Lecture 10
Some Continuous Probability Distributions II
Lecture Information

Ceng272 *Statistical Computations* at May 03, 2010

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Contents

Normal Approximation to the Binomial I

- Poisson distribution can be used to approximate binomial probabilities when *n* is quite large and *p* is very close to 0 or 1.

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Normal Approximation to the Binomial I

- Poisson distribution can be used to approximate binomial probabilities when n is quite large and p is very close to 0 or 1.
- Normal distribution not only provide a very accurate approximation to binomial distribution when n is large and p is not extremely close to 0 or 1,
- But also provides a fairly good approximation even when n is small and p is reasonably close to $\frac{1}{2}$.

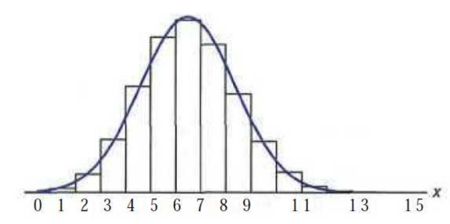

Figure: Normal approximation of $b(x; 15, 0.4)$.

Normal Approximation to the Binomial II

- **Theorem 6.2**:

If X is a binomial random variable with mean $\mu = np$ and variance $\sigma^2 = npq$, then the limiting form of the distribution of

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- $P(7 \le X \le 9)$

$$\sum_{x=7}^{9} b(x; 15, 0.4) = \sum_{x=0}^{9} b(x; 15, 0.4) - \sum_{x=0}^{6} b(x; 15, 0.4)$$

$$= 0.9662 - 0.6098 = 0.3564$$

$$\mu = np = 15 * 0.4 = 6, \ \sigma^2 = 15 * 0.4 * 0.6 = 3.6, \ \sigma = 1.897$$

$$z_1 = \frac{6.5 - 6}{1.897} = 0.26, \ and \ z_2 = \frac{9.5 - 6}{1.897} = 1.85$$

$$P(7 \le X \le 9) \approx P(0.26 < Z < 1.85) = P(Z < 1.85) - P(Z < 0.26)$$

$$= 0.9687 - 0.6026 = 0.3652$$

Normal Approximation to the Binomial III

$P(X = 4) = b(4; 15, 0.4) = 0.1268$

$$z_1 = \frac{3.5 - 6}{1.897} = -1.32, \ and\ z_2 = \frac{4.5 - 6}{1.897} = -0.79$$

$$P(X = 4) \approx P(3.5 < X < 4.5) = P(-1.32 < Z < -0.79)$$
$$= P(Z < -0.79) - P(Z < -1.32)$$

$$= 0.2148 - 0.0934 = 0.1214$$

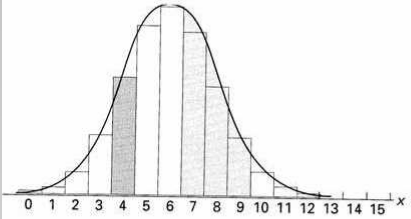

Figure: Normal approximation of $b(x; 15, 0.4)$ and $\sum_{x=7}^{9} b(x; 15, 0.4)$.

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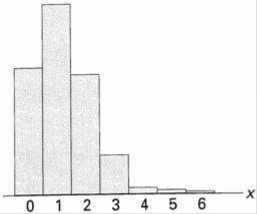

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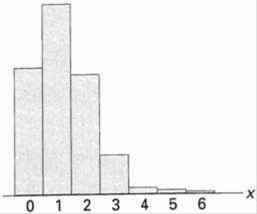

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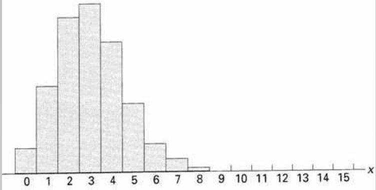

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Normal Approximation to the Binomial V

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- Let X be a binomial random variable with parameters n and p.
- Then X has approximately a normal distribution with mean $\mu = np$ and variance $\sigma^2 = npq$ and

$$P(X \leq x) = \sum_{k=0}^{x} b(k; n, p)$$

$$\approx \textit{area under normal curve to the left of } x + 0.5$$

$$= P\left(Z \leq \frac{x + 0.5 - np}{\sqrt{npq}}\right)$$

and the approximation will be good if np and nq are greater than or equal to 5.

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$$\mu = np = 100 * 0.4 = 40$$

$$\sigma = \sqrt{100 * 0.4 * 0.6} = 4.899$$

$$z_1 = \frac{29.5 - 40}{4.899} = -2.14$$

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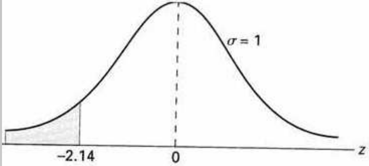

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$$\mu = np = 80 * \frac{1}{4} = 20$$

$$\sigma = \sqrt{80 * \frac{1}{4} * \frac{3}{4}} = 3.873$$

$$z_1 = \frac{24.5 - 20}{3.873} = 1.16,$$

$$z_2 = \frac{30.5 - 20}{3.873} = 2.71$$

$$P(25 \leq X \leq 30) = \sum_{x=25}^{30} b(x; 80, \frac{1}{4})$$

$$\approx P(1.16 < Z < 2.71)$$

$$= 0.9966 - 0.8770 = 0.1196$$

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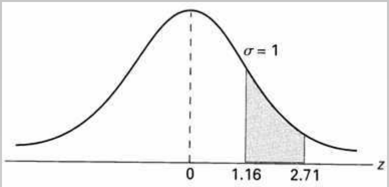

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The **gamma function** is defined by

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx, \ for \ \alpha > 0$$

with

$$\Gamma(n) = (n-1)(n-2)\ldots\Gamma(1),$$

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- also

$$\Gamma(n+1) = n\Gamma(n) = n!$$

$$\Gamma(1/2) = \sqrt{\pi} \; exception$$

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- If its density function is given by

$$f(x) = \begin{cases} \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}, & x > 0 \\ 0, & elsewhere \end{cases}$$

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  where $\alpha > 0$ and $\beta > 0$
- The mean and variance of the gamma distribution are (Proof is in Appendix A.28)

$$\mu = \alpha\beta \textit{ and } \sigma^2 = \alpha\beta^2$$

Gamma and Exponential Distributions III

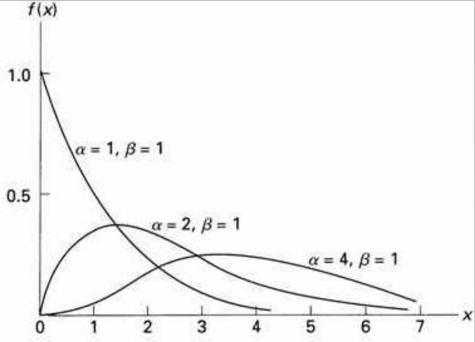

Figure: Gamma Distributions.

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- The exponential distribution has a single tail. The single parameter β determines the shape of the distribution.

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- The probability that the length of time until the first event will exceed x is the same as the probability that no Poisson events will occur in x.

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- Differentiate the cumulative distribution function for the exponential distribution

$$f(x) = \lambda e^{-\lambda x} \ \textit{with} \ \lambda = 1/\beta$$

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- The equipment failure often conforms to this Poisson process, β is called mean time between failures.
- Many equipment breakdowns do follow the Poisson process, and thus the exponential distribution does apply.
- Other applications include survival times in bio-medical experiments and computer response time.

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- Solution:

$$P(X \geq 2) = \sum_{x=2}^{5} b(x; 5, 0.2) = 1 - \sum_{x=0}^{1} b(x; 5, 0.2)$$

$$= 1 - 0.7373 = 0.2627$$

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- **Example 6.18**: Suppose that telephone calls arriving at a switchboard follow a Poisson process with an average of 5 calls coming per minute.
- What is the probability that up to a minute will elapse until 2 calls have come in to the switchboard?
- Solution:

The Poisson process applies with time until 2 Poisson events following a gamma distribution with $\beta = 1/5$ and $\alpha = 2$.
Let represent the time in minutes that transpires before 2 calls come.

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$$P(X \le 1) = 25 \int_0^1 x e^{-5x} dx$$

$$= 1 - e^{-5*1}(1 + 5) = 0.96$$

Gamma and Exponential Distributions IX

TABLE A.24 The Incomplete Gamma Function: $F(x;\alpha) = \int_0^x \frac{1}{\Gamma(\alpha)} y^{\alpha-1} e^{-y} dy$

x \ α	1	2	3	4	5	6	7	8	9	10
1	0.632	0.264	0.080	0.019	0.004	0.001	0.000	0.000	0.000	0.000
2	0.865	0.594	0.323	0.143	0.053	0.017	0.005	0.001	0.000	0.000
3	0.950	0.801	0.577	0.353	0.185	0.084	0.034	0.012	0.004	0.001
4	0.982	0.908	0.762	0.567	0.371	0.215	0.111	0.051	0.021	0.008
5	0.993	0.960	0.875	0.735	0.560	0.384	0.238	0.133	0.068	0.032
6	0.998	0.983	0.938	0.849	0.715	0.554	0.394	0.256	0.153	0.084
7	0.999	0.993	0.970	0.918	0.827	0.699	0.550	0.401	0.271	0.170
8	1.000	0.997	0.986	0.958	0.900	0.809	0.687	0.547	0.407	0.283
9		0.999	0.994	0.979	0.945	0.884	0.793	0.676	0.544	0.413
10		1.000	0.997	0.990	0.971	0.933	0.870	0.780	0.667	0.542
11			0.999	0.995	0.985	0.962	0.921	0.857	0.768	0.659
12			1.000	0.998	0.992	0.980	0.954	0.911	0.845	0.758
13				0.999	0.996	0.989	0.974	0.946	0.900	0.834
14				1.000	0.998	0.994	0.986	0.968	0.938	0.891
15					0.999	0.997	0.992	0.982	0.963	0.930

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Let $y = x/\beta$, and $x = \beta y$

$$\Rightarrow P(X \leq 60) = \int_0^6 \frac{y^4 e^{-y}}{\Gamma(5)} dy$$

$$= F(6;5) = 0.715, \; \textit{see Appendix A.24}$$

Chi-Squared Distribution

- **Chi-Squared Distribution** ($\alpha = \nu/2$ and $\beta = 2$, special gamma distribution): The continuous random variable X has a chi-squared distribution, with ν degrees of freedom, if its density function is given by

$$f(x;\nu) = \begin{cases} \frac{1}{2^{\nu/2}\Gamma(\nu/2)} x^{\nu/2-1} e^{-x/2}, & x > 0 \\ 0, & elsewhere \end{cases}$$

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- The chi-squared distribution plays a vital role in statistical inference.
- Topics dealing with sampling distributions, analysis of variance and nonparametric statistics involve extensive use of the chi-squared distribution.
- **Theorem 6.4**:

  The mean and variance of the chi-squared distribution are

$$\mu = \nu \text{ and } \sigma^2 = 2\nu$$

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- The lognormal only extends to ∞ but is 0 for $x < 0$.

Lognormal Distribution II

- **Theorem 6.5**:

The mean and variance of the lognormal distribution are

$$\mu = e^{\mu+\sigma^2/2} \text{ and } \sigma^2 = e^{2\mu+\sigma^2} * (e^{\sigma^2} - 1)$$

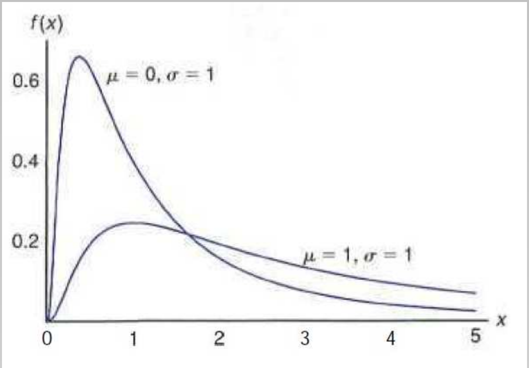

Figure: Lognormal Distributions.

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Here,we use the Φ notation to denote the cumulative distribution function of the standard normal distribution.

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- Solution:

$$P(Z < z_1) = 0.05 \Rightarrow z_1 = -1.645$$

$ln(x)$ has a normal distribution with $\mu = 5.149$ and $\sigma = 0.737$

$$\frac{ln(x) - 5.149}{0.737} = -1.645$$

$$\Rightarrow ln(x) = 0.737 * (-1.645) + 5.149 = 3.937$$

$$\Rightarrow x = 51.265$$

5% of the locomotives will have lifetime less than 51.265 thousand miles