Lecture 2

Introduction to Statistics and Data Analysis II

Lecture Information

Ceng272 *Statistical Computations* at February 22, 2010

Dr. Cem Özdoğan
Computer Engineering Department
Çankaya University

Contents

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- The small probability suggests that the process indeed have a long-run defective exceeding 5%.
- Probability aids in translation of sample information into conclusions.

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- Would the data set indicate that nitrogen is effective? We observed:
  - Four nitrogen observations are larger than any of the no-nitrogen observations (see underlined elements in Table 1).
  - Most of the no-nitrogen observations appear to be below the center of the data (see underlined element in Table 1).

Role of Probability III

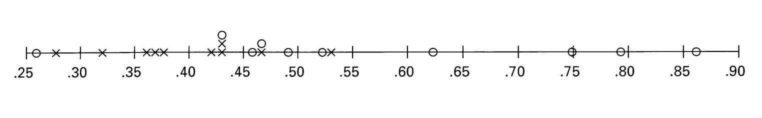

Figure: Stem weight data. (o: the with nitrogen data. x: the without nitrogen data.

Role of Probability III

Table: Observation of nitrogen influences.

No nitrogen	Nitrogen
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0.28	0.47
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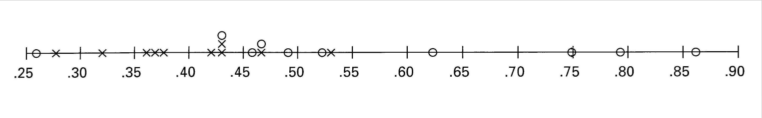

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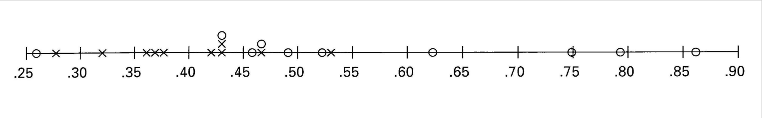

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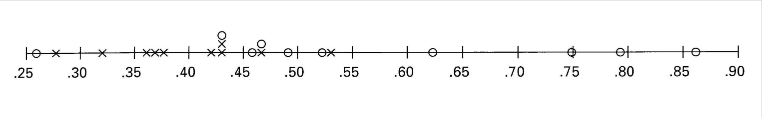

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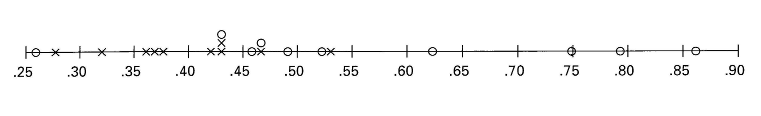

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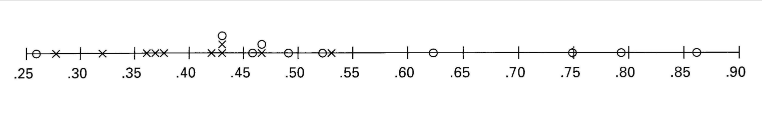

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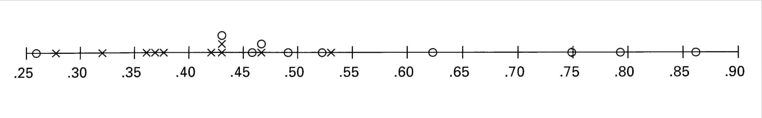
- That would be strong evidence that the use of nitrogen does have influence.



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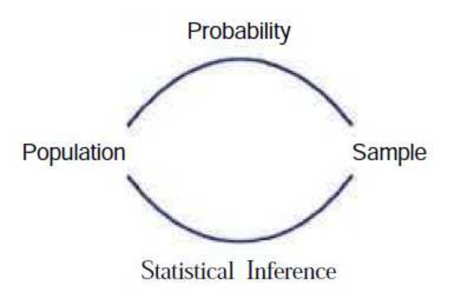

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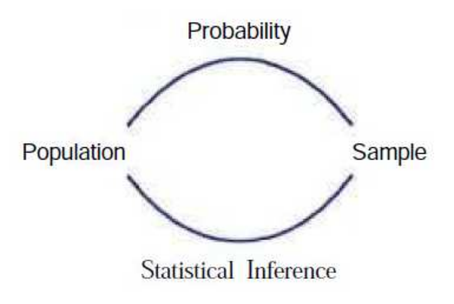
- For a probability problem, we can draw conclusions about characteristics of hypothetical data taken from the population based on known features of the population. (deductive in nature)



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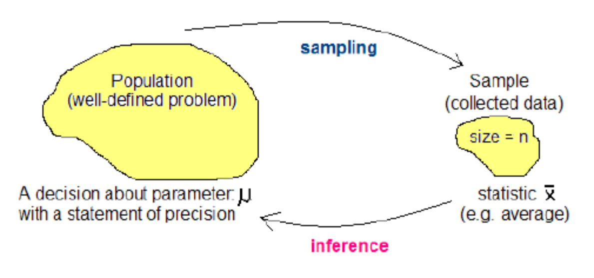

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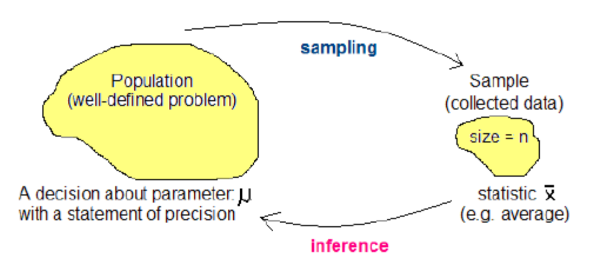

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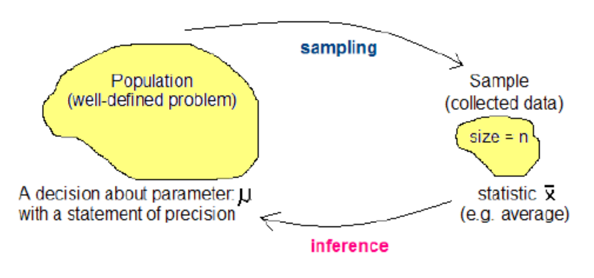

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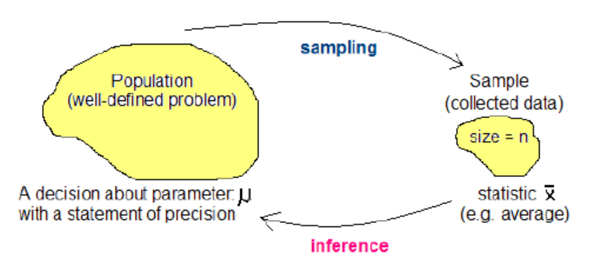

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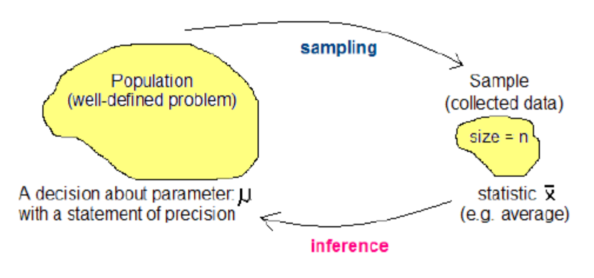
- Usually the above two procedures are called collectively statistical inference



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  - Biased sample confined the population and thus the inferences need to be confined to the limited population.

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    - Separate random samples of families could be chosen from each group.

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  - In a completely randomized design, 100 patients are assigned randomly to placebo and 100 to the active drug.

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- **Example 1.3**. A corrosion study to determine if coating of an aluminium reduces the amount of corrosion.

Table: Data for Example 1.3

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  - The corrosion data are averages of 2 specimens.

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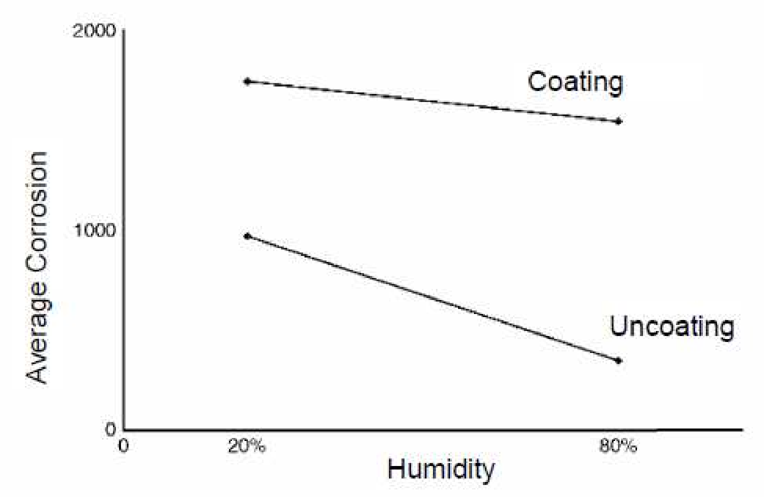

Figure: Corrosion results for Example 1.3.

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Random Assignment IV

- Consider the variability around the average:
  - The use of the chemical corrosion coating procedures appears to reduce corrosion if two corrosion values at each treatment combination are close together.
  - If each corrosion value is an average of two values that are widely dispersed, then this variability wash away any information we obtain.
- Three concepts are illustrated:
  - Random assignment of treatment combination to experiment units.
  - The use of sample average in summarizing sample information.
  - The need for consideration of measures variability in the analysis of sample sets.

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- **Sample Median -** middle value in the observations of ordered data set. It divides a data set into two equal parts, denoted by $\tilde{x}$;

$$\tilde{x} = \left\{ \begin{array}{cc} x_{(n+1)/2} & \text{if } n \text{ is odd}, \\ \frac{1}{2}(x_{n/2} + x_{n/2+1}) & \text{if } n \text{ is even, average of two middle observations} \end{array} \right\}$$

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- For example, if the data set is the following: 1.7, 2.2, 3.11, 3.9, and 14.7. The sample mean is 5.12 and the sample median is 3.9.

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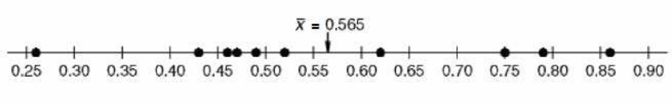

Figure: Sample mean as a centroid of the "with nitrogen" stem weight.

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Table: Observation of nitrogen influences.

No nitrogen	Nitrogen
0.32	**0.26**
0.53	0.43
0.28	0.47
0.37	0.49
0.47	0.52
0.43	0.75
0.36	0.79
0.42	**0.86**
0.38	0.62
0.43	0.46

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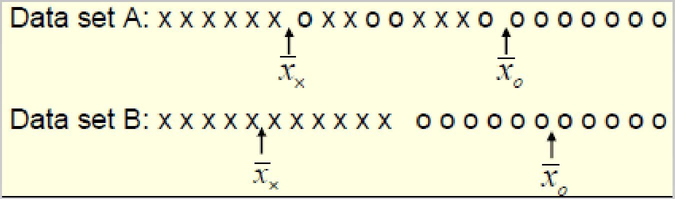

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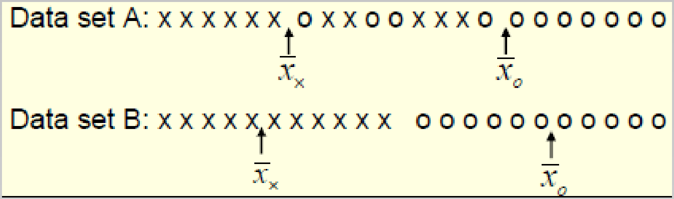

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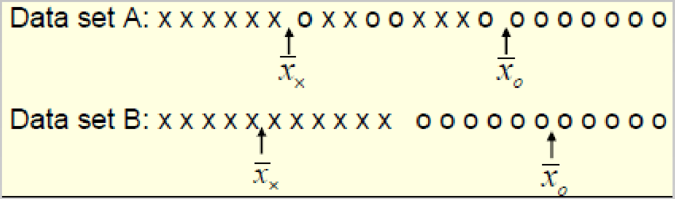

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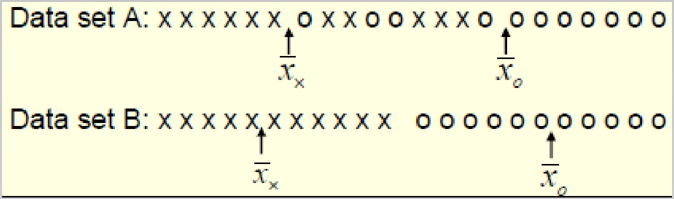

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- **Example 1.4**. An engineer is interested in testing the “bias” in a pH meter. Data are collected on the meter by measuring the pH of a neutral substance (pH = 7.0). A sample of size 10 is taken with results given by

  7.07 7.00 7.10 6.97 7.00 7.03 7.01 7.01 6.98 7.08

$$\bar{x} = 7.0205$$
$$\sigma^2 = 0.001939$$
$$\sigma = \sqrt{0.001939} = 0.0440$$

  with 9 degrees of freedom.

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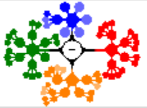

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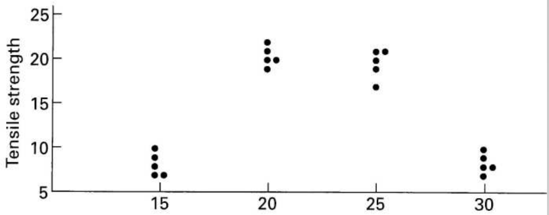

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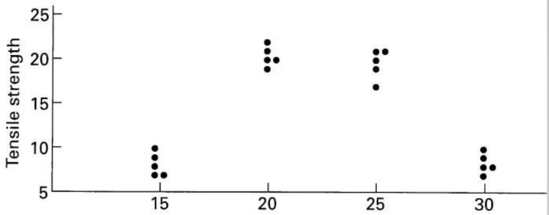

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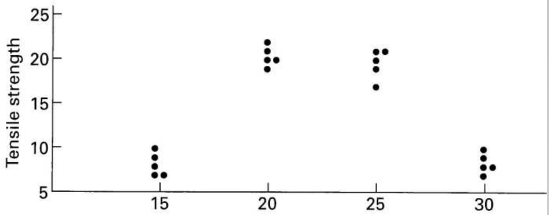

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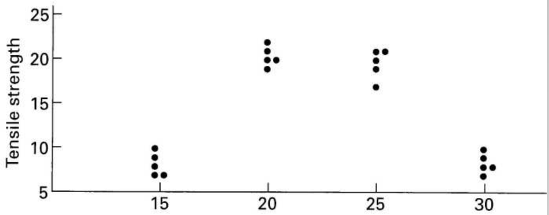

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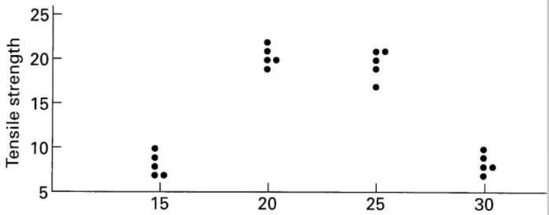

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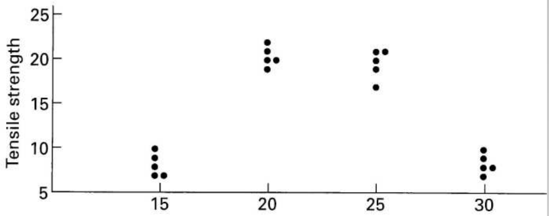

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- A selection of a model represents a fundamental assumption upon which the resulting statistical inference is based.
- Often, plots (graphics)can illustrate information that allows the results of the formal statistical inference to be better communicated to the scientist or engineer, and teach the analyst something not retrieved from the formal analysis.

Graphical Methods and Data Description I

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- A **Stem-and-leaf** plot, a combined tabular and graphic display, can be used to study the behavior of the mass statistical data.
- Example: the following table show the life of 40 car batteries

2.2	4.1	3.5	4.5	3.2	3.7	3.0	2.6
3.4	1.6	3.1	3.3	3.8	3.1	4.7	3.7
2.5	4.3	3.4	3.6	2.9	3.3	3.9	3.1
3.3	3.1	3.7	4.4	3.2	4.1	1.9	3.4
4.7	3.8	3.2	2.6	3.9	3.0	4.2	3.5

Figure: Table of Car Battery Life (in years).

Graphical Methods and Data Description II

Table: Stem (integer part)-and-Leaf (decimal part) Plot of Battery Life.

Stem	Leaf	Frequency
1	69	2
2	25669	5
3	0011112223334445567778899	25
4	11234577	8

Table: Double-Stem-and-Leaf Plot of Battery Life.

Stem	Leaf	Frequency
1.	69	2
2*	2	1
2.	5669	4
3*	001111222333444	15
3.	5567778899	10
4*	11234	5
4.	577	3

Graphical Methods and Data Description III

Table: Relative Frequency Distribution of Battery Life.

Class Interval	Class Midpoint	Frequency *f*	Relative Frequency
1.5-1.9	1.7	2	0.05
2.0-2.4	2.2	1	0.025
2.5-2.9	2.7	4	0.100
3.0-3.4	3.2	15	0.375
3.5-3.9	3.7	10	0.250
4.0-4.4	4.2	5	0.125
4.5-4.9	4.7	3	0.075

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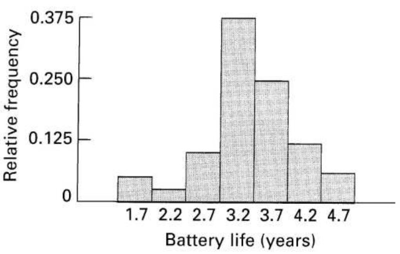

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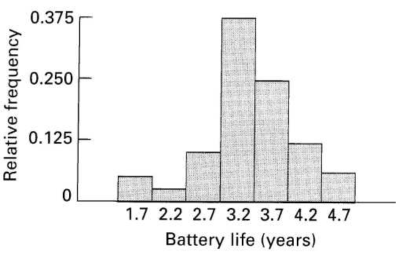

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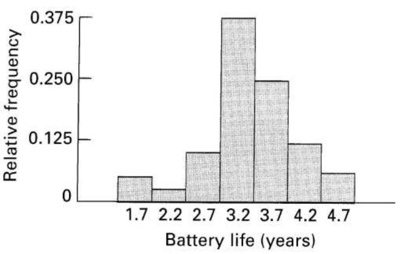

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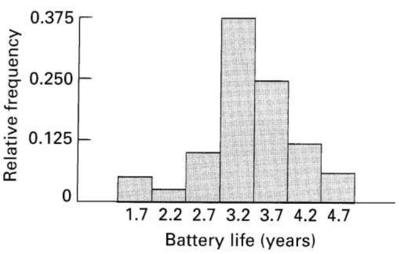

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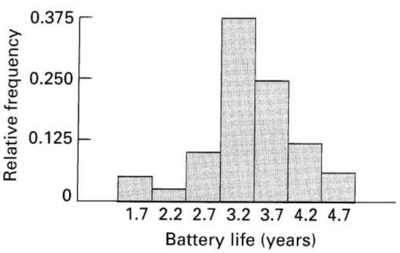

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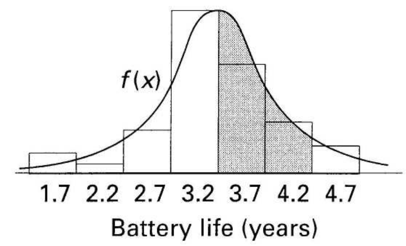

Figure: Estimating frequency distribution.

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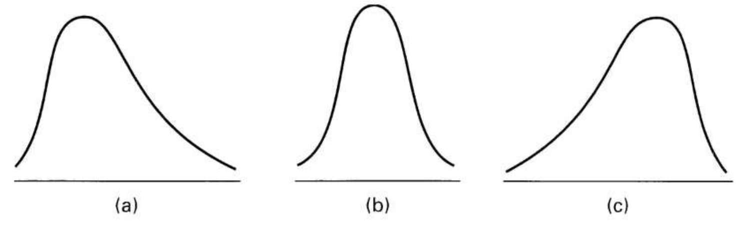

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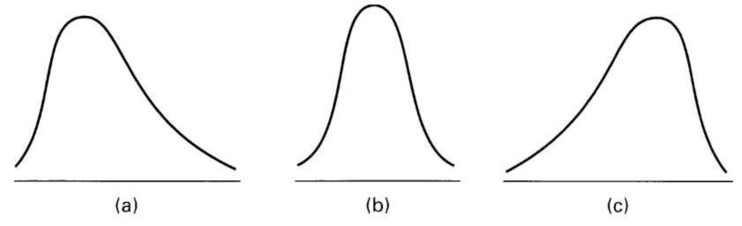

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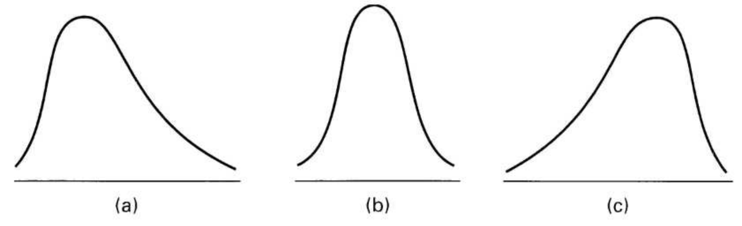

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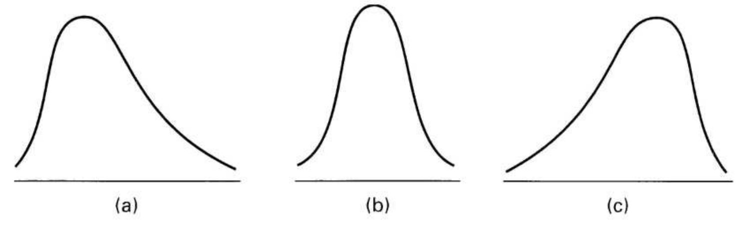

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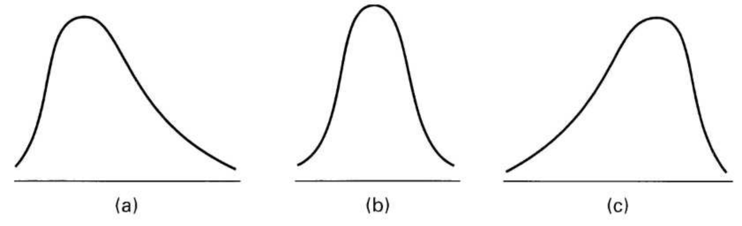

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