# Chapter 24 - Electric Potential

A force is conservative? —
- thus, has an associated electrical potential energy
- Path independence (i.e. Gravitational force)
- Apply the principle of the conservation of mechanical energy.
  - height $\leftrightarrow$ equipotential surfaces

## Electric Potential Energy

Charged particles $\rightarrow$ Electrostatic force acting btw them $\rightarrow$ Assign an electric potential energy

$W = \vec{F} \cdot \vec{d}$ ; $\vec{F} = q\vec{E}$

system has a configuration $\rightarrow$ initial state, $i \Rightarrow U_i$
a change
final state, $f \Rightarrow U_f$

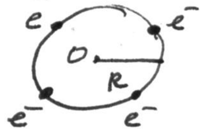
$\Delta U = U_f - U_i = -W$
Change in potential energy

Since electrostatic force is conservative $\Rightarrow$ path independence which means that work done on the particle is the same for all paths.

## Electric Potential

- The potential energy per unit charge $\left(\frac{U}{q}\right)$ is independent of the charge, $q$.
- Characteristic only of the electric field.

Example



$V$ at $O$?
$\vec{E}$ at $O$?

scalar

$V = -4 \times \frac{1}{4\pi\epsilon_0} \frac{e}{R}$ :)

$\vec{E} = 0$ : vector quantity
symmetry cancelled

$V = \frac{U}{q}$ : Electric Potential. A scalar not a vector
(OR Potential)
Potential energy per unit charge

- Electric Potential Difference btw two points, $\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \boxed{\frac{\Delta U}{q}} = \boxed{\frac{-W}{q}}$

SLN

Take $U_i = 0$ at infinity then $V$ at infinity is equal to zero.

$V = -\frac{W_\infty}{q}$ — work done by $\vec{E}$ on a charged particle to move from infinity to point $f$.

Unit of potential

$\frac{\text{Joule}}{\text{Coulomb}} \equiv 1 \text{ Volt}$

- Redefine the unit of $\vec{E}\left(\frac{N}{C}\right)$: $1 \frac{N}{C} = 1 \frac{N}{J/V} = 1 \frac{NV}{Nm} = 1 \, V/m$
- Work to move an electron through 1V is called one electron-volt (1eV)

Magnitude of the work: $q\Delta V \Rightarrow 1\,eV = e(1V) = (1.6 \times 10^{-19} C)(1 J/C)$

$\Rightarrow \boxed{1\,eV = 1.6 \times 10^{-19} J}$

# Work Done by an Applied Force

a charged particle, $q$ } in an $\vec{E}$ → experiences force } move from point $i$ to $f$ } Change in KE $\Delta K = K_f - K_i = W_{app} + W$

force: Applied one / From $\vec{E}$ itself } $W_{app}$: work done by the applied force; $W$: work done by $\vec{E}$

if $K_f = K_i \Rightarrow W_{app} = -W$

~~stationary~~ particle is stationary before and after move. } $\Delta U = -W$, $\Delta U = W_{app}$ } $\Delta U = U_f - U_i = \boxed{W_{app} = q\Delta V}$ — (+), (−) or zero

# Equipotential Surfaces

- Adjacent points that have the same electric potential form an equipotential surface.
- Work done on a particle on a given equipotential surface is zero. $-W = q\Delta V = 0 \rightsquigarrow \Delta V = 0$

SLN Fig. 24-2, Fig. 24-3

## ① Calculating the Potential from the Field

What is the potential difference btw two points ① and ② in an $\vec{E}$?

SLN Fig. 24-4 · Consider a positive charge $q_0$ moving btw points ① and ② ($i$, $f$) along the path shown

- $dW = \vec{F} \cdot d\vec{s}$, $dW = q\vec{E} \cdot d\vec{s}$ } Differential work } $W = q_0 \int_1^2 \vec{E} \cdot d\vec{s}$ (Total work) $\rightarrow \frac{W}{q_0} = \int_1^2 \vec{E} \cdot d\vec{s} \Rightarrow \boxed{V_f - V_i = -\int_1^2 \vec{E} \cdot d\vec{s}}$ $\quad \Delta V = \frac{-W}{q}$
- Since $\vec{F}$ is conservative, all paths btw 1-2 yield the same work.

## ② Potential due to a Point Charge

Consider a positive charged particle $q$ and a positive test charge, $q_0$ at point P at a distance R from the fixed charged particle. What is the potential at point P? SLN Fig. 24-6

$i$: $q_0$ at R; $f$: $q_0$ at ∞ } $V_i = V$, $V_f = V_\infty = 0$ } $V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$ } $0 - V_i = -\int_R^\infty E\,ds \underbrace{\cos\theta}_{1 \text{ since } \theta = 0°}$, $ds \sim dr$

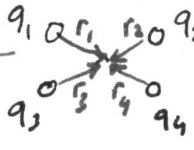
$\rightarrow V_i = V(\text{at } R) = +\int_R^\infty \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} dr \rightarrow \boxed{V = +\frac{1}{4\pi\varepsilon_0} \frac{q}{R}}$ → + charge produces +V; − charge produces −V

## ③ Potential due to a Group of Point Charges

Net Potential. Superposition principle. Potential is equal to sum of potential resulting from each charge at the given point

$\boxed{V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}}$ ($n$ ~ # charge)

Example square center




$V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \frac{q_4}{r_4}\right)$

④ Potential due to an Electric Dipole  $\uparrow \vec{p} \sim qd$

SLN Fig. 24-10 Potential at an arbitrary point P, $V = \sum_{i=1}^{2} V_i$

- At point P: + charge sets a +V, − charge sets a −V } $\underbrace{V = V_{(+)} + V_{(-)}}$ : Net potential

$$\Rightarrow V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{r_{(-)} - r_{(+)}}{r_{(-)}\, r_{(+)}}\right)$$

Since naturally occurring dipoles are small, $r \gg d$
$\Rightarrow r_{(-)} - r_{(+)} \sim d\cos\theta$
$r_{(-)}\, r_{(+)} \sim r^2$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2} \Rightarrow \boxed{V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}}$$ ; p: magnitude of electric dipole moment

SLN Fig. 24-11

⑤ Potential due to a Continuous Charge Distribution

Not point charges anymore. But Continuous charge distribution.

$q \longrightarrow V$ $\qquad dq \rightarrow dV \quad dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$

Line of Charge:

Then, integrate over the entire charge distribution to find the potential, V

non-conducting thin rod.

$$V = \int dV = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$$

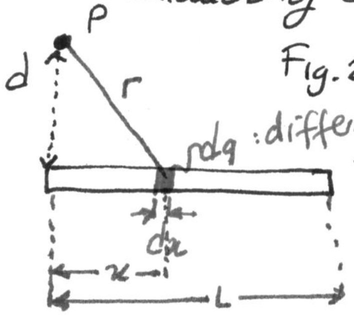



Fig. 24-12

What is V at point P? positive charge linear density, $\lambda$ length, L

Start by

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} \rightarrow V = \int dV$$ } $dq = \lambda dx$, $r = \sqrt{x^2+d^2}$

$$\Rightarrow V = \int_0^L \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(x^2+d^2)^{1/2}} \Rightarrow \boxed{V = \frac{1}{4\pi\varepsilon_0}\ln\left[\frac{L + (L^2+d^2)^{1/2}}{d}\right]}$$

Charged Disk:

non-conducting (plastic) disc

SLN Fig. 24-13


What is V at point P? surface charge density, $\sigma$

Start by

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} \rightarrow V = \int dV$$ } $dq = \sigma dA = \sigma 2\pi R' dR'$, $r = \sqrt{R'^2 + z^2}$

$$\Rightarrow V = \frac{1}{4\pi\varepsilon_0}\int_0^R \frac{\sigma 2\pi R' dR'}{\sqrt{R'^2+z^2}} \Rightarrow V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R'^2} - z\right)$$

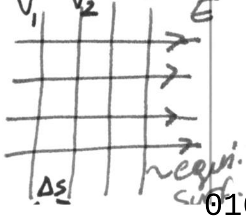
Calculating the Field from the Potential

SLN Fig. 24-14

$\frac{\Delta U}{q_0} = \Delta V$, $\Delta U = -W$
$W = \vec{F}\cdot\Delta\vec{s}$, $W = q_0\vec{E}\cdot d\vec{s}$ } $-q_0 dV$, $q_0\vec{E}\cdot d\vec{s}$ } $-q_0 dV = q_0 \underbrace{E\cos\theta}_{E_s} ds$ $\Rightarrow \boxed{E_s = \frac{-dV}{ds}}$ or $\frac{\partial V}{\partial s}$

in general $E_{x,y,z} = -\frac{\partial V}{\partial x,y,z}$

$E = \frac{-\overbrace{\Delta V}^{(V_2 - V_1)}}{\Delta s}$



# Electric Potential Energy of a System of Point Charges

SLN Fig.24-15

When we bring $q_2$ from infinity to a point near $q_1$, we must do a work since $q_1$ exerts electrostatic force on $q_2$.

That work should be equal to $q_2 V$, where $V$ is the potential that is $\underbrace{\text{created by } q_1}$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r} \Rightarrow \boxed{U = W = q_2 V = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}}$$

| $U$ | $q_1, q_2$ |
|---|---|
| (+) ← | same charges |
| (−) ← | opposite charges |