İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy102 Physics II
Midterm Examination
November 11, 2021 17:00 – 18:30
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

INSTRUCTOR:

DURATION: 90 minutes

◇ **Answer all the questions.**
◇ **Write the solutions explicitly and clearly. Use the physical terminology.**
◇ **You are allowed to use Formulae Sheet.**
◇ **Calculator is allowed.**
◇ **You are not allowed to use any other electronic equipment in the exam.**

Question	Grade	Out of
1A		15
1B		15
2		20
3		20
4		20
5		20
TOTAL		110

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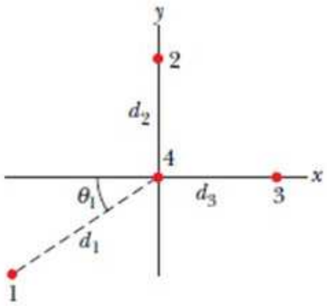
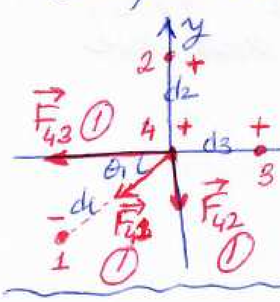
1. A) In figure, all four particles are fixed in the $xy-$plane, and $q_1 = -3.20 \times 10^{-19}\ C$, $q_2 = +3.20 \times 10^{-19}\ C$, $q_3 = +6.40 \times 10^{-19}\ C$, $q_4 = +3.20 \times 10^{-19}\ C$, $\theta_1 = 35.0°$, $d_1 = 3.00\ cm$ and $d_2 = d_3 = 2.00\ cm$.



What are the magnitude and direction of the net electrostatic force on particle 4 due to the other three particles?

Target particle: 4



$q_1 = -3.2 \times 10^{-19}\ C$

$q_2 = |q_1|$

$q_3 = 2|q_1|$

$q_4 = |q_1|$

$\theta_1 = 35°$

$d_1 = 3 \times 10^{-2}\ m$

$d_2 = d_3 = 2 \times 10^{-2}\ m$

$$\vec{F}_{4net} = \sum_{i=1}^{3} \vec{F}_{4i} = \vec{F}_{41} + \vec{F}_{42} + \vec{F}_{43}$$ 2

x & y-components

$F_{4,net,x} = F_{43,x} + F_{41,x}$

$F_{4,net,y} = F_{42,y} + F_{41,y}$

$$\Rightarrow F_{4net,x} = -F_{43} - |F_{41}| \cos 35 = -k\frac{|q_4||q_3|}{d_3^2} - k\frac{|q_4||q_1|}{d_1^2}\cos 35$$ 2

$$F_{4net,y} = -F_{42} - |F_{41}| \sin 35° = -k\frac{|q_4||q_2|}{d_2^2} - k\frac{|q_4||q_1|}{d_1^2}\sin 35°$$ 1

$$= -(8.99 \times 10^{9}\ Nm^2/C^2)(3.2 \times 10^{-19}\ C)^2\left(\frac{2}{(2 \times 10^{-2}\ m)^2} + \frac{\cos 35}{(3 \times 10^{-2}\ m)^2}\right) = \boxed{-5.44 \times 10^{-24}\ N}$$ 2 1

x-component

$$F_{4net,y} = -(8.99 \times 10^{9}\ Nm^2/C^2)(3.2 \times 10^{-19}\ C)^2\left(\frac{1}{(2 \times 10^{-2}\ m)^2} + \frac{\sin 35}{(3 \times 10^{-2}\ m)^2}\right) = \boxed{-2.89 \times 10^{-24}\ N}$$ 2 1

y-component

$$F_{4net} = \sqrt{F_{4net,x}^2 + F_{4net,y}^2} = \sqrt{(-5.44 \times 10^{-24}\ N)^2 + (-2.89 \times 10^{-24}\ N)^2} = \boxed{6.16 \times 10^{-24}\ N}$$ 1 1

magnitude

$$\tan\theta = \frac{F_{4net,y}}{F_{4net,x}} = \frac{-2.89 \times 10^{-24}}{-5.44 \times 10^{-24}} \rightsquigarrow \theta = \tan^{-1}\frac{-2.89}{-5.44} = \boxed{27.98 \approx 28°}$$ 1 1 1

$\boxed{\text{III. quadrant } \theta = 208°}$

angle

B) The density of conduction electrons in aluminum is $2.1 \times 10^{29}\ m^{-3}$. What is the drift velocity in an aluminum conductor that has a $2.0\ \mu m$ by $3.0\ \mu m$ rectangular cross section and when a 32.0 mA current flows through the conductor?

$n = 2.1 \times 10^{29}\ m^{-3}$

$i = 32 \times 10^{-3}\ A$

$A = (2 \times 10^{-6}\ m)(3 \times 10^{-6}\ m)$

$v_d = ?$

$\vec{J} = ne\vec{v}_d$

3 $J = nev_d$

3 $J = \frac{i}{A}$

$\frac{i}{A} = nev_d$

$$\Rightarrow v_d = \frac{i}{Ane} \overset{4}{=} \frac{32 \times 10^{-3}\ A}{(2 \times 10^{-6}\ m)(3 \times 10^{-6}\ m)(2.1 \times 10^{29}\ m^{-3})(1.602 \times 10^{-19}\ C)}$$

$$= \boxed{0.016\ m/s}$$

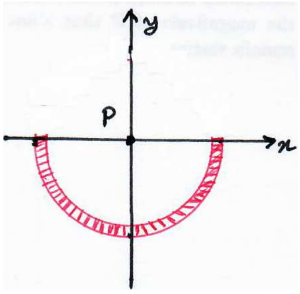
3 2

$$\frac{A}{m^2 m^{-3} C} \sim \frac{C/s}{m^2 m^{-3} C} \sim m/s$$

unit check

2. Semicircular wire shown in figure below has a non-uniform charge distribution $\lambda(\theta) = \lambda_0 Cos\theta$.

Find the electric field at point P in unit vector notation and in terms of total charge Q.
(Hint: $\int Cos^2 axdx = x/2 + Sin2ax/4a$)



$dE = \frac{k\,dq}{R^2} = \frac{k\lambda R d\theta}{R^2} = \frac{k\lambda d\theta}{R} \left\{ \lambda = \lambda_0 \cos\theta \right\} = \frac{k\lambda_0 \cos\theta}{R} d\theta$ 2 2

x-components are cancelling due to symmetry 2

$dE_y = |d\vec{E}|\cos\theta = \frac{k\lambda_0 \cos^2\theta \, d\theta}{R}$ 2 2 $\left\{ E_y = \int dE_y \right.$

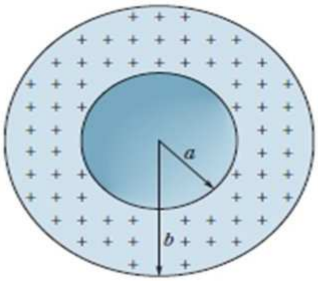
$E_y = E = \frac{k\lambda_0}{R}\int_{-\pi/2}^{\pi/2} \cos^2\theta \, d\theta = \frac{k\lambda_0}{R}\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right)\Bigg|_{-\pi/2}^{\pi/2}$ 2

$= \frac{k\lambda_0}{R}\frac{\pi}{2} \Rightarrow \vec{E} = \frac{k\lambda_0 \pi}{2R}(\hat{j})$ 2

in terms of Q $\left\{ Q = \int \lambda ds = \int_{-\pi/2}^{\pi/2} \lambda_0 \cos\theta R d\theta = \lambda_0 R \sin\theta \Big|_{-\pi/2}^{\pi/2} = 2\lambda_0 R \right.$ 2

$\Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{2R}\frac{\pi}{2R}\hat{j} = \boxed{\frac{Q}{16\pi\epsilon_0 R^2}\hat{j}}$ 2

3. Figure shows a spherical shell with uniform volume charge density $\rho = (1.56 \times 10^{-9}\ C/m^3$, inner radius $a = 10\ cm$, and outer radius $b = 2.00a$.

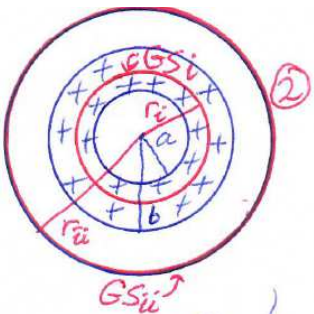


What is the magnitude of the electric field at radial distances

i $r = 1.5a$

ii $r = 3.00b$

Hints: Use Gauss' Law. Volume of the spherical shell: $\frac{4}{3}\pi(b^3 - a^3)$.



$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$ 2 , $\rho = \frac{q}{V} = \frac{q_{enc}}{V_{enc}}$ 2 , $\vec{E} \parallel \vec{A}$ 2

$\rho = 1.56 \times 10^{-19}\ C/m^3$
$a = 10 \times 10^{-2}\ m$
$b = 2a$
$r_i = 1.5a$
$r_{ii} = 3b$
$V_{ss} = \frac{4}{3}\pi(b^3 - a^3)$

i) $\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \rightsquigarrow E 4\pi r_i^2 = \frac{q_{enc}}{\varepsilon_0}$ $\}$ $q_{enc} = ?$

GS_i

$\rightsquigarrow q_{enc} = \rho V_{enc} = \rho \frac{4}{3}\pi(r_i^3 - a^3)$ 2

$\rightsquigarrow E 4\pi r_i^2 = \rho \frac{4}{3}\frac{\pi(r_i^3 - a^3)}{\varepsilon_0}$ $\}$ $r_i = 1.5a$

$\Rightarrow E = \frac{\rho\, 4/3\pi}{4\pi\varepsilon_0}\ \frac{((1.5a)^3 - a^3)}{1.5a^2} = \frac{\rho a}{3\varepsilon_0}\left(\frac{2.375}{2.25}\right)$ 2 1

$E(r=1.5a) = \frac{(1.56 \times 10^{-9}\ C/m^3)(10 \times 10^{-2}\ m)}{3 * 8.85 \times 10^{-12}\ C^2/(Nm^2)}\left(\frac{2.375}{2.25}\right) = \boxed{6.20\ N/C}$ 1

ii) $E 4\pi r_{ii}^2 = \frac{q_{enc}}{\varepsilon_0} \rightsquigarrow q_{enc} = \rho \frac{4}{3}\pi(b^3 - a^3)$ 2

GS_{ii} $\rightsquigarrow E 4\pi r_{ii}^2 = \rho \frac{4}{3}\frac{\pi(b^3 - a^3)}{\varepsilon_0}$ $\}$ $r_{ii} = 3b$

$\Rightarrow E = \frac{\rho\, 4/3\pi}{4\pi\varepsilon_0}\ \frac{b^3 - a^3}{(3b)^2} = \frac{\rho\, 7a}{108\,\varepsilon_0}$ 1 1

2 $E(r=3b) = \frac{(1.56 \times 10^{-9}\ C/m^3)\ 7\ (10 \times 10^{-10}\ m)}{108 * 8.85 \times 10^{-12}\ C^2/(Nm^2)} = \boxed{1.14\ N/C}$

4. The electric potential at points in an xy plane is given by $V = 4x^2 - 2y^3$. **In unit vector notations**, what is the electric field at point (1m, 2m)?

$V(x,y) = 4x^2 - 2y^3$ & $E_s = -\frac{\partial V}{\partial s}$

$\vec{E} = -\frac{\partial V}{\partial x}\hat{\imath} - \frac{\partial V}{\partial y}\hat{\jmath} = -8x\,\hat{\imath} + 6y^2\,\hat{\jmath}$

5 5

$\vec{E}(x = 1m, y = 2m) = \boxed{-8\hat{\imath} + 24\hat{\jmath}}$

5 5

5. In figure below, the parallel plate capacitor of plate area $2 \times 10^{-2}\ m^2$ is filled with two dielectric slabs , each with thickness 2.00 mm. One slab has dielectric constant 3.00, and the other, 4.00 . **How much charge** does the 7.00V battery store on the capacitor?

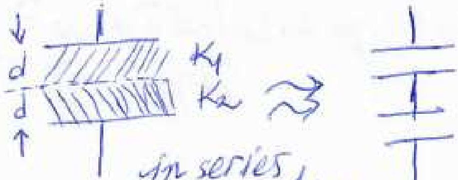
$A = 2 \times 10^{-2}\ m^2$

$d = 2 \times 10^{-3}\ m$

$K_1 = 3$ & $K_2 = 4$

$V = 7\ V$

$q = ?$



in series connection 4

$C_1 = K_1 C_0 \overset{3}{=} K_1 \varepsilon_0 \frac{A}{d} = 3\varepsilon_0 A/d$ 1

$C_2 = K_2 C_0 \overset{3}{=} K_2 \varepsilon_0 \frac{A}{d} = 4\varepsilon_0 A/d$ 1

1

$\rightsquigarrow C_{eqv} = \frac{C_1 C_2}{C_1 + C_2} = \frac{12}{7}\varepsilon_0 A/d = \frac{12}{7}\left(8.85 \times 10^{-12}\ C^2/(Nm^2)\right) \frac{2 \times 10^{-2}\ m^2}{2 \times 10^{-3}\ m}$

$= \underline{\underline{1.52 \times 10^{-10}\ F}}$ 2 1

$\frac{C^2}{Nm^2}\frac{m^2}{m} \sim F \sim \frac{Q}{V} = \frac{Q}{U/Q} = \frac{Q^2}{Fx}$

unit check

$C_{eqv} = \frac{Q}{V} \rightsquigarrow q = C_{eqv} V$ 1

$= (1.52 \times 10^{-10}\ F)\ 7V = \boxed{1.06 \times 10^{-9}\ C}$ 2 1