İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy102 Physics II
Midterm Examination
November 10, 2022 17:00 – 18:30

Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

INSTRUCTOR:

DURATION: 90 minutes

◇ **Answer all the questions.**
◇ **Write the solutions explicitly and clearly. Use the physical terminology.**
◇ **You are allowed to use Formulae Sheet.**
◇ **Calculator is allowed.**
◇ **You are not allowed to use any other electronic equipment in the exam.**

Question	Grade	Out of
1A		15
1B		15
2		20
3		20
4		20
5		20
TOTAL		110

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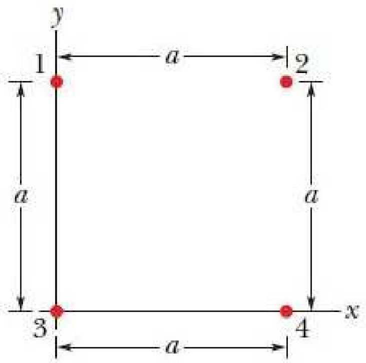
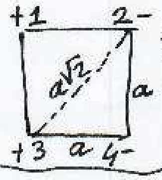
1. A) In Figure, four particles form a square. The particles have charges $q_1 = 100\ nC$, $q_2 = -100\ nC$, $q_3 = 200\ nC$, $q_4 = -200\ nC$, and distance $a = 5.0\ cm$.



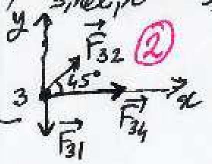
i What are the x and y components of the net electrostatic force on particle 3?

ii If the charges were $q_1 = q_4 = Q$ and $q_2 = q_3 = q$. What is Q/q if the net electrostatic force on particles 1 and 4 is zero?

$q_1 = 100\times10^{-9}C$
$q_2 = -q_1$
$q_3 = 200\times10^{-9}C$
$q_4 = -q_3$
$a = 5\times10^{-2}m$



i) $F_{3,net,x}$ & $F_{3,net,y}$? $\vec{F}_{3,net} = \sum_{i=1}^{3} \vec{F}_{3i} = \vec{F}_{31}+\vec{F}_{32}+\vec{F}_{34}$ (2)



$\rightsquigarrow F_{3,net,x} = |\vec{F}_{34}| + |\vec{F}_{32}| \cos 45$ (1)

$\rightsquigarrow F_{3,net,y} = |\vec{F}_{32}| \sin 45 - |\vec{F}_{31}|$ (1)

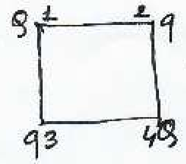
$$F_{3,net,x} = k\frac{|q_3||q_4|}{a^2} + k\frac{|q_3||q_2|}{(a\sqrt{2})^2}\frac{\sqrt{2}}{2} = \frac{k|q_3|}{a^2}\left(|q_4| + \frac{|q_2|}{2}\frac{\sqrt{2}}{2}\right) = \frac{8.99\times10^{9} Nm^2/C^2 (200\times10^{-9}C)}{(5\times10^{-2}m)^2}\left(|-200\times10^{-9}C| + |-100\times10^{-9}C|\frac{\sqrt{2}}{2}\right)$$
$$= \boxed{0.169N} \quad (1)(1)$$

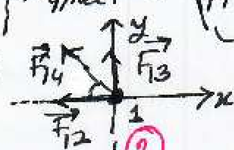
$$F_{3,net,y} = \frac{k|q_3|}{a^2}\left(\frac{|q_2|}{2}\frac{\sqrt{2}}{2} - |q_1|\right) = \frac{8.99\times10^{9} Nm^2/C^2 (200\times10^{-9}C)}{(5\times10^{-2}m)^2}\left(|100\times10^{-9}C|\frac{\sqrt{2}}{4} - |100\times10^{-9}C|\right)$$
$$= \boxed{-0.046N} \quad (1)(1)$$

ii) $q_1 = q_4 = Q$, $q_2 = q_3 = q$, $Q/q = ?$

$|\vec{F}_{1net}| = 0 \rightsquigarrow F_{1net,x} = 0$ & $F_{1net,y} = 0$ (1)

$|\vec{F}_{4net}| = 0$ $(|\vec{F}_{14}|\cos 45 + |\vec{F}_{12}|)(-\hat{i})$ $(|\vec{F}_{13}| + |\vec{F}_{14}|\sin 45)(\hat{j})$





(2)

$$0 = \frac{k|q_1|}{a^2}\left(\frac{|q_4|}{2}\frac{\sqrt{2}}{2} + |q_2|\right) = \frac{kQ}{a^2}\left(Q\frac{\sqrt{2}}{4} + q\right) \quad (1)$$

$$\rightsquigarrow \frac{Q}{q} = -\frac{4}{\sqrt{2}} = -2\sqrt{2} = \boxed{-2.83} \quad (1)$$

B) The density of conduction electrons in aluminum is $2.1 \times 10^{29}\ m^{-3}$. What is the drift velocity in an aluminum conductor that has a 2.0 μm by 3.0 μm rectangular cross section and when a 32.0 mA current flows through the conductor?

$n = 2.1 \times 10^{29}\ m^{-3}$

$i = 32 \times 10^{-3}\ A$

$A = (2 \times 10^{-6}\ m)(3 \times 10^{-6}\ m)$

$v_d = ?$

$\vec{J} = ne\vec{v}_d$

3 $J = nev_d$

3 $J = \frac{i}{A}$

$\frac{i}{A} = nev_d$

$$\Rightarrow v_d = \frac{i}{Ane} \overset{4}{=} \frac{32 \times 10^{-3} A}{(2 \times 10^{-6} m)(3 \times 10^{-6} m)(2.1 \times 10^{29} m^{-3})(1.602 \times 10^{-19} C)}$$

$$= \boxed{0.016\ m/s}$$

3 2

$$\frac{A}{m^2 m^{-3} C} \sim \frac{C/s}{m^2 m^{-3} C} \sim m/s$$

unit check

2. At some instant the velocity components of an electron moving between two charged parallel plates are $v_x = 3 \times 10^5\ m/s$ and $v_y = 5.0 \times 10^3\ m/s$. Suppose the electric field between the plates is given by $\vec{E} = (180N/C)\hat{j}$. In unit-vector notation, what are

i the electron's acceleration in that field

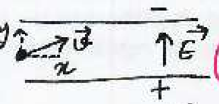
ii the electron's velocity when its x coordinate has changed by 2.4 cm?

$\bar{e}$: electron

$v_x = 3\times10^5\,m/s = v_{0x}$

$v_y = 5\times10^3\,m/s = v_{0y}$

$\vec{E} = 180\,N/C\,\hat{j}$



(2) i) $a = ?$ $\vec{F}_E = q\vec{E} = (1.6\times10^{-19}C)(180\,N/C)(-\hat{j})$ (2)

$= 288\times10^{-19}N\,(-\hat{j})$ (1) (1)

$\Rightarrow m_e\vec{a} = \vec{F}_E$ (2) $\rightarrow \vec{a} = \frac{288\times10^{-19}N}{9.109\times10^{-31}kg}(-\hat{j}) = \boxed{3.16\times10^{13}\,m/s^2\,(-\hat{j})}$ (1) (1)

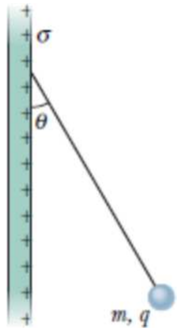
ii) $\Delta x = x - x_0 = 2.4\times10^{-2}m$ & no force acting on x direction (2)

$\Rightarrow v_x = v_{0x}$ (1) & $v_y = v_{0y} + at$ (1) $\Rightarrow \frac{\Delta x}{\Delta t} = v_x \rightarrow \frac{\Delta x}{v_x} = \Delta t$

$\Rightarrow \Delta t = \frac{2.4\times10^{-2}m}{3\times10^5 m/s} = 8\times10^{-8}s$ (1) (1) $\rightarrow v_y = 5\times10^3 m/s - (3.16\times10^{13} m/s^2)(8\times10^{-8}s) = \boxed{-2.52\times10^6 m/s}$ (1) (1)

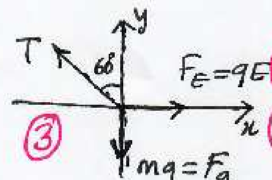
$\Rightarrow \boxed{\vec{v} = 3\times10^5 m/s\,\hat{i} + 2.52\times10^6 m/s\,(-\hat{j})}$ (2)

3. A small, nonconducting ball of mass $m = 2 \times 10^{-6}\ kg$ and charge $q = 4.0 \times 10^{-8}\ C$ (distributed uniformly through its volume) hangs from an insulating thread that makes an angle $\theta = 60°$ with a vertical, uniformly charged **nonconducting sheet** (shown in cross section).



Considering the gravitational force on the ball and assuming the sheet extends far vertically and into and out of the page, **calculate the surface charge density** σ **of the sheet.** (**Hint: The ball is in equilibrium (stationary)**.)

$m = 2\times10^{-6}\,kg$ (non-conducting)
$q = 4\times10^{-8}\,C$ uniform distribution
non-conducting sheet, $\sigma = ?$ (if hanged)
$\cancel{E} = \frac{\sigma}{2\varepsilon_0}$



3

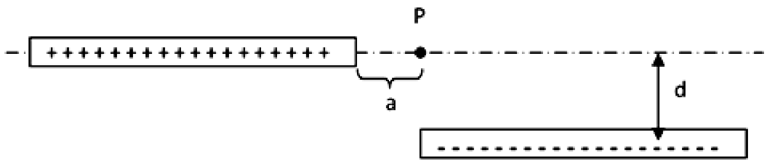
3 $T\cos 60° - mg = ma_y = 0$
3 $qE - T\sin 60° = ma_x = 0$
hangs ⇝ stationary

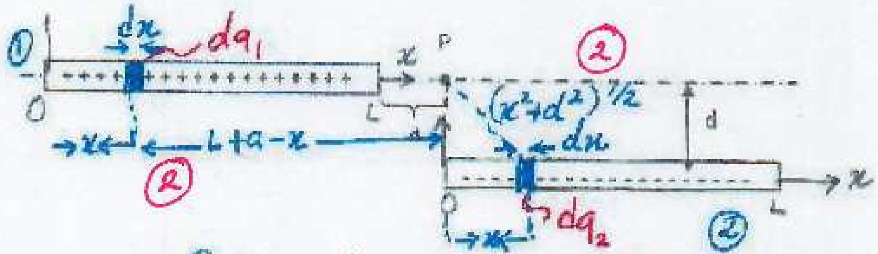
now, eliminate T

$\rightsquigarrow qE - \left(\frac{mg}{\cos 60°}\right)\sin 60° = 0$ 3 $\rightsquigarrow qE = mg\tan 60° \rightsquigarrow q\frac{\sigma}{2\varepsilon_0} = mg\tan 60° \rightsquigarrow \sigma = \frac{2mg\varepsilon_0 \tan 60°}{q}$

$\rightsquigarrow \sigma = \frac{2mg\varepsilon_0\tan 60°}{q}$ 3 $= \frac{2(2\times10^{-6}kg)(9.8\,m/s^2)(8.85\times10^{-12}C^2/Nm^2)\tan 60°}{(4\times10^{-8}C)}$ 2 $= \boxed{15\times10^{-9}\,C/m^2}$ 2 1

4. Two very thin non-conducting rods are placed together as shown. Both rods have lengths of L and they carry uniform charges of $+q$ and $-q$ over their lengths. Find the potential at point P at a distance a and d from the positively and negatively charged rods as shown. Don't perform integration.





$$V_{1\,at\,P} = \int dV = \int \frac{1}{4\pi\varepsilon_0}\frac{dq_1}{r_1}$$

$dq_1 = \lambda\, dx$ (3)

$r_1 = L + a - x$

$$V_{2\,at\,P} = \int dV = \int \frac{1}{4\pi\varepsilon_0}\frac{dq_2}{r_2}$$

(3) $dq_2 = -\lambda\, dx$

$r_2 = \sqrt{x^2 + d^2}$

$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$ (2)

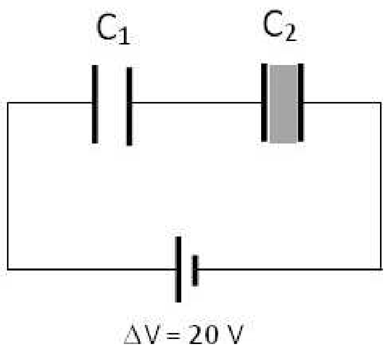
$dq = ?$

$dq = \lambda\, dx$ (2)

$$\boxed{V_{tot} = V_{1\,at\,P} + V_{2\,at\,P}} = \boxed{\frac{\lambda}{4\pi\varepsilon_0}\left(\int_0^L \frac{dx}{(L+a-x)} - \int_0^L \frac{dx}{\sqrt{x^2+d^2}}\right)}$$

(3) (3)

5. The parallel plate capacitors in the given circuit have the same plate area A and plate separation d. The capacitance of the air-filled capacitor is $C_1 = 6.0\mu\ F$. A dielectric slab of dielectric constant $\kappa = 2.0$ is placed between the plates of the second capacitor as shown. The voltage across the combination of capacitors is $\Delta V = 20\ V$ and the capacitors are fully charged.



i Find the equivalent capacitance of the combination of capacitors.

ii Calculate the energy stored in each capacitor.

iii Calculate the electric field in the second capacitor if the area of the capacitor is $100\ cm^2$.

i) Capacitors C_1 & C_2 are in series. $C = \kappa \frac{\varepsilon_0 A}{d}$ (1)

C_1 : air filled $\rightarrow \kappa = 1 \Rightarrow C_1 = \varepsilon_0 \frac{A}{d} = 6.0\mu F$ (1)

C_2 : dielectric slab $\rightarrow \kappa = 2 \Rightarrow C_2 = \kappa C_1 = 12.0\mu F$

$\Rightarrow \frac{1}{C_{eqv}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow \frac{1}{C_{eqv}} = \frac{1}{6\mu F} + \frac{1}{12\mu F} = \frac{18}{72\mu F} \Rightarrow \boxed{C_{eqv} = 4\mu F}$

(1) (2) (1) (1)

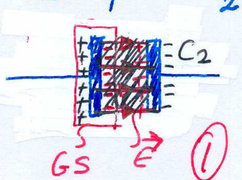
ii) In series $\rightarrow$ same charge on both capacitors (1)

(1) $q = q_1 = q_2$ & $\Delta V = 20\ V$ } $C_{eqv} = \frac{q}{\Delta V} \rightarrow q = C_{eqv}\Delta V = (4\mu F)(20V) \boxed{= 80\mu C}$ (1)

$U = \frac{q^2}{2C}$ { $U_1 = \frac{(80\mu C)^2}{2(6.0\mu F)} \boxed{= 533.3\mu J}$ & $U_2 = \frac{(80\mu C)^2}{2\times(12\mu F)} \boxed{= 266.6\mu J}$

(1) (1) (1) (1)

iii)



(1)

$\varepsilon_0 \oint \kappa \vec{E}\cdot d\vec{A} = q$ (1)

$\Rightarrow E = \frac{q}{\varepsilon_0 \kappa A} = \frac{80\times10^{-6}\ C}{(8.85\times10^{-12}\ F/m)(2.0)(10^{-2}\ m^2)} \boxed{= 4.52\times10^{8}\ V/m}$

$A = 100\ cm^2 = 10^{-2}\ m^2$

(2) (1) (1)