# Chapter 22 - Electric Fields

How the chaged particles "know" the presence of othe particle?
How these particles interact at a distance althoug they do not touch?

The answer is Electric Field! $\vec{E}$ Particle 2 pushes on particle 1 not by touching it by means of the electric field produced by particle 2.

- Define $\vec{E}$
- How to calculate it for various arragements of chaged particles.

## The Electric Field

Temperature, Pressure : Scalar fields
Electric field is a vector field < Distribution of vectors around a chaged object; place a test charge near by

SLN Fig 22-1

Define the electric field $\vec{E}$ at point P due to the chaged object as $\left.\right\}$ $\boxed{\vec{E} = \dfrac{\vec{F}}{q_0}}$ $|\vec{E}| = \dfrac{|\vec{F}|}{q_0}$ magnitude

Unit N/C

same as $\vec{F}$ ~ direction

$\vec{E}$ exists independently of the test charge.

SLN

## Electric Field Lines

To visualize patterns in $\vec{E}$.

- direction of line ⇒ direction of $\vec{E}$
- number of lines per unit area ⇒ magnitude of $\vec{E}$ < lines: close → large $|\vec{E}|$; lines: away → small $|\vec{E}|$
- extends away from positive chage
- toward negative charge

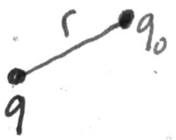
SLN Fig 22-2 sphere of uniform negative charge
Fig 22-3 nonconducting sheet with uniformly distributed positive charge

SLN Fig 22-4
Fig 22-5 : Electric Dipole
$\vec{E}$ is tangent to the field line

## $\vec{E}$ Due to a Point Charge

To find $\vec{E}$ due to a point charge at any point a distance r $\left.\right\}$ put a test charge $q_0$

$\vec{F} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q q_0}{r^2}\hat{r}$, $\boxed{\vec{E} = \dfrac{\vec{F}}{q_0} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}\hat{r}}$




Then, electrostatic force acting on $q_0$;

magnitude : $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}$

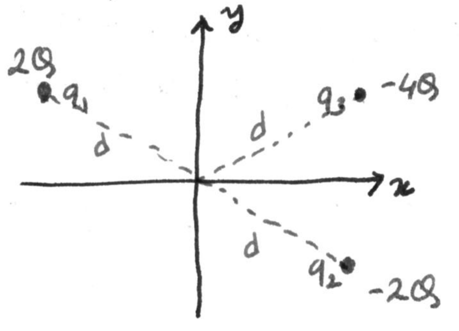
direction : same as the direction of $\vec{F}$

More than one point charge? Net force, $\vec{F}_0$, acting on test charge, $q_0$?

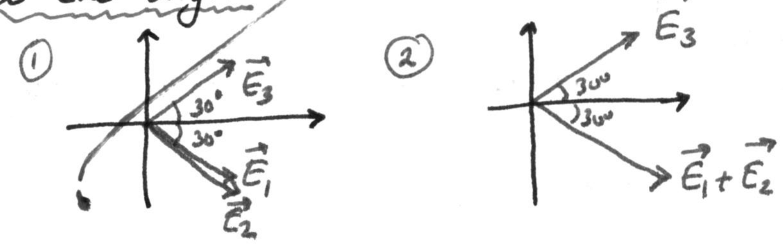
$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \dots + \vec{F}_{0n} \quad \left\{ \begin{matrix} \text{net} \\ \vec{E} \end{matrix} \right| \quad \vec{E} = \frac{\vec{F}_0}{q_0} = \underbrace{\vec{E}_1 + \vec{E}_2 + \dots + \vec{E}_n}_{\text{principle of superposition}}$$

(acting one by one in pairs)

## Example




Find the net $\vec{E}$ at the origin




in magnitude $\quad E_1 + E_2 = \frac{1}{4\pi\varepsilon_0}\frac{|2Q|}{d^2} + \frac{1}{4\pi\varepsilon_0}\frac{|-2Q|}{d^2}$ ; $E_3 = \frac{1}{4\pi\varepsilon_0}\frac{|-4Q|}{d^2}$

in vector $\quad \vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$

$\vec{E}_{net,x} = \vec{E}_{1,x} + \vec{E}_{2,x} + \vec{E}_{3,x}$

$\vec{E}_{net,y} = \vec{E}_{1,y} + \vec{E}_{2,y} + \vec{E}_{3,y}$

$\rightarrow \vec{E}_{net,x} = E_1 \cos 30\, \hat{i} + E_2 \cos 30\, \hat{i} + E_3 \cos 30\, \hat{i}$

$\vec{E}_{net,y} = E_1 \sin 30(-\hat{j}) + E_2 \sin 30(-\hat{j}) + E_3 \sin 30(\hat{j})$

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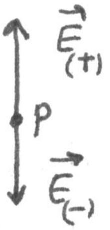
$$\vec{E}_{net,x} = \frac{1}{4\pi\varepsilon_0}\frac{8Q}{d^2}\frac{\sqrt{3}}{2}\,\hat{i} = \boxed{\frac{6.93\,Q}{4\pi\varepsilon_0 d^2}\,\hat{i}}$$

$$\vec{E}_{net,y} = \frac{1}{4\pi\varepsilon_0}(-2Q - 2Q + 4Q)\sin 30\,\hat{j} = \boxed{0\,\hat{j}}$$

## $\vec{E}$ Due to an Electric Dipole

Two charged particles of magnitude $q$ but opposite sign seperated by a distance $d$. SLN Fig 22-8 $\vec{E}$ due to dipole at a point P, a distance $z$ from the midpoint of the dipole and on the axis through the particles (dipole axis)




$$\underbrace{E = E_{(+)} - E_{(-)}}_{\text{at point P (magnitude)}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(-)}^2} \qquad \left\{ \begin{matrix} r_{(+)} = z - \frac{d}{2} \\ r_{(-)} = z + \frac{d}{2} \end{matrix} \right.$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{(z - d/2)^2} - \frac{1}{(z + d/2)^2}\right)$$

$$\boxed{E = \frac{q}{4\pi\varepsilon_0 z^3}\left(\frac{d}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2}\right)}$$

at large distances $\frac{d}{2z} \ll 1$ $(z \gg d)$ $\Rightarrow \boxed{E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3} = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}}$

⊕
$\uparrow \vec{p}$
⊖ $p$ : electric dipole moment

Direction of $\vec{p}$: $- \rightarrow +$
specify orientation of a dipole

# $\vec{E}$ Due to a Line of Charge

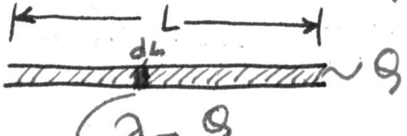
A single point charge (discrete) ! not realistic. Usually we have charge distributions

Many closely spaced point charges spread
- along a line
- over a surface
- within a volume

} continuous.

Now, we need ~charge density~ rather than ~total charge~

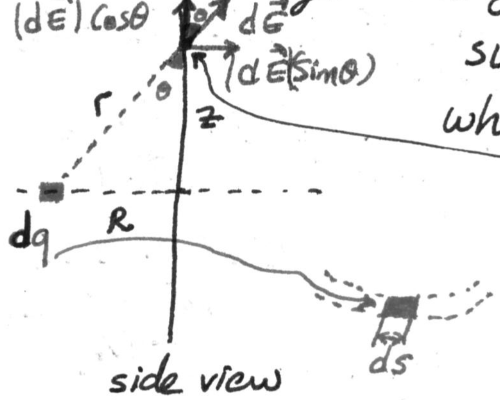
$\lambda, \sigma, \rho$ (linear, surface, volume) charge density.   SLN Table 22-




$\lambda = \frac{Q}{L}$

$\lambda = \frac{dq}{dL}$

Example: A ring of radius $R$ with a uniform positive linear charge density. (Plastic or insulator so that charges are fixed, not moving)




SLN Fig 22-10

what is $\vec{E}$ at point $P$

Previously

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

$$\frac{dq}{ds} = \lambda \text{ charge per unit length}$$

now, we have $dq$ and we should find $d\vec{E}$.

$dq$: charge of the differential element

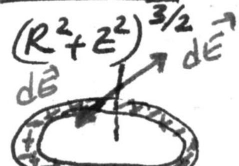
$ds$: length of any differential element

$\Rightarrow d\vec{E}$ at point P. Now, $dq$ can be treated as point charge.

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{(R^2+z^2)}$$ : First step is completed

Second step is to calculate total electric field.

$$|\vec{E}| = \int dE\cos\theta \quad \left\{ \cos\theta = \frac{z}{r} = \frac{z}{(R^2+z^2)^{1/2}} \right\} \quad E = \int_0^{\overbrace{2\pi R}^{\text{length of ring}}} \frac{1}{4\pi\varepsilon_0}\frac{z\lambda ds}{(R^2+z^2)^{3/2}}$$



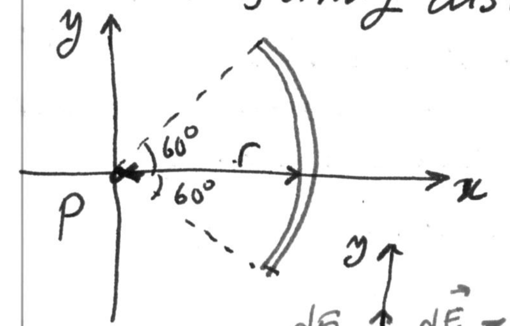

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\lambda z}{(R^2+z^2)^{3/2}}\int_0^{2\pi R} ds = \frac{z\,\overbrace{\lambda\, 2\pi R}^{q}}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}}$$

$$\Rightarrow \boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{qz}{(R^2+z^2)^{3/2}}}$$ charged ring

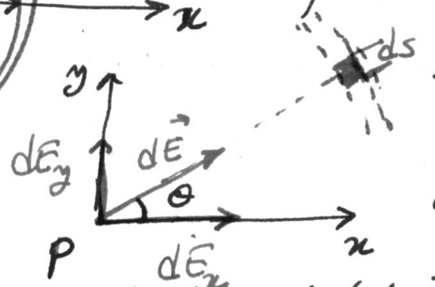
when $z \gg R$ $\quad E = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2}$

At very large distances, ring looks like a point charge.

**Example** A $120°$ circular arc rod of radius $r$ with having a uniformly distributed charge $-Q$. SLN Fig. 22-11




What is $\vec{E}$ at point P? in terms of $\theta$ and $r$




Firs step: Find $dE$ with knowns!

$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{r}$, $|d\vec{E}| = dE$ magnitude

$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}$, $\lambda = \frac{dq}{ds}$, $dq = \lambda ds$

$\Rightarrow dE = \frac{1}{4\pi\epsilon_0}\frac{\lambda ds}{r^2}$ : First Step is comple

Second Step: Calculate total $\vec{E}$ — y-components of $d\vec{E}$ are cancelled out.

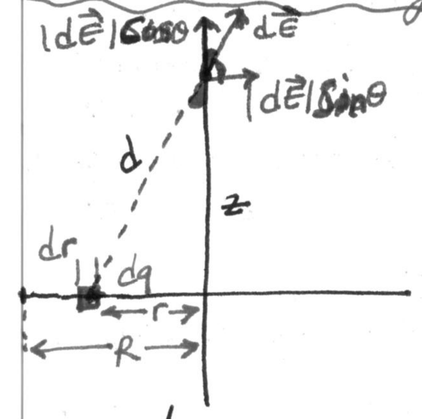
$\vec{E} = E_x\hat{i} + E_y\hat{j}$, $|\vec{E}| = E_x = E$

$E = \int dE_x = \int dE\cos\theta = \int \frac{1}{4\pi\epsilon_0}\frac{\lambda ds}{r^2}\cos\theta$ $\Big\}$ $ds = r d\theta$


$= \frac{1}{4\pi\epsilon_0}\frac{\lambda}{r}\int^{60°}\cos\theta\, d\theta \Rightarrow \frac{\lambda}{4\pi\epsilon_0}\Big[\sin\theta\Big]_{-60°}^{60°} = \frac{\lambda}{4\pi\epsilon_0 r}\left[\sin 60° - \sin(-60°)\right]$

$\Rightarrow \boxed{E = \frac{1.73\lambda}{4\pi\epsilon_0 r}}$ : Total $\vec{E}$. Not finished. $\lambda$ should be removed $\Big|$ $\lambda = \frac{Q}{\frac{2\pi}{3}r} \Rightarrow \boxed{E = \frac{0.83Q}{4\pi\epsilon_0 r^2}}$ $\boxed{\vec{E} = \frac{0.83Q}{4\pi\epsilon_0 r^2}\hat{i}}$

## $\vec{E}$ Due to a Charged Disk




side view

What is $\vec{E}$ at point P?

$\frac{dq}{dA} = \sigma = \frac{dq}{2\pi r dr}$

$d = \sqrt{r^2+z^2}$

$\cos\theta = \frac{z}{d}$

$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{d^2} = \frac{1}{4\pi\epsilon_0}\frac{\sigma dA}{d^2}$

$dE = \frac{1}{4\pi\epsilon_0}\frac{\sigma 2\pi r dr}{(r^2+z^2)}$ : $1^{st}$ step

$\Rightarrow 2^{nd}$ step: $\cancel{E=}\int dE\cos\theta = dE_z$

$\cancel{dE} = \frac{1}{4\pi\epsilon_0}\sigma 2\pi \int \frac{r\,dr}{(r^2+z^2)}\frac{z}{(r^2+z^2)^{1/2}}$

$dE_z = \frac{\sigma z}{4\epsilon_0}\int \frac{2r dr}{(r^2+z^2)^{3/2}}$

$3^{rd}$ step: Calculate total $\vec{E}$

$E = \int dE_z = \frac{\sigma z}{4\epsilon_0}\int_0^R (z^2+r^2)^{-3/2}(2r)dr$ $\Big\{$ $\int X^M dX = \frac{X^{M+1}}{M+1}$ $\Big\}$ $X = (z^2+r^2)$, $M = -3/2$, $dX = (2r)dr$

$E = \frac{\sigma z}{4\epsilon_0}\left[\frac{(z^2+r^2)^{-1/2}}{-1/2}\right]_0^R$

$\boxed{E = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)}$ charged disk $\Big\{$ if $R \to \infty \Rightarrow \boxed{E = \frac{\sigma}{2\epsilon_0}}$ infinite sheet

## A Point Charge in an $\vec{E}$

A charged particle located to $\vec{E}$. what happens? An electrostatic force acts on the particle: $\boxed{\vec{F}=q\vec{E}}$

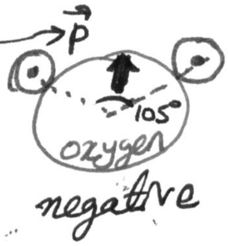
$\vec{E}$ : External electric field
$q$ : charge of the particle

* The direction of $\vec{F}$ and $\vec{E}$ are the same if charge, $q$, is positive
" " " opposite " " " negative

SLN. Example

## A dipole in an $\vec{E}$

The behavior of a dipole in an $\vec{E}$ can be described completely in terms of $\vec{E}$ and $\vec{p}$!
No need any information about dipole's structure




A dipole in a uniform external $\vec{E}$.

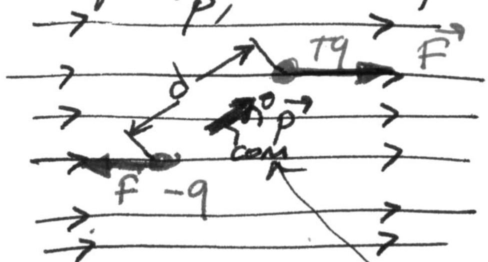
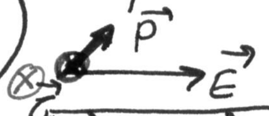



Fig. 22-19

- Forces act in opposite directions.
- Net force on the dipole is zero.
- But, forces produce a net torque, $\vec{\tau}$ on the dipole about its com.

$\tau = F d \sin\theta$

$\tau = p E \sin\theta$




$p = qd$ & $F = qE$

$$\boxed{\begin{aligned} \vec{\tau} &= \vec{p}\times\vec{E} \\ U &= -\vec{p}\cdot\vec{E} \end{aligned}}$$

SLN potential energy of a dipole.