3.33 + 7.50 = 12.5\,\mu F$$

13. An electron has an initial velocity of 2×10^6 m/s in the +x direction. It enters a uniform electric field E = 425 N/C which is in the +y direction. What is the ratio of the y-component of the velocity of the electron to the x-component of the velocity after traveling 7 cm in the +x direction in the field?
(Take elementary charge $1.6\text{x}10^{-19}$ C, take mass of electron $9.1\text{x}10^{-31}$ kg).



$q = -e$, $e = 1.6\times10^{-19}$ C

$\vec{v}_0 = 2\times10^6\,\hat{i}$ m/s.

$\vec{E} = 425\,\hat{j}$ N/C

$x = 7\text{cm} \rightarrow \dfrac{v_y}{v_x} = ?$

$x = v_x . t$

$0.07 = (2\times10^6).t$

$t = 3.5\times10^{-8}$ s

$v_y = v_{0y} - at$, $a = \dfrac{qE}{m}$

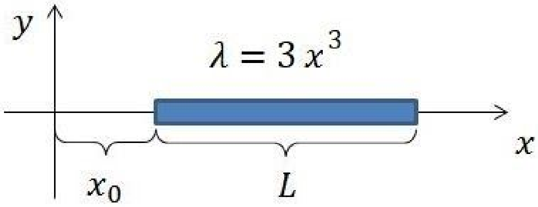
$v_y = 0 - \left(\dfrac{qE}{m}\right)t$

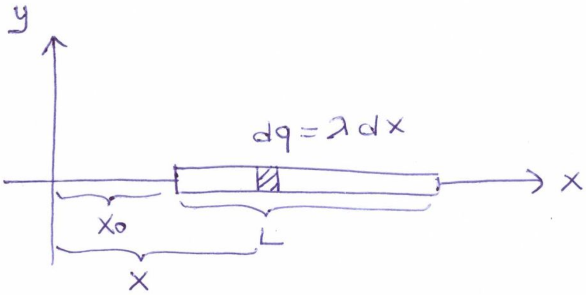
$v_y = -\dfrac{(1.6\times10^{-19})\,425}{9.1\times10^{-31}}\,(3.5\times10^{-8})$

$v_y = -2.62\times10^6$ m/s

$$\frac{v_y}{v_x} = \frac{-2.62\times\cancel{10^6}}{2\times\cancel{10^6}} = -1.31$$

14. A non-uniform positive line charge of length L=2 m is put along the x-axis as shown in the figure, where x_0=0.5 m. The linear charge density is given by $\lambda(x)=3x^3$ C/m^4. Find the magnitude of the total electric potential, V, created by the line charge at the origin by solving the relevant integral.
(Take Coulomb's constant $k=9\times10^9$ Nm^2/C^2).





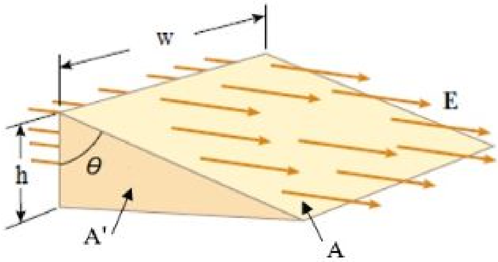
L = 2m
x_0 = 0.5m
$\lambda(x) = 3x^3$ C/m^4
V = ?

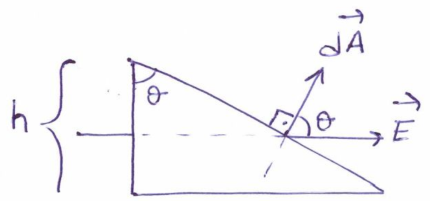
$$dV = k\frac{dq}{x} = k\frac{\lambda\,dx}{x} = k\frac{3x^{\cancel{3}\,2}dx}{\cancel{x}}$$

$$\int dV = 3k\underbrace{\int_{x_0}^{x_0+L} x^2dx}_{\frac{x^3}{3}} = \cancel{3}k\frac{x^3}{\cancel{3}}\Big|_{0.5}^{2.5} = (9\times10^9)(2.5^3-0.5^3)$$

$$V = 1.40\times10^{11}\ \text{V}$$

15. Consider a closed triangular box with height h=10 cm, width w=30 cm and angle Θ=50° is rest within a horizontal electric field of magnitude E=7x10^4 N/C as shown in figure. Calculate the electric flux through the inclined surface (A).





$$\Phi = \int \vec{E} \cdot d\vec{A}$$

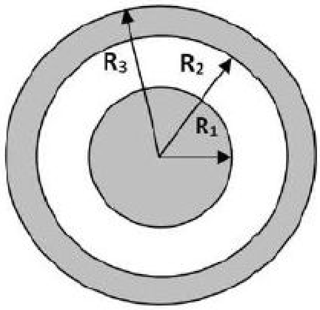
$$\Phi = \int E\, dA \cos\theta$$

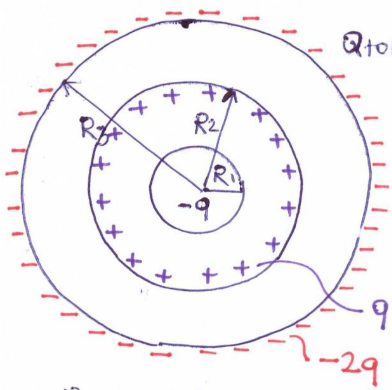
$$\Phi = E A \cos\theta \,, \quad A = w \frac{h}{\cos\theta}$$

$$\Phi = E \frac{wh}{\cancel{\cos\theta}} \cancel{\cos\theta}$$

$$\Phi = (7 \times 10^4)\, 0.3\ 0.1 = 2100\ Nm^2/C$$

16. Consider a solid nonconducting sphere with a uniform charge density of ρ and a charge of -q, inside a conducting spherical shell which has a charge of -q. See the figure below. Suppose that R_1=5 cm, R_2=15 cm and R_3=30 cm. Furthermore, suppose that q=3 nC. Find the charge density on the outer surface of the shell. (Take Coulomb's constant k=8.99x10 ^{9} N m^2 /C^2 and π = 3.14). Your result must be in nC/m^2. Include 1 digit after the decimal point and maximum of 5% of error is accepted in your answer.





$Q_{total} = -q$

$\sigma_{outer\ surf.} = ?$

$k = 8.99\times10^9\ Nm^2/C^2\ ,\ \pi = 3.14$

$R_1 = 5cm\ ,\ R_2 = 15cm,\ R_3 = 30cm$

$q = 3nC$

$R_2 < r < R_3 \rightarrow q_{enc} = 0$

$-q + Q_{\substack{inner\\surface}} = 0$

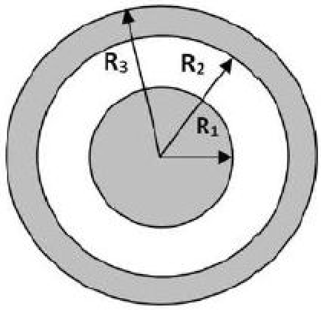
$Q_{\substack{inner\\surf.}} \boxed{= q}$

$Q_{total} = Q_{\substack{inner\\surf.}} + Q_{\substack{outer\\surf.}}$

$-q = q + Q_{\substack{outer\\surf.}} \rightarrow Q_{\substack{outer\\surf}} \boxed{= -2q}$

$$\sigma = \frac{Q}{A} = \frac{-2q}{4\pi R_3^2} = \frac{-2(3\times10^{-9})}{4\cdot(3.14)(0.3)^2} = -5.31\ nC/m^2$$

17. Consider a solid nonconducting sphere with a uniform charge density of ρ and a charge of -q, inside a conducting spherical shell which has a charge of -q. See the figure below. Suppose that R_1=10 cm, R_2=15 cm and R_3=25 cm. Furthermore, suppose that q=3 nC. Using Gauss' Law find the magnitude of the electric field in the region with radius r=4 cm.
(Take Coulomb's constant k=9x10^9 N m^2 /C^2)



$R_1 = 10cm$

$-q \ , \ q = 3nC$

$E(r = 4cm) = ?$

$$\oint \vec{E} \cdot d\vec{a} = \frac{q_{enc}}{\varepsilon_0}$$

$$q_{enc} \Rightarrow \rho(r) = \rho(R_1)$$

$$\frac{q_{enc}}{\frac{4}{\cancel{3}}\pi r^3} = \frac{q}{\frac{4}{\cancel{3}}\pi R_1^3}$$

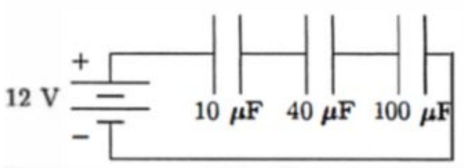
$$q_{enc} = q\frac{r^3}{R_1^3}$$

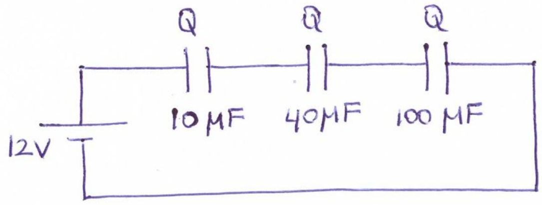
$$E\, 4\pi \cancel{r^2} = \frac{1}{\varepsilon_0} q \frac{r^{\cancel{3}}}{R_1^3}$$

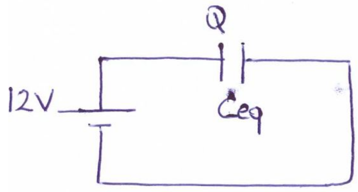
$$E = k\frac{q\,r}{R_1^3} = (9.10^9)\frac{(3\times10^{-9})(0.04)}{(0.1)^3}$$

$$= 1.08 \times 10^3 \ N/C$$

18. A 10.0 μF capacitor, a 40.0 μF capacitor, and a 100.0 μF capacitor are connected in series. A 12 V battery is connected across this combination. What is the potential difference across the 100.0 μF capacitor?







$$\frac{1}{C_{eq}} = \frac{1}{10} + \frac{1}{40} + \frac{1}{100}$$

$$C_{eq} = 7.41 \ \mu F$$

$$Q = V C = (12)(7.41) = 88.9 \ \mu C$$

$$V = \frac{Q}{C} = \frac{88.9 \ \mu C}{100 \ \mu F} = 0.889 \ V$$

19. The electric potential at points in an xy plane is given by $V = 4x^2 - 2y^3$. What is the magnitude of the electric field at point (1m, 2m)?

$V = 4x^2 - 2y^3, \quad E(1,2) = ?$

$E_x = -\frac{\partial V}{\partial x} = -8x$

$E_y = -\frac{\partial V}{\partial y} = +6y^2$

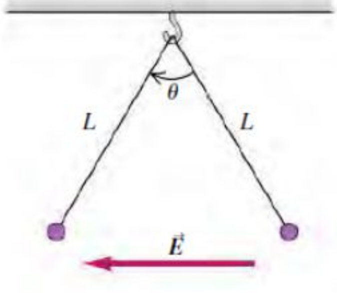
$\rightarrow \vec{E} = -8x\,\hat{i} + 6y^2\,\hat{j}$

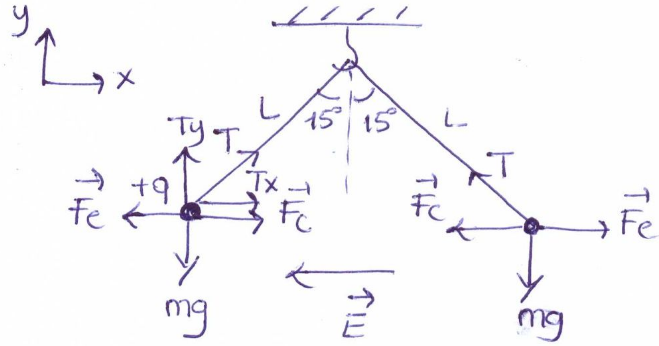
$\vec{E}(1,2) = -8\,\hat{i} + 24\,\hat{j}$

$E(1,2) = \sqrt{(-8)^2 + 24^2}$

$= 25.3\ V/m$

20. Two tiny spheres of mass 5 mg carry charges of equal magnitude, 70 nC, but opposite sign. They are tied to the same ceiling hook by light strings of length 550 mm. When a horizontal uniform electric field E that is directed to the left is turned on, the spheres hang at rest with the angle θ between the strings equal to 30° as seen in the figure below. What is the magnitude E of the field?





$m = 5\,mg$

$q = 70\,nC$

$L = 550\,mm$

$\theta = 30°$

$E = ?$

$\vec{F_e} = qE \quad , \quad F_c = k\dfrac{q^2}{(2L\sin 15°)^2}$

$\Sigma F_y = 0$

$T\cos 15° = mg$

$T = \dfrac{mg}{\cos 15°}$

$\Sigma F_x = 0$

$F_e = T\sin 15° + F_c$

$qE = \dfrac{mg}{\cos 15°}\sin 15° + k\dfrac{q^2}{(2L\sin 15°)^2}$

$E = \dfrac{mg}{q}\tan 15° + k\dfrac{q}{(2L\sin 15°)^2}$

$E = \dfrac{(5\times10^{-6})(9.81)}{(70\times10^{-9})}\tan 15° + (9\times10^{9})\dfrac{(70\times10^{-9})}{(2.0.55\sin 15°)^2}$

$E = 7.96\times10^{3}\,N/C$