

i. Find the normal eqns for  $x = \alpha y + \beta$ , by using least-square approximation.

$$S = \sum (x_i - \alpha y_i - \beta)^2 \begin{cases} \frac{\partial S}{\partial \alpha} = 0 = \sum 2(x_i - \alpha y_i - \beta)(-y_i) \\ \frac{\partial S}{\partial \beta} = 0 = \sum 2(x_i - \alpha y_i - \beta)(-1) \end{cases}$$

then so-called normal eqns

$$\Rightarrow \begin{cases} \alpha \sum y_i^2 + \beta \sum y_i = \sum x_i y_i \\ \alpha \sum y_i + \beta N = \sum x_i \end{cases}$$

ii. Find the normal eqns for  $y = a x^b$ , by using least-square approximation

$$\ln y = \ln a + b \ln x \rightarrow Y = A + b X$$

$$S = \sum (Y - A - bX)^2 \begin{cases} \frac{\partial S}{\partial A} = 0 \\ \frac{\partial S}{\partial b} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} b \sum X_i^2 + A \sum X_i = \sum X_i Y_i \\ b \sum X_i + A N = \sum Y_i \end{cases}$$