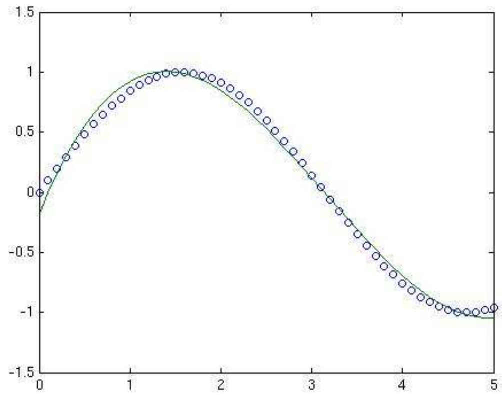

Figure 1: plot(x,y,'o',x,f,'-').

1 Interpolation and Curve Fitting

1. The MATLAB procedure for polynomial least-squares is *polyfit*. Study the following example;

```
x =(0:0.1:5)';  % x from 0 to 5 in steps of 0.1
y = sin(x);  % get y values
p = polyfit(x,y,3);  % fit a cubic to the data
f = polyval(p,x);  % evaluate the cubic on the x data
plot(x,y,'o',x,f,'-')  % plot y and its approximation f
```

   Solution:

```
>> x =(0:0.1:5)';
>> y = sin(x);
>> p = polyfit(x,y,3)
p =    0.0919   -0.8728    1.8936   -0.1880
>> f = polyval(p,x);
>> plot(x,y,'o',x,f,'-');
```

2. For the given data points;

x	Y
0.000	1.500
0.142	1.495
0.285	1.040
0.428	0.821
0.571	1.003
0.714	0.821
0.857	0.442
1.000	0.552

- to which we will fit $y(x) = \alpha e^{\beta x}$
  Hint: First, we should compute a new table with $z(x) = ln y(x)$

x	z
0.000	
0.142	
0.285	
0.428	
0.571	
0.714	
0.857	
1.000	

- Construct the normal equations
  Hints: $A = ln\alpha$ and $C = \beta$
- Solve these normal equations to find A and C
- Convert back to the original variables
- Plot Y vs x and y vs x then compare them.
- **Soln:** $y(x) = 1.561e^{-1.132x}$

Solution:

```
>> Y=[1.5 1.495 1.04 0.821 1.003 0.821 0.442 0.552]'
Y =
    1.5000
    1.4950
    1.0400
    0.8210
```

```
    1.0030
    0.8210
    0.4420
    0.5520
>> z=log(Y)
z =
    0.4055
    0.4021
    0.0392
   -0.1972
    0.0030
   -0.1972
   -0.8164
   -0.5942
```

x	z
0.000	0.4055
0.142	0.4021
0.285	0.0392
0.428	−0.1972
0.571	0.0030
0.714	−0.1972
0.857	−0.8164
1.000	−0.5942

$$
\begin{aligned}
&y(x) = \alpha e^{\beta x} \\
&lny(x) = ln\alpha + \beta x \\
&z = A + Cx \\
&S = \textstyle\sum_{i=1}^{N} (z_i - Cx_i - A)^2 \\
&\tfrac{\partial S}{\partial C} = 0 = \textstyle\sum_{i=1}^{N} 2(z_i - Cx_i - A)(-x_i) \\
&\tfrac{\partial S}{\partial A} = 0 = \textstyle\sum_{i=1}^{N} 2(z_i - Cx_i - A)(-1)
\end{aligned}
$$

Dividing each of these equations by -2 and expanding the summation, we get the so-called *normal equations*

$$
\begin{aligned}
C\sum x_i^2 + A\sum x_i &= \sum x_i z_i \\
C\sum x_i + AN &= \sum z_i
\end{aligned}
$$

```
>> x=[0 0.142 0.285 0.428 0.571 0.714 0.857 1]';
>> Y=[1.5 1.495 1.04 0.821 1.003 0.821 0.442 0.552]';
>> z=log(Y);
```

```
>> sum(x'*x)
ans =    2.8549
>> sum(x')
ans =    3.9970
>> sum(x'*z)
ans =   -1.4491
>> sum(z')
ans =   -0.9553
>> A=[ 2.8549 3.997; 3.997 8]
A =
    2.8549    3.9970
    3.9970    8.0000
>> B=[-1.4491 -0.9553]'
B =
   -1.4491
   -0.9553
>> X=uptrbk(A,B)
X =
   -1.1328
    0.4466
```

so; we obtained $C = -1.1328$ and $A = 0.4466$, we should convert back to the original variables

```
>> exp(0.4466)
ans =    1.5630
```

we have

$$z = 0.4466 - 1.1328x, \Rightarrow y = 1.563 * e^{-1.1328x}$$

For plotting;

```
>> y=1.5630*exp(-1.1328*x)
y =
    1.5630
    1.3308
    1.1317
    0.9625
    0.8185
    0.6961
    0.5920
    0.5035
>> plot(x,Y,'o',x,y,'-')
```

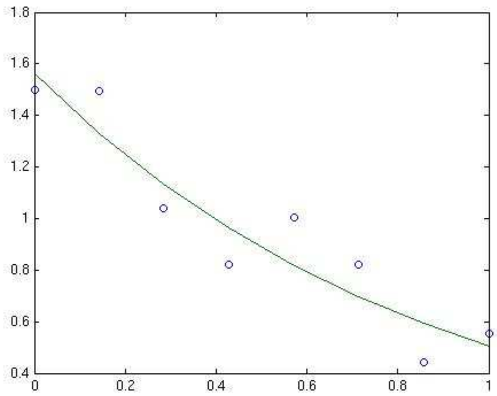

Figure 2: plot(x,Y,'o',x,y,'-').

3. Apply the procedure given in the first item by using the data set in the previous item.

   - **Hints:**
     - fit a cubic to the data
     - evaluate the cubic on the x data
   - Plot by $plot(x, Y,' o', x, f,' -')$
   - Compare this least-square polynomial with the function used in the previous item.

   Solution:

```
>> x=[0 0.142 0.285 0.428 0.571 0.714 0.857 1]';
>> Y=[1.5 1.495 1.04 0.821 1.003 0.821 0.442 0.552]';
>> p = polyfit(x,Y,3)
p =   -0.3476    0.9902   -1.6946    1.5518
>> f = polyval(p,x)
f =
    1.5518
    1.3301
    1.1412
    0.9806
    0.8423
```

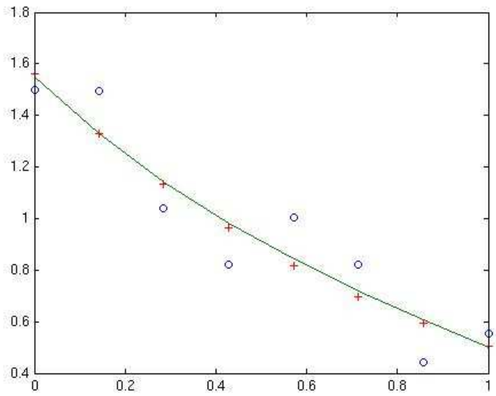

Figure 3: plot(x,Y,'o',x,f,'-',x,y,'+').

```
    0.7201
    0.6080
    0.4998
>> plot(x,Y,'o',x,f,'-');
>> plot(x,Y,'o',x,f,'-',x,y,'+')
```