İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy102 Physics II
Midterm Examination
November 11, 2021 17:00 – 18:30
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

INSTRUCTOR:

DURATION: 90 minutes

◇ **Answer all the questions.**
◇ **Write the solutions explicitly and clearly. Use the physical terminology.**
◇ **You are allowed to use Formulae Sheet.**
◇ **Calculator is allowed.**
◇ **You are not allowed to use any other electronic equipment in the exam.**

Question	Grade	Out of
1A		15
1B		15
2		20
3		20
4		20
5		20
TOTAL		110

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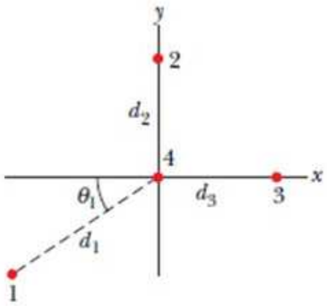
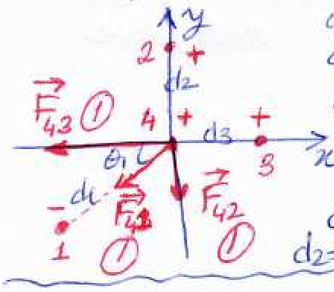
1. A) In figure, all four particles are fixed in the $xy-$plane, and $q_1 = -3.20 \times 10^{-19}\ C$, $q_2 = +3.20 \times 10^{-19}\ C$, $q_3 = +6.40 \times 10^{-19}\ C$, $q_4 = +3.20 \times 10^{-19}\ C$, $\theta_1 = 35.0°$, $d_1 = 3.00\ cm$ and $d_2 = d_3 = 2.00\ cm$.



What are the magnitude and direction of the net electrostatic force on particle 4 due to the other three particles?

Target particle: 4



$q_1 = -3.2 \times 10^{-19}\ C$

$q_2 = |q_1|$

$q_3 = 2|q_1|$

$q_4 = |q_1|$

$\theta_1 = 35°$

$d_1 = 3 \times 10^{-2}\ m$

$d_2 = d_3 = 2 \times 10^{-2}\ m$

$$\vec{F}_{4net} = \sum_{i=1}^{3} \vec{F}_{4i} = \vec{F}_{41} + \vec{F}_{42} + \vec{F}_{43} \quad (1)$$

x & y-components

$F_{4,net,x} = F_{43,x} + F_{41,x}$

$F_{4,net,y} = F_{42,y} + F_{41,y}$

$$\Rightarrow F_{4net,x} = -F_{43} - |F_{41}| \cos 35 = -k\frac{|q_4||q_3|}{d_3^2} - k\frac{|q_4||q_1|}{d_1^2}\cos 35 \quad (1)$$

$$F_{4net,y} = -F_{42} - |F_{41}| \sin 35° = -k\frac{|q_4||q_2|}{d_2^2} - k\frac{|q_4||q_1|}{d_1^2}\sin 35° \quad (1)$$

$$= -(8.99 \times 10^9\ Nm^2/C^2)(3.2 \times 10^{-19}\ C)^2 \left(\frac{2}{(2 \times 10^{-2}\ m)^2} + \frac{\cos 35}{(3 \times 10^{-2}\ m)^2}\right) = \boxed{-5.44 \times 10^{-24}\ N} \quad (1)\ (1)$$

x-component

$$F_{4net,y} = -(8.99 \times 10^9\ Nm^2/C^2)(3.2 \times 10^{-19}\ C)^2 \left(\frac{1}{(2 \times 10^{-2}\ m)^2} + \frac{\sin 35}{(3 \times 10^{-2}\ m)^2}\right) = \boxed{-2.89 \times 10^{-24}\ N} \quad (1)\ (1)$$

y-component

$$F_{4net} = \sqrt{F_{4net,x}^2 + F_{4net,y}^2} = \sqrt{(-5.44 \times 10^{-24}\ N)^2 + (-2.89 \times 10^{-24}\ N)^2} = \boxed{6.16 \times 10^{-24}\ N} \quad (1)\ (1)$$

magnitude

$$\tan\theta = \frac{F_{net,y}}{F_{net,x}} = \frac{-2.89 \times 10^{-24}}{-5.44 \times 10^{-24}} \quad (1)\ (1)$$

$$\Rightarrow \theta = \tan^{-1}\frac{-2.89}{-5.44} = \boxed{27.98 \approx 28°} \quad (1)$$

angle

$\boxed{\text{III. quadrant } \theta = 208°}$

B) The density of conduction electrons in aluminum is $2.1 \times 10^{29}\ m^{-3}$. What is the drift velocity in an aluminum conductor that has a 2.0 μm by 3.0 μm rectangular cross section and when a 32.0 mA current flows through the conductor?

$n = 2.1 \times 10^{29}\ m^{-3}$

$i = 32 \times 10^{-3}\ A$

$A = (2 \times 10^{-6}\ m)(3 \times 10^{-6}\ m)$

$v_d = ?$

$\vec{J} = ne\vec{v}_d$

3 $J = nev_d$

3 $J = \frac{i}{A}$

$\frac{i}{A} = nev_d$

$$\Rightarrow v_d = \frac{i}{Ane} = \text{4}\ \frac{32 \times 10^{-3}\ A}{(2 \times 10^{-6}\ m)(3 \times 10^{-6}\ m)(2.1 \times 10^{29}\ m^{-3})(1.602 \times 10^{-19}\ C)}$$

$$= \boxed{0.016\ m/s}$$

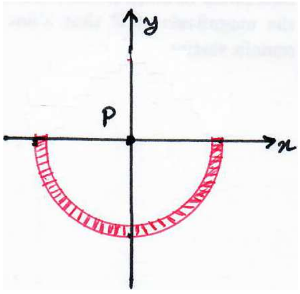
3 2

$$\frac{A}{m^2 m^{-3} C} \sim \frac{C/s}{m^2 m^{-3} C} \sim m/s$$

unit check

2. Semicircular wire shown in figure below has a non-uniform charge distribution $\lambda(\theta) = \lambda_0 Cos\theta$.

Find the electric field at point P in unit vector notation and in terms of total charge Q.
(Hint: $\int Cos^2 axdx = x/2 + Sin2ax/4a$)



$dE = \frac{k\,dq}{R^2}$ 2 $= \frac{k\lambda R\,d\theta}{R^2} = \frac{k\lambda\,d\theta}{R} \left\{\lambda = \lambda_0 \cos\theta\right\} = \frac{k\lambda_0 \cos\theta}{R}\,d\theta$ 2

x-components are cancelling due to symmetry 2

$dE_y = |d\vec{E}|\cos\theta = \frac{k\lambda_0 \cos^2\theta\,d\theta}{R}$ 2 2 $\left\{ E_y = \int dE_y \right.$

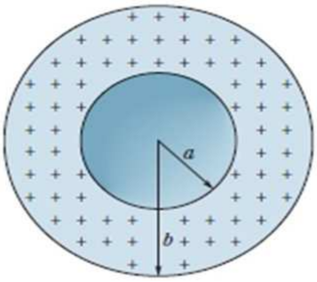
$E_y = E = \frac{k\lambda_0}{R}\int_{-\pi/2}^{\pi/2} \cos^2\theta\,d\theta = \frac{k\lambda_0}{R}\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right)\Big|_{-\pi/2}^{\pi/2}$ 2

$= \frac{k\lambda_0}{R}\frac{\pi}{2} \Rightarrow \vec{E} = \frac{k\lambda_0 \pi}{2R}(\hat{j})$ 2

in terms of Q $\left\{ Q = \int \lambda\,ds = \int_{-\pi/2}^{\pi/2} \lambda_0 \cos\theta\,R\,d\theta = \lambda_0 R \sin\theta \Big|_{-\pi/2}^{\pi/2} = 2\lambda_0 R \right.$ 2

$\Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{2R}\frac{\pi}{2R}\hat{j} = \boxed{\frac{Q}{16\pi\epsilon_0 R^2}\hat{j}}$ 2

3. Figure shows a spherical shell with uniform volume charge density $\rho = (1.56 \times 10^{-9}\ C/m^3$, inner radius $a = 10\ cm$, and outer radius $b = 2.00a$.

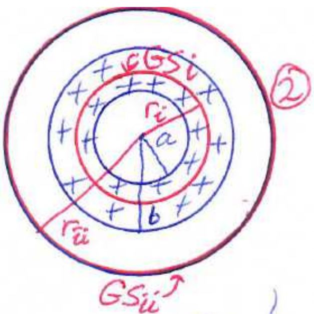


What is the magnitude of the electric field at radial distances

i $r = 1.5a$

ii $r = 3.00b$

Hints: Use Gauss' Law. Volume of the spherical shell: $\frac{4}{3}\pi(b^3 - a^3)$.



$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$ (2), $\rho = \frac{q}{V} = \frac{q_{enc}}{V_{enc}}$ (2), $\vec{E} \parallel \vec{A}$ (2)

$\rho = 1.56 \times 10^{-9}\ C/m^3$
$a = 10 \times 10^{-2}\ m$
$b = 2a$
$r_i = 1.5a$
$r_{ii} = 3b$
$V_{ss} = \frac{4}{3}\pi(b^3 - a^3)$

i) $\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \rightsquigarrow E 4\pi r_i^2 = \frac{q_{enc}}{\varepsilon_0}$ $\Big\}\ q_{enc} = ?$

GS_i

$\rightsquigarrow q_{enc} = \rho V_{enc} = \rho \frac{4}{3}\pi (r_i^3 - a^3)$ (2)

$\rightsquigarrow E 4\pi r_i^2 = \rho \frac{4}{3} \frac{\pi (r_i^3 - a^3)}{\varepsilon_0}$ $\Big\}\ r_i = 1.5a$

$\Rightarrow E = \frac{\rho\, 4/3\,\pi}{4\pi\varepsilon_0} \frac{((1.5a)^3 - a^3)}{1.5a^2} = \frac{\rho a}{3\varepsilon_0}\left(\frac{2.375}{2.25}\right)$ (2) (1)

$E(r = 1.5a) = \frac{(1.56 \times 10^{-9}\ C/m^3)(10 \times 10^{-2}\ m)}{3 \times 8.85 \times 10^{-12}\ C^2/(Nm^2)}\left(\frac{2.375}{2.25}\right) = \boxed{6.20\ N/C}$ (1)

ii) $E 4\pi r_{ii}^2 = \frac{q_{enc}}{\varepsilon_0} \rightsquigarrow q_{enc} = \rho \frac{4}{3}\pi(b^3 - a^3)$ (2)

GS_{ii} $\rightsquigarrow E 4\pi r_{ii}^2 = \rho \frac{4}{3} \frac{\pi (b^3 - a^3)}{\varepsilon_0}$ $\Big\}\ r_{ii} = 3b$

$\Rightarrow E = \frac{\rho\, 4/3\,\pi}{4\pi\varepsilon_0} \frac{b^3 - a^3}{(3b)^2} = \frac{\rho\, 7a}{108\,\varepsilon_0}$ (1) (1)

$E(r = 3b) = \frac{(1.56 \times 10^{-9}\ C/m^3)\, 7 (10 \times 10^{-10}\ m)}{108 \times 8.85 \times 10^{-12}\ C^2/(Nm^2)} = \boxed{1.14\ N/C}$ (2)

4. The electric potential at points in an xy plane is given by $V = 4x^2 - 2y^3$. **In unit vector notations**, what is the electric field at point (1m, 2m)?

$$V(x,y) = 4x^2 - 2y^3 \quad \& \quad E_s = -\frac{\partial V}{\partial s}$$

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} = -8x\,\hat{i} + 6y^2\hat{j}$$

5 5

$$\vec{E}(x = 1m, y = 2m) = \boxed{-8\hat{i} + 24\hat{j}}$$

5 5

5. In figure below, the parallel plate capacitor of plate area $2 \times 10^{-2}\ m^2$ is filled with two dielectric slabs , each with thickness 2.00 mm. One slab has dielectric constant 3.00, and the other, 4.00 . **How much charge** does the $7.00V$ battery store on the capacitor?

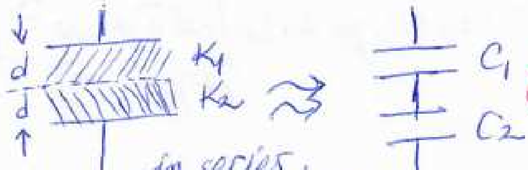
$A = 2\times10^{-2}\ m^2$

$d = 2\times10^{-3}\ m$

$K_1 = 3$ & $K_2 = 4$

$V = 7\ V$

$q = ?$



in series connection (4)

$C_1 = K_1 C_0 \overset{(3)}{=} K_1 \varepsilon_0 \frac{A}{d} = 3\varepsilon_0 A/d$ (1)

$C_2 = K_2 C_0 \overset{(3)}{=} K_2 \varepsilon_0 \frac{A}{d} = 4\varepsilon_0 A/d$ (1)

(1)

$\rightarrow C_{eqv} = \frac{C_1 C_2}{C_1 + C_2} = \frac{12}{7}\varepsilon_0 A/d = \frac{12}{7}\left(8.85\times10^{-12}\ C^2/(Nm^2)\right)\frac{2\times10^{-2}\ m^2}{2\times10^{-3}\ m}$

$= 1.52\times10^{-10}\ F$ (2) (1)

$\frac{C^2}{Nm^2}\frac{m^2}{m} \sim F \sim \frac{Q}{V} = \frac{Q}{U/Q} = \frac{Q^2}{Fx}$

unit check

$C_{eqv} = \frac{Q}{V} \rightsquigarrow q = C_{eqv} V$ (1)

$= (1.52\times10^{-10}\ F)\ 7V = \boxed{1.06\times10^{-9}\ C}$ (2) (1)

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Phy102 Physics II
Midterm Examination
November 03, 2019 15:30 – 17:30
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

INSTRUCTOR:

DURATION: 120 minutes

◇ **Answer all the questions.**
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◇ **You are allowed to use Formulae Sheet.**
◇ **Calculator is allowed.**
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1A		15
1B		15
2		20
3		20
4		20
5		20
TOTAL		110

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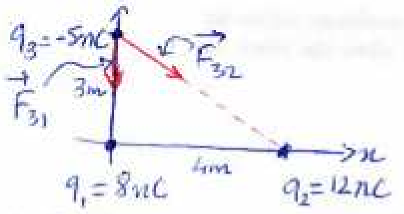
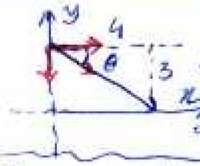
1. A) A point charge $q_1 = 8\ nC$ is at the origin and a second point charge $q_2 = 12\ nC$ is on the x-axis at x=4 m. Find the net electric force they exert on $q_3 = -5\ nC$ located on the y-axis at y=3.0 m in vector notation, magnitude and angle.



$$\vec{F}_{3,net} = \vec{F}_{31} + \vec{F}_{32}$$

$$|\vec{F}_{31}| = k\frac{|q_3||q_1|}{r_{31}^2} = \left(9\times10^{9}\frac{Nm^2}{C^2}\right)\frac{|5\times10^{-9}C||8\times10^{-9}C|}{(3m)^2}$$

$$= \underline{4\times10^{-8}N} \rightarrow \underline{\vec{F}_{31} = 4\times10^{-8}N(-\hat{j})}$$



$\cos\theta = \frac{4}{5} = 0.8$

$\sin\theta = \frac{3}{5} = 0.6$

$$|\vec{F}_{32}| = k\frac{|q_3||q_2|}{r_{32}^2} = \left(9\times10^{9}\frac{Nm^2}{C^2}\right)\frac{|-5\times10^{-9}C||12\times10^{-9}C|}{(4m)^2}$$

$$= \underline{2.16\times10^{-8}N} \rightsquigarrow \vec{F}_{32} = ?$$

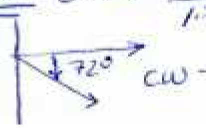
$$F_{32,x} = |\vec{F}_{32}|\cos\theta = 2.16\times10^{-8}N\times0.8 = 1.73\times10^{-8}N$$

$$F_{32,y} = |\vec{F}_{32}|\sin\theta = 2.16\times10^{-8}N\times0.6 = 1.3\times10^{-8}N$$

$$\underline{\vec{F}_{32} = 1.7\times10^{-8}N\hat{i} + 1.3\times10^{-8}N(-\hat{j})}$$

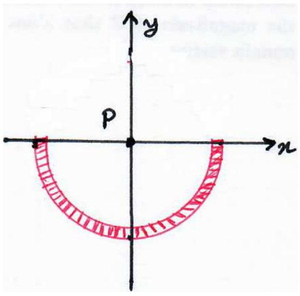
$$\Rightarrow \vec{F}_{3,net} = (4.0\times10^{-8}(-\hat{j}) + 1.73\times10^{-8}N\hat{i} + 1.3\times10^{-8}(-\hat{j}))N = \underline{1.73\times10^{-8}N\hat{i} - 5.3\times10^{-8}\hat{j}}$$

$$|\vec{F}_{3,net}| = \sqrt{(1.73\times10^{-8}N)^2 + (-5.3\times10^{-8}N)^2} = \underline{5.6\times10^{-8}N} \quad \theta = \tan^{-1}\frac{-5.3}{1.73} \simeq -72^\circ$$



B) Semicircular wire shown in figure below has a non-uniform charge distribution $\lambda(\theta) = \lambda_0 Cos\theta$.

Find the electric field at point P in unit vector notation and in terms of total charge Q.
(Hint: $\int Cos^2 axdx = x/2 + Sin2ax/4a$)



$dE = k\frac{dq}{R^2} = k\frac{\lambda R d\theta}{R^2} = \frac{k\lambda d\theta}{R} \left\{ \lambda = \lambda_0 \cos\theta \right\} = \frac{k\lambda_0 \cos\theta}{R} d\theta$ (2) (2)

x-components are cancelling due to symmetry (2)

$dE_y = |d\vec{E}|\cos\theta = \frac{k\lambda_0 \cos^2\theta d\theta}{R}$ (2) (2) $\left\{ E_y = \int dE_y \right.$

$E_y = E = \frac{k\lambda_0}{R}\int_{-\pi/2}^{\pi/2} \cos^2\theta d\theta = \frac{k\lambda_0}{R}\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right)\Big|_{-\pi/2}^{\pi/2}$ (2)

$= \frac{k\lambda_0}{R}\frac{\pi}{2} \rightsquigarrow \vec{E} = \frac{k\lambda_0 \pi}{2R}(\hat{j})$ (2)

in terms of Q $\left\{ Q = \int \lambda ds = \int_{-\pi/2}^{\pi/2} \lambda_0 \cos\theta R d\theta = \lambda_0 R \sin\theta \Big|_{-\pi/2}^{\pi/2} = 2\lambda_0 R \right.$ (2)

$\Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{2R}\frac{\pi}{2R}\hat{j} = \boxed{\frac{Q}{16\pi\epsilon_0 R^2}\hat{j}}$ (2)

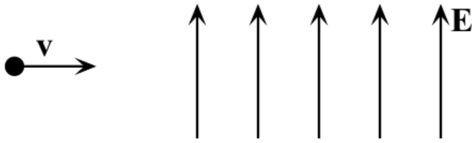
2. A proton moves at $4.5 \times 10^5\ m/s$ in the horizontal direction. It enters a uniform vertical electric field with a magnitude of $9.6 \times 10^3\ N/C$.

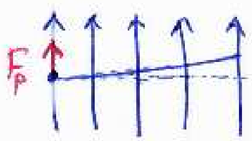
Ignoring any gravitational effects, find

i the time required for the proton to travel 5 cm horizontally,

ii the vertical displacement during that time,

iii the horizontal and vertical components of the velocity after the proton has traveled 5 cm horizontally.





$v = 4.5\times10^5\ m/s$ & $E = 9.6\times10^3\ N/C$ (uniform)

$v = v_{0x}$ & $v_{0y} = 0$ } Constant $\vec{E} \rightsquigarrow$ constant

$a = a_y$ & $a_x = 0$ } acceleration ← force

$qE = ma$

i) $v_x = v_{0x} = v = \frac{\Delta x}{\Delta t} \rightsquigarrow \Delta t = \frac{5\times10^{-2}\ m}{4.5\times10^{5}\ m/s} = 1.11\times10^{-7}\ s = \underline{111\ ns}$

ii) $a_y m_p = q_p E \rightsquigarrow a_y = \frac{q_p E}{m_p} = \frac{(1.6\times10^{-19}C)(9.6\times10^{3}N/C)}{(1.67\times10^{-27}kg)} = 9.21\times10^{11}\ m/s^2$

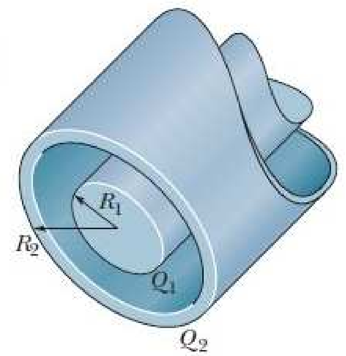
$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \rightsquigarrow y - y_0 = h = \frac{1}{2}a_y t^2 = \frac{1}{2}(9.21\times10^{11}\ m/s^2)(1.11\times10^{-7}s)^2$

$= 5.68\times10^{-3}\ m = \underline{5.68\ mm}$

iii) $v_x = v_{0x} = 4.5\times10^5\ m/s$

$v_y = v_{0y} + a_y t = a_y t = (9.21\times10^{11}\ m/s^2)(1.11\times10^{-7}s) = \underline{1.02\times10^5\ m/s}$

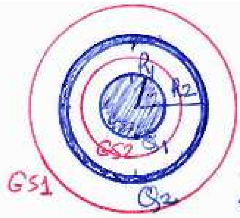
3. Figure below shows a section of a conducting rod of radius $R_1 = 1.30\ mm$ and length $L = 11.00\ m$ inside a thin-walled coaxial conducting cylindrical shell of radius $R_2 = 10.0R_1$ and the (same) length L. The net charge on the rod is $Q_1 = +3.40 \times 10^{-12}\ C$; that on the shell is $Q_2 = -2.00Q_1$



i What are the magnitude E and direction (radially inward or outward) of the electric field at radial distance $r = 2.00R_2$?

ii What are E and the direction at $r = 5.00R_1$?

iii What is the charge on the interior and exterior surface of the shell?



$R_1 = 1.30 \times 10^{-3} m$
$R_2 = 10.0 R_1 = 1.30 \times 10^{-2} m$
$L = 11.00 m$

$Q_1 = +3.40 \times 10^{-12} C$ (on rod)
$Q_2 = -2Q_1 = -6.80 \times 10^{-12} C$ (on shell)
$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$ & cylindrical Gaussian surface

i) GS1: $r = 2R_2 \rightsquigarrow E\, 2\pi r L = \frac{Q_1 + Q_2}{\varepsilon_0} \Rightarrow E = \frac{3.4 \times 10^{-12} C - 6.8 \times 10^{-12} C}{2\pi\, 2 \times 1.30 \times 10^{-2} m\, 11 m\, \varepsilon_0}$

$\vec{E}$ $\hat{r}_1$ $\rightsquigarrow E = -0.214\ N/C \rightsquigarrow |\vec{E}| = 0.214\ N/C$ & inward

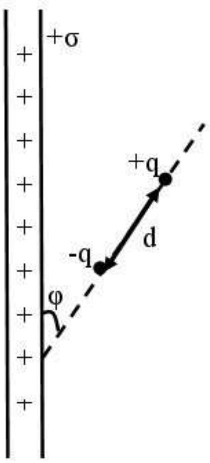
ii) GS2: $r = 5R_1 \rightsquigarrow E\, 2\pi r L = \frac{Q_1}{\varepsilon_0} \rightsquigarrow E = \frac{3.4 \times 10^{-12} C}{2\pi\, 5 \times 1.30 \times 10^{-3} m\, 11 m\, 8.85 \times 10^{-12} C^2/Nm^2}$

$\hat{r}_2$ $\vec{E}$ $\rightsquigarrow E = 0.855\ N/C \rightsquigarrow |\vec{E}| = 0.855\ N/C$ & outward

iii) Q_1 (rod, outer) $\quad -Q_1$ (shell, inner) $\quad Q_2 - (-Q_1)$ (shell, outer) $\rightsquigarrow 3.4 \times 10^{-12} C \quad \underline{\underline{-3.4 \times 10^{-12} C}} \quad \underbrace{-6.8 \times 10^{-12} C - (-3.4 \times 10^{-12} C)}_{\underline{\underline{-3.4 \times 10^{-12} C}}}$

sum up to $-6.8 \times 10^{-12} C$

4. An electric dipole of two opposite charges of magnitude $q = 1.50\ \mu C$, separated by a distance $d = 1.20\ cm$ is placed near an infinitely large plane of charge of uniform charge density $\sigma = 1.77\ \mu C/m^2$. The axis of the electric dipole makes an angle of $\varphi = 37°$ with the plane, as shown in the figure.

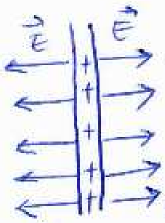


i Find the magnitude of the electric field due to the plane. Show its direction on the figure.

ii Calculate the magnitude of the electric dipole moment. Show its direction on the figure.

iii Calculate the magnitude of the torque acting on the electric dipole. Show its direction on the figure.

iv How much work must be done by an external agent to turn the electric dipole by 90° in clockwise direction?



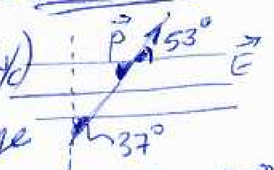
i) $\sigma = 1.77\times10^{-6}\ C/m^2$ & non-conducting plane (uniform charge density)

$\Rightarrow |\vec{E}| = \frac{\sigma}{2\varepsilon_0} = \frac{1.77\times10^{-6}\ C/m^2}{2\times 8.85\times10^{-12}\ C^2/Nm^2} = \underline{10^5\ N/C}$ magnitude

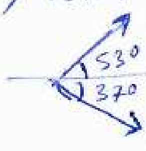
ii) $p = qd$ $\Big\{ \ominus \xrightarrow[\vec{p}]{} \oplus$ (separated by d) $\Big|$ $p = 1.50\times10^{-6}\ C \times 1.20\times10^{-2}\ m = \underline{1.8\times10^{-8}\ Cm}$

iii) $\vec{\tau} = \vec{p}\times\vec{E} \rightsquigarrow |\vec{\tau}| = |\vec{p}||\vec{E}|\sin 53 = (1.8\times10^{-8}\ Cm)(10^5\ N/C)\sin 53$

$= 1.44\times10^{-3}\ Nm$ ⊗ into the page



iv) rotation in cw, 90°



$W_{ext} = -\Delta U = -U_f + U_i = U_i - U_f \ \{ U = \vec{p}\cdot\vec{E}$

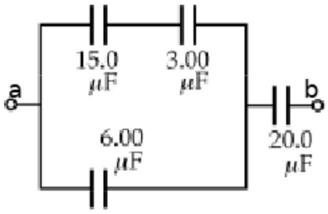
$= \vec{p}_i\cdot\vec{E} - \vec{p}_f\cdot\vec{E}$

$= |\vec{p}||\vec{E}|\cos 53 - |p||E|\cos 37°$

$= (1.8\times10^{-8}\ Cm)(10^5\ N/C)(\cos 53° - \cos 37°)$

$= \underline{-3.6\times10^{-4}\ J}$ $\quad J \equiv Nm$

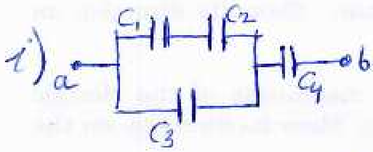
5. Four capacitors are connected as shown in Figure.



i Find the equivalent capacitance between points a and b.

ii Calculate the charge on each capacitor if $\Delta V_{ab} = 15.0\ V$.

1)



$C_{12} = \frac{C_1C_2}{C_1+C_2} = \frac{(15\times10^{-6}F)(3\times10^{-6}F)}{18\times10^{-6}F} = 2.5\times10^{-6}F$

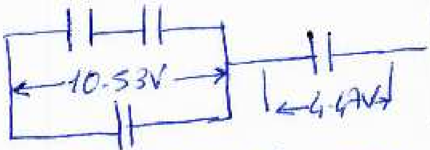
$C_{123} = C_{12}+C_3 = 2.5\times10^{-6}F + 6\times10^{-6}F = 8.5\times10^{-6}F$

$C_{eqv} = C_{1234} = \frac{C_{123}C_4}{C_{123}+C_4} = \frac{(8.5\times10^{-6}F)(20\times10^{-6}F)}{28.5\times10^{-6}F} = 5.97\times10^{-6}F = 5.97\mu F$

ii) $C = \frac{Q}{V} \rightarrow Q_{eqv} = Q_{1234} = C_{eqv}V = 5.97\times10^{-6}F\times15V = \boxed{89.47\mu C}$ 4

$\rightarrow Q_4 = Q_{123} = Q_{eqv} = 89.47\mu C \rightarrow V_4 = \frac{89.47\mu C}{20\mu F} = 4.47\ V \leftarrow$ 4

$\rightarrow$



$10.53V \leftarrow$ 3 $\rightarrow Q_3 = C_3V_3 = \boxed{63.18\mu C}$ 3

$Q_{12} = C_{12}V = (2.5\times10^{-6}F)\ 10.53V = Q_1 = Q_2$

$\rightarrow Q_1 = Q_2 = \boxed{2.63\mu C}$

$\Rightarrow V_1 = \frac{Q_1}{C_1} = \frac{2.63\mu C}{15\mu F} = 1.75V \leftarrow$ 1

$V_2 = \frac{Q_1}{C_2} = 8.78V \leftarrow$ 2

İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy102 Physics II
Midterm Examination
November 06, 2018 16:30 – 18:30
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

DURATION: 120 minutes

◇ **Answer all the questions.**
◇ **Write the solutions explicitly and clearly. Use the physical terminology.**
◇ **You are allowed to use Formulae Sheet.**
◇ **Calculator is allowed.**
◇ **You are not allowed to use any other electronic equipment in the exam.**

Question	Grade	Out of
1A		15
1B		15
2		20
3		20
4		20
5		20
TOTAL		110

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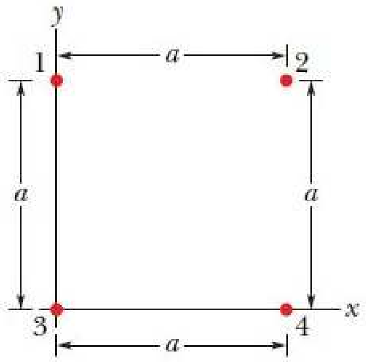
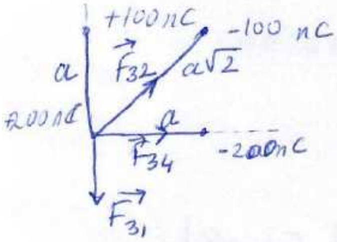
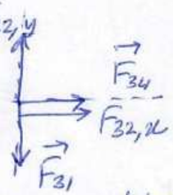
1. A) In Figure, four particles form a square.



The particles have charges $q_1 = -q_2 = 100\ nC$ and $q_3 = -q_4 = 200\ nC$, and distance $a = 5.0\ cm$. What are the x and y components of the net electrostatic force on particle 3?



$$\vec{F}_{3,net} = \sum_{i=1}^{3} \vec{F}_{3i} = \vec{F}_{31} + \vec{F}_{32} + \vec{F}_{34}$$



$$F_{3,net,x} = F_{31,x} + F_{32,x} + F_{34,x}$$

$$F_{3,net,y} = F_{31,y} + F_{32,y} + F_{34,y}$$

$$F_{3,net,x} = |F_{32}| \cos 45 + |F_{34}| = k \frac{|q_3||q_2|}{a^2 2} \cos 45 + k \frac{|q_3||q_4|}{a^2}$$

$$= 8.99 \times 10^9 \frac{Nm^2}{C^2} \frac{(200 \times 10^{-9} C)}{(5 \times 10^{-2} m)^2} \left(\frac{100 \times 10^{-9} C}{2} \frac{\sqrt{2}}{2} + 200 \times 10^{-9} C \right)$$

$$\boxed{= 0.17\ N}$$

$$F_{3,net,y} = |F_{32}| \sin 45 - |F_{31}| = k \frac{|q_3||q_2|}{a^2 2} \sin 45 - k \frac{|q_3||q_1|}{a^2}$$

$$= 8.99 \times 10^9\ Nm^2/C^2 \frac{(200 \times 10^{-9} C)}{(5 \times 10^{-2} m)^2} \left(\frac{100 \times 10^{-9} C}{2} \frac{\sqrt{2}}{2} - 100 \times 10^{-9} C \right)$$

$$\boxed{= -0.046\ N}$$

B) In Figure (a), particle 1 (of charge q_1) and particle 2 (of charge q_2) are fixed in place on an x-axis, 8.00 cm apart. Particle 3 (of charge $q_3 = +8.00 \times 10^{-19}\ C$) is to be placed on the line between particles 1 and 2 so that they produce a net electrostatic force $F_{3,net}$ on it.

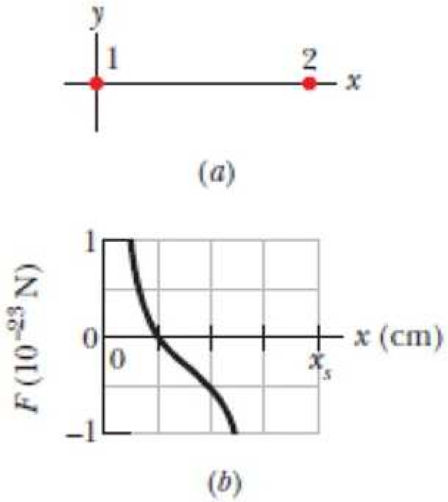
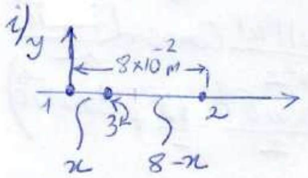


Figure (b) gives the x component of that force versus the coordinate x at which particle 3 is placed. The scale of the x axis is set by $x_s = 8.0\ cm$.

i What is the sign of charge q_1?

ii What is the ratio q_2/q_1?

i)



if ⊖ ⊕ ⊖ F_{31} F_{32} ✓ but Figure(b)

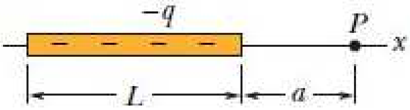
if ⊕ ⊕ ⊕ F_{32} F_{31} ✓ when $x>2$ repulsive force (positive value) → q_1 should be (+)

ii) $F_{3,net}(x=2) = 0 \rightarrow |F_{32}(x=2)| = |F_{31}(x=2)|$

$$k\frac{|q_3||q_2|}{(8-x)^2} = k\frac{|q_3||q_1|}{x^2}$$ when $x = 2\times10^{-2}\ m$

$$\frac{q_2}{(6\times10^{-2}\ m)^2} = \frac{q_1}{(2\times10^{-2}\ m)^2} \rightarrow \boxed{\frac{q_2}{q_1} = 9}$$

2. In the figure below, a nonconducting rod of length $L = 8.15\ cm$ has a charge $q = -4.23\ fC$ uniformly distributed along its length.



i What is the linear charge density of the rod?

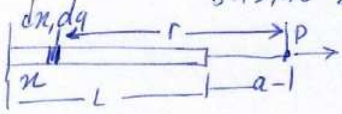
ii What are the magnitude and direction (relative to the $+x$-axis) of the electric field produced at point P, at distance $a = 12.0\ cm$ from the rod?

iii What is the electric field magnitude produced at distance $a = 50.0\ cm$ by the rod?

iv What is the electric field magnitude produced at distance $a = 50.0\ cm$ by <u>a particle of charge</u> $q = -4.23\ fC$ that replaces the rod?

i) $\lambda = \frac{Q}{L} = \frac{-4.23 \times 10^{-15} C}{8.15 \times 10^{-2} m} = \underline{\underline{-5.19 \times 10^{-14} C/m}}$

ii)



$\rightsquigarrow r = L + a - x$

$dE = k\frac{dq}{r^2} = k\frac{\lambda dx}{(L+a-x)^2}$ $\Big\}$ $E = \int_0^L dE$

$E_p = k\lambda \int_0^L \frac{dx}{(L+a-x)^2} = k\lambda \frac{1}{(L+a-x)}\Big|_0^L = k\lambda\left(\frac{1}{a} - \frac{1}{L+a}\right)$

$= 8.99 \times 10^9 \frac{Nm^2}{C^2}\left(-5.19 \times 10^{-14} \frac{C}{m}\right)(-\hat{\imath})\left(\frac{L}{a(L+a)}\right) = 4.67 \times 10^{-4} \frac{Nm}{C}$

$\Rightarrow$ $L = 8.15 \times 10^{-2} m$, $a = 12 \times 10^{-2} m$ $\Big\}$ $E_p = 4.67 \times 10^{-4} Nm/C \left(\frac{8.15 \times 10^{-2} m}{(12 \times 10^{-2} m)(20.15 \times 10^{-2} m)}\right) = \boxed{1.57 \times 10^{-3} \frac{N}{C}}$

iii) $L = 8.15 \times 10^{-2} m$, $a = 50 \times 10^{-2} m$ $\Big\}$ $E_p = 4.67 \times 10^{-4} Nm/C \left(\frac{8.15 \times 10^{-2} m}{(50 \times 10^{-2} m)(58.15 \times 10^{-2} m)}\right) = \boxed{1.31 \times 10^{-4} N/C}$

iv) Point charge: $E_p = k\frac{|q|}{r^2} = 8.99 \times 10^9 \frac{Nm^2}{C^2} \frac{4.23 \times 10 \times 10^{-15} C}{(50 \times 10^{-2} m)^2} = \boxed{1.54 \times 10^{-4} N/C}$

3. An infinitely long cylindrical insulating shell of inner radius a and outer radius b has a uniform volume charge density ρ. A line of uniform linear charge density λ, is placed along the axis of the shell. Determine the electric field in the following regions:

   i $r < a$

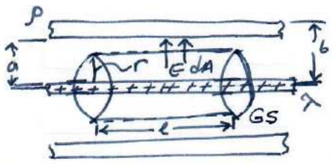
   ii $a < r < b$

   iii $r > b$

i) $r < a$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(2\pi r l) = \frac{Q_{enc}}{\varepsilon_0}$$



$Q_{line} = \lambda l$

$Q_{clyinder} = \phi$ (shell theorem)

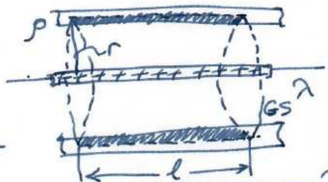
$\rightsquigarrow Q_{enc} = \lambda l + \phi$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

ii) $a < r < b$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(2\pi r l) = \frac{Q_{enc}}{\varepsilon_0}$$



$Q_{line} = \lambda l$

$Q_{cylnder} = \rho * \text{Volume}$
$= \rho * (\pi r^2 l - \pi a^2 l)$
$= \pi l \rho (r^2 - a^2)$

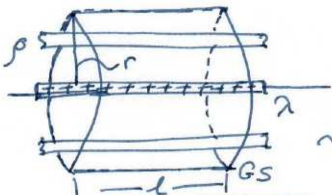
$\rightsquigarrow Q_{enc} = \lambda l + \pi l \rho (r^2 - a^2)$

$$\boxed{E = \frac{\lambda + \rho\pi(r^2 - a^2)}{2\pi r}}$$

iii) $r > b$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(2\pi r l) = \frac{Q_{enc}}{\varepsilon_0}$$



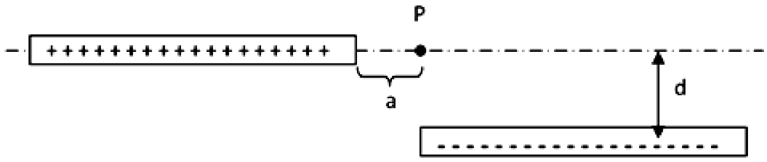
$Q_{line} = \lambda l$

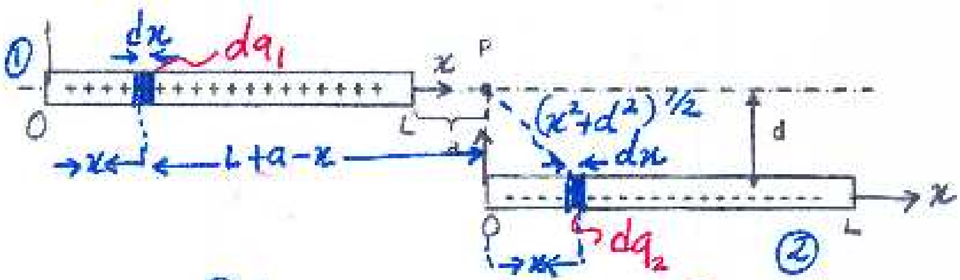
$Q_{cylnder} = \rho(\pi b^2 l - \pi a^2 l)$

$\rightsquigarrow Q_{enc} = \lambda l + \rho l \pi (b^2 - a^2)$

$$\boxed{E = \frac{\lambda + \rho\pi(b^2 - a^2)}{2\pi r}}$$

4. Two very thin non-conducting rods are placed together as shown. Both rods have lengths of L and they carry uniform charges of $+q$ and $-q$ over their lengths. Find the potential at point P at a distance a and d from the positively and negatively charged rods as shown. Don't perform integration.





$$V_{1\,at\,P} = \int dV = \int \frac{1}{4\pi\epsilon_0}\frac{dq_1}{r_1}$$

$dq_1 = \lambda\, dx$

$r_1 = L+a-x$

$$V_{2\,at\,P} = \int dV = \int \frac{1}{4\pi\epsilon_0}\frac{dq_2}{r_2}$$

$dq_2 = -\lambda\, dx$

$r_2 = \sqrt{x^2+d^2}$

$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$

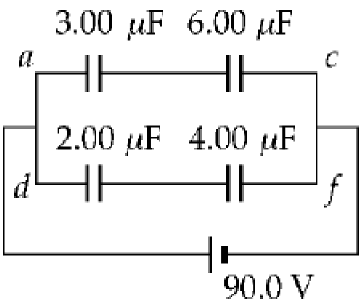
$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{r}$

$dq = ?$

$dq = \lambda\, dx$

$$V_{tot} = V_{1\,at\,P} + V_{2\,at\,P} = \boxed{\frac{\lambda}{4\pi\epsilon_0}\left(\int_0^L \frac{dx}{(L+a-x)} - \int_0^L \frac{dx}{\sqrt{x^2+d^2}}\right)}$$

5. For the system of capacitors shown in Figure,



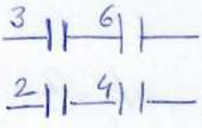
find

i the equivalent capacitance of the system,

ii the potential across each capacitor,

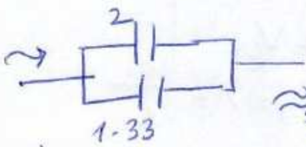
iii the charge on each capacitor.

i) $C_{eqv} = ?$



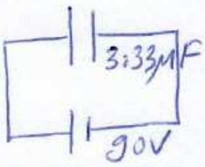
$\sim \frac{1}{C_{ac}} = \frac{1}{3\mu F} + \frac{1}{6\mu F} \Rightarrow C_{ac} = 2\mu F$

$\sim \frac{1}{C_{df}} = \frac{1}{2\mu F} + \frac{1}{4\mu F} \Rightarrow C_{df} = 1.33\mu F$

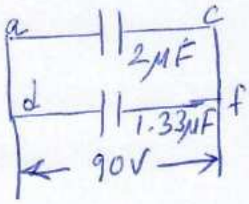


$\Rightarrow C_{eqv} = 3.33\mu F$

ii)



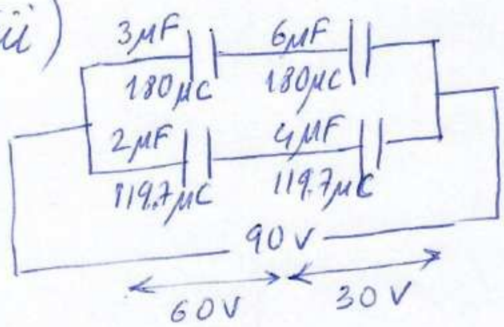
$C = \frac{Q}{V} \sim Q = C_{eqv} * V = (3.33\times10^{-6} F)\, 90V = 299.7\mu C$ (total charge)



$Q_{ac} = (2\mu F)\, 90V = 180\mu C = q_a = q_c$

$Q_{df} = (1.33\mu F)\, 90V = 119.7\mu C = q_d = q_f$

iii)



$V_a = \frac{q_a}{C_a} = \frac{180\mu C}{3\mu F} = \boxed{60V}$

$V_b = \frac{180\mu C}{6\mu F} = \boxed{30V}$

$V_c = \frac{119.7\mu C}{2\mu F} \sim \boxed{60V}$

$V_d = \frac{119.7\mu C}{4\mu F} \cong \boxed{30V}$