İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy102 Physics II
Final Examination
January 09, 2023 17:00 – 18:30
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

INSTRUCTOR:

DURATION: 90 minutes

◇ **Answer all the questions.**
◇ **Write the solutions explicitly and clearly. Use the physical terminology.**
◇ **You are allowed to use Formulae Sheet.**
◇ **Calculator is allowed.**
◇ **You are not allowed to use any other electronic equipment in the exam.**
◇ **I declare hereby that I fulfilled the requirements for the attendance according to the University regulations and I accept that my examination will not be valid otherwise.**

Question	Grade	Out of
1A		15
1B		15
2		20
3		20
4		20
5		20
TOTAL		110

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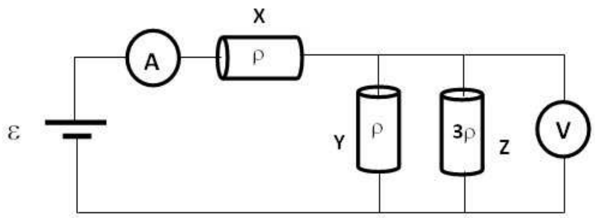
1. A) The circuit containing three cylindrical resistors, namely X, Y and Z, which obey Ohm's Law is shown in the figure below. The resistors which have length of L and cross-sectional area of A are connected to an ideal battery of emf ε. As shown an ammeter is connected in series while voltmeter is connected to ends of resistor Z. The resistors X and Y have a resistivity ρ and the resistor Z has a resistivity 3ρ.



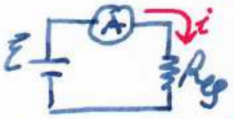
i Find the current i through the ammeter.

ii Find the reading of voltmeter.

Express your result in terms of given quantities and constants (ε, A, ρ, L). **(Hint:** Resistance is related to resistivity; $R = \rho\frac{L}{A}$)

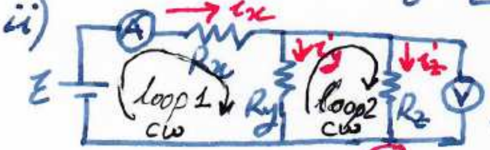
i) $\frac{1}{R_{yz}} = \frac{1}{R_y} + \frac{1}{R_z}$ (2) $\to R_{yz} = \frac{R_y R_z}{R_y + R_z} \Rightarrow R_{eq} = R_x + R_{yz} = R_x + \frac{R_y R_z}{R_y + R_z}$ (2)

where $R_x = R_y = \rho\frac{L}{A}$ & $R_z = 3\rho\frac{L}{A} \Rightarrow R_{eq} = \rho\frac{L}{A} + \frac{\rho\frac{L}{A}\cdot 3\rho\frac{L}{A}}{\rho\frac{L}{A} + 3\rho\frac{L}{A}} = \frac{7}{4}\rho\frac{L}{A}$ (2)



$\to i = \frac{\varepsilon}{R_{eq}} = \boxed{\frac{4}{7}\frac{\varepsilon A}{\rho L}} = i_x$ (1) $\to i_x = i_y + i_z$

ii)



Loop1: (2) $\varepsilon - i_x R_x - i_y R_y = 0 \to i_y = \frac{\varepsilon - i_x R_x}{R_y}$

loop2: (2) $i_y R_y - i_z R_z = 0 \to i_z = i_y \frac{R_y}{R_z}$

(2)

$\Rightarrow i_y = \frac{\varepsilon - i_x R_x}{R_y}$ & $i_z = \left(\frac{\varepsilon - i_x R_x}{R_y}\right)\frac{R_y}{R_z} = \frac{\varepsilon - i_x R_x}{R_z}$ (1)

$\Rightarrow V = i_z R_z = \left(\frac{\varepsilon - i_x R_x}{R_z}\right) R_z = \varepsilon - i_x R_x = \varepsilon - \left(\frac{4}{7}\frac{\varepsilon A}{\rho L}\right)\rho\frac{L}{A} = \varepsilon - \frac{4\varepsilon}{7}$

$\boxed{V = \frac{3\varepsilon}{7}}$ (1)

B) **What uniform magnetic field**, applied **perpendicular** to a beam of electrons moving at $1.30 \times 10^6\ m/s$, is required to make the electrons travel **in a circular arc** of radius of 0.35 m? **(Hint:** Centripetal Force; $F_c = m\frac{v^2}{R}$)

$v = 1.3 \times 10^6\ m/s$

$R = 0.35 m$

$e = 1.602 \times 10^{-19} C$ ($\equiv |q|$)

$m_e = 9.109 \times 10^{-31} kg$

$B = ?$

$F_c = m\frac{v^2}{R}$ & $F_B = |q| v B \sin\theta$

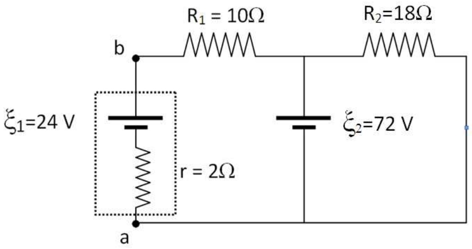
$|q| v B \sin 90° = m_e v^2/R$ (5)

$\Rightarrow B = \frac{m_e v}{e R}$ (5)

$= \frac{(9.109 \times 10^{-31} kg)(1.3 \times 10^6 m/s)}{(1.602 \times 10^{-19} C)(0.35 m)}$

$= \boxed{2.11 \times 10^{-5} T}$ (5)

2. Consider circuit as shown in figure which consists of two batteries. One of the following batteries has an internal resistance r, while the other battery is an ideal battery.

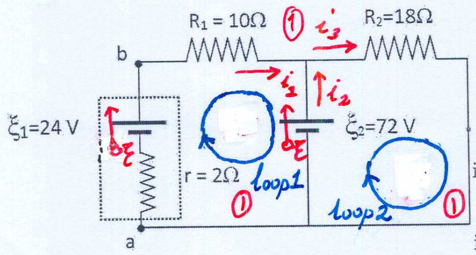


Calculate;

i Currents through each battery,

ii Total power dissipated by resistors.

iii Potential difference between points a and b, V_{ab},



i Currents through each battery,

ii Potential difference between points a and b, V_{ab},

iii Total power supplied by batteries,

iv Total power dissipated by resistors.

i) (1) loop 1: $-i_1 r + \xi_1 - i_1 R_1 - \xi_2 = 0 \Rightarrow -2i_1 + 24 - 10i_1 - 72 = 0$ (2)

(2) loop 2: $\xi_2 - i_3 R_2 = 0 \Rightarrow 72 - 18 i_3 = 0$ (2) $\quad -12 i_1 = 48$

(3) $i_1 + i_2 = i_3$ (1) $\Rightarrow -4A + i_2 = 4A$ (1) $\rightarrow$ $i_2 = 8A$ $\quad$ $i_3 = 4A$ (1) $\quad$ $i_1 = -4A$ (1) opposite direction

Three unknowns ($i_{1,2,3}$), three equations

$i_1 = -4A$: Through battery 1

$i_2 = 8A$: Through battery 2

$i_3 = 4A$: Through Resistor 3

ii) $V_{ab} = V_b - V_a$ (2) $\quad$ $i_1 = 4A$, loop 1

$V_a + i_1 r + \xi_1 = V_b$

$V_b - V_a = 4A\,2\Omega + 24V$ $= 32V$ (1)

iii) $P = i\xi$

Battery 1: $P_1 = i_1 \xi_1 = (4A)(24V)$ $= -96W$ (1.5)

Battery 2: $P_2 = i_2 \xi_2 = (8A)(72V)$ $= 576W$ (1.5)

$P_1 + P_2 = 480W$

iv) $P = i^2 R$

Resistor 1: $P'_1 = i_1^2 R_1 = (4A)^2(10\Omega) = 160\,W$ → R_1 (1)

Resistor 2: $P'_2 = i_3^2 R_2 = (4A)^2(18\Omega) = 288W$ → R_2 (1)

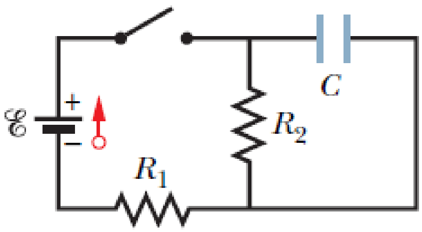
Internal Resistor r: $P'_r = i_1^2 r = (4A)^2(2\Omega) = 32\,W$ → r (1)

$P_1 + P_2 = P'_1 + P'_2 + P'_r$

$480W = 480W$

3

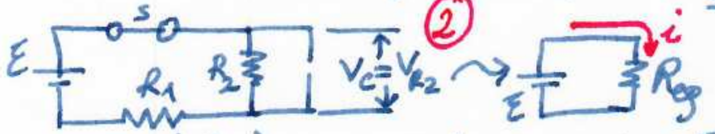
3. In Figure given below, $R_1 = 8.0 \times 10^3\ \Omega$, $R_2 = 10.0 \times 10^3\ \Omega$, $C = 6 \times 10^{-7}\ F$, and the ideal battery has emf $\epsilon = 12.0\ V$. First, the switch is closed a long time so that the steady state is reached. Then the switch is opened at time $t = 0$.



What is the current in resistor 2 at $t = 2.00 \times 10^{-3}\ s$?

$R_1 = 8\ k\Omega$, $R_2 = 10\ k\Omega$, $C = 0.6\ \mu F$, $\mathcal{E} = 12\ V$

initially, C acts as a connecting wire ①
after a long time, C acts as a broken wire ②
① Charging stage ② fully charged
$\Rightarrow V_C = V_{R_2} = iR_2$ ②



$i = \frac{\mathcal{E}}{R_1 + R_2}$ ②

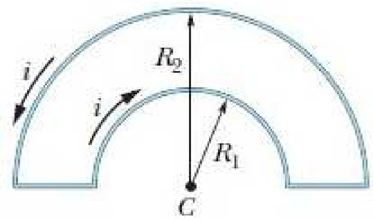
$\rightarrow V_{R_2} = \left(\frac{\mathcal{E}}{R_1 + R_2}\right) R_2 = \frac{12V}{(8k\Omega + 10k\Omega)}\ 10k\Omega = \boxed{\frac{20}{3}\ V}\ (= V_C)$ when fully charged ③ ②

③ switch is opened $\rightarrow t = 0$ & $V_C = V_0 = \frac{20}{3} V$ ②
discharging through R_2, $V = V_0 \exp(-t/R_2C)$ ③

$\rightarrow V = \left(\frac{20}{3} V\right) \exp\left(-\frac{2\ ms}{(10k\Omega)(0.6\mu F)}\right) = \left(\frac{20}{3} V\right) \exp\left(-\frac{2 \times 10^{-3} s}{10 \times 10^3 \Omega\ 0.6 \times 10^{-6} F}\right)$

$V(t = 2ms) = \boxed{4.78V}$ ② $\rightarrow i_{R_2} = \frac{V}{R_2}$ ① $= \frac{4.78V}{10k\Omega} = \boxed{4.77 \times 10^{-4} A}$ ② ①

4. In Figure, two semicircular arcs have radii $R_2 = 2.6\ cm$ and $R_1 = 1.05\ cm$, carry current $i = 0.0937\ A$, and share the same center of curvature C.



What are the

i magnitude

ii direction (into or out of the page, why?)

of the net magnetic field at C?
Hint: Use Biot-Savart Law.

Biot-Savart law: $d\vec{B} = \frac{\mu_0}{4\pi}\, i\, \frac{d\vec{s}\times\hat{r}}{r^2}$ → R, $d\phi$, $d\vec{s}$, $\hat{r}$, $\hat{r} \perp d\vec{s}$

$dB = \frac{\mu_0}{4\pi}\, \frac{i\, ds \sin 90^\circ}{R^2} = \frac{\mu_0}{4\pi}\, \frac{i\, ds}{R^2}$ {where $ds = R\,d\phi$} → $B = \int dB$

→ $B = \frac{\mu_0}{4\pi}\, i \int_0^{\phi} \frac{R\,d\theta}{R^2} = \frac{\mu_0 i}{4\pi R}\,\phi$ (4) {where ϕ is arc angle}

$R_1 = 1.05\times10^{-2}\ m$
$R_2 = 2.6\times10^{-2}\ m$
$i = 0.0937\ A$
$\phi = 180^\circ \equiv \pi$

i) $B = B_1 + B_2 = \frac{\mu_0 i}{4\pi}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\pi$ (4)

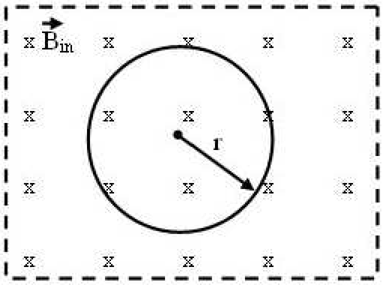
→ $B = \frac{(4\pi\times10^{-7}\ Tm/A)\,0.0937\,A}{4\pi}\left(\frac{1}{1.05\times10^{-3}\ m} - \frac{1}{2.6\times10^{-3}\ m}\right)\pi$ (4)

$= \boxed{1.67\times10^{-6}\ T}$ (2) (2)

ii) into the page (4)

$\vec{B}_1$ (into the page) & $|\vec{B}_1| > |\vec{B}_2|$
$\vec{B}_2$ (out of the page)

5. In figure below, the magnetic flux through the circular loop of radius $r = 2.0\ m$ increases according to the relation $\Phi_B = 3t^2 + 3t$, where Φ_B is in Webers and t is in seconds.



i Find the magnitude of the induced emf, ξ in the circular loop at $t = 2.0\ s$.

ii What is the magnitude and direction of the induced current in the circular loop at $t = 2.0\ s$ if the loop has a total resistance of $R = 30\ \Omega$?

i) $\phi_B(t) = 3t^2 + 3t$: increasing flux ⇒ induced $\mathcal{E}, i$ should oppose

$\mathcal{E} = -N \frac{d\phi_B}{dt}$ (5) $\rightarrow |\mathcal{E}| = \frac{d}{dt}(3t^2+3t)\Big|_{t=2s} = 6t+3\Big|_{t=2s} = \boxed{15\ V}$ (3) (5)

(induced; $N = 1$)

ii) $i = \frac{\mathcal{E}}{R}$ (2) $= \frac{15\ V}{30\ \Omega} = \boxed{0.5 A}$ (2)

$\vec{B}_{applied}$ ⊗ ~ in to the page

$\vec{B}_{induced}$ ⊙ ← since it should oppose

by Right Hand Rule ~ direction of induced current is ccw (3)