Ceng 375 Numerical Computing Midterm Nov 30, 2007 12.40–14.30 Good Luck!

1. (40 pts) Consider the function:

 $f(x) = 2x - 6\log(x)$

Plot of the function is given at the following figure;



- i (20 pts) Use three iterations of Newton's method to estimate only one of the roots of this function. *Hint:* $\int \frac{1}{x} dx = \log(x)$
- ii (10 pts) Estimate the error in your answer to part i.
- iii (10 pts) Approximately how many iterations of the bisection method would have been required to achieve the same error?*Hint: Take the interval as ((initial+1)-initial)*

2. (40 pts) The following table and figure are given as the population growth of Turkey between years of 1927 and 2000



- i (10 pts) What is the relationship that the graph suggests? Use least squares method to find out the necessary parameters of this suggested formula.
- ii (5 pts) Estimate the population at the years of 1995, 2007 and 2010 with least squares method.
- iii (10 pts) Fit a cubic (P_3) polynomial to the given data.
- iv (5 pts) Estimate the population at the years of 1995, 2007 and 2010 with fitted polynomial.
- v (10 pts) Compare your results for both least squares and interpolated polynomial methods.

3. (40 pts) For the given data points;

$$\begin{array}{cccc}
x & y \\
\hline
2.1 & -12.4 \\
4.1 & 7.3 \\
\hline
7.1 & 10.1
\end{array}$$

- (a) (20 pts) Write out the Lagrangian polynomial from this table
 - i confirm that it reproduces the y's for each x-value.
 - ii interpolate with it to estimate y at x = 3.
 - iii extrapolate with it to estimate y at x = 8.
- (b) (10 pts) Suppose in previous item that the y-value for x = 4.1 is mistakenly entered as 7.2 rather than 7.3. Repeat the previous item with this incorrect value. How much difference does this make?
- (c) (10 pts) Expand the Lagrangian polynomials in the previous items to get the quadratics in the form $ax^2 + bx + c$. How different are the values for a,b, and c?