Ceng 375 Numerical Computing
Midterm
Nov 12, 2009 14.40–16.30
Good Luck!

Each question is 25 pts.

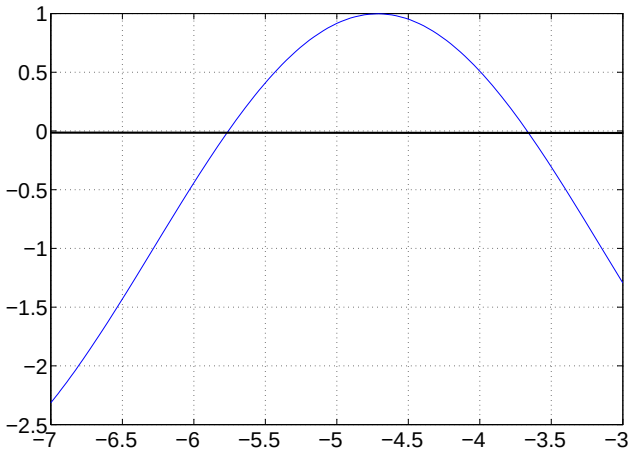
1. The following function is given

$$f(x) = 3 * sin(x) - e^x/4 - 1$$

This nonlinear equation ($f(x) = 0$) is solved by using four methods, namely *Bisection*, *Regula Falsi*, *Newton's*, *Muller's* methods. See the following MATLAB commands;

```
>> f = inline ( ' 2*sin( x) - exp ( x)/4 -1');
>> df = inline ( ' 2*cos( x) - exp ( x)/4');
>> fplot(f,[-7 -3]); grid on;
>> format short e
>> bisect(f,-7,-5,fzero(f,[-7 -5]),1e-5);
>> regula(f,-7,-5,fzero(f,[-7 -5]),1e-5,eps,20);
>> newton(f,df,-7,fzero(f,[-7 -5]),1e-5,eps,20);
>> muller(f,-7,-6,-5,fzero(f,[-7 -5]),1e-5,eps,20);
```

Plot of the function is given at the following figure;



Then, the following tables are obtained.

Table 1: Obtained **root** values at each iteration for all of four methods.

iteration	Bisection	Regula	Newton	Muller
1	-6.0000e+00	-5.5672e+00	-5.4650e+00	-5.7134e+00
2	5.5000e+00	-5.7373e+00	-5.8008e+00	-5.7604e+00
3	-5.7500e+00	-5.7575e+00	-5.7596e+00	-5.7591e+00
4	-5.8750e+00	-5.7590e+00	-5.7591e+00	-5.7591e+00
5	-5.8125e+00	-5.7591e+00	-5.7591e+00	-
6	-5.7812e+00	-5.7591e+00	-	-

Table 2: Obtained **function** values at each iteration for all of four methods.

iteration	Bisection	Regula	Newton	Muller
1	-4.4179e-01	3.1174e-01	4.5882e-01	7.8084e-02
2	4.1006e-01	3.7524e-02	-7.3051e-02	-2.2184e-03
3	1.5762e-02	2.8928e-03	-8.1042e-04	4.1882e-06
4	-2.0681e-01	2.0926e-04	-1.0968e-07	-4.0674e-11
5	-9.3753e-02	1.5061e-05	-2.2204e-15	-
6	-3.8525e-02	1.0836e-06	-	-

i Analyze these tables. Is the convergence sustained for the each methods? For the sustained ones; at which iteration and why?

ii If the exact value is given as $-5.7591e+00$, fill the following table for two methods. Choose two methods and use scientific notation with five significant figures.;

iteration	$Error_1$	$Error_2$	$Error\ Ratio_1$	$Error\ Ratio_2$
1				
2				
3				
4				
5				
6				

iii What can you say about the speed of convergences for each method?

iv Which method is the best one? Why?

2. Consider the same function with the previous question:

   i Find the root in the interval of $[-5, -3]$ with Newton's method. *Hint: Newton's method uses the algorithm:*

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

ii Estimate the error at the last iteration in your answer to part i. *Hint: To estimate the error, compute one more iteration.*

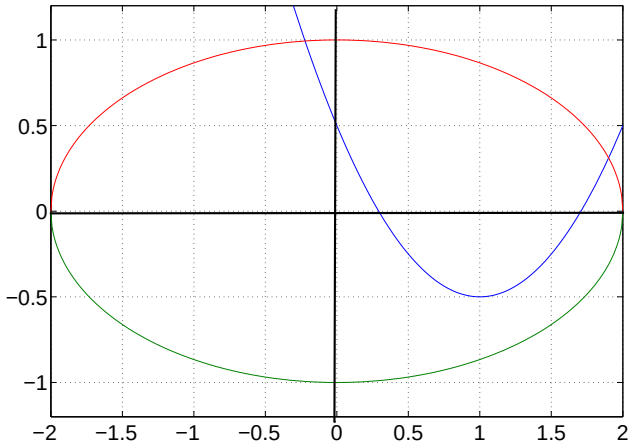
iii Approximately how many iterations of the bisection method would have been required to achieve the error value of $1e-5$?

3. Consider this pair of equations:

$$x^2 + 4y^2 = 4$$

$$y - x^2 + 2x = 0.5$$

Plot of the system is given at the following figure;



Solve this system by iteration. *Start with something like* $y =$??? *and proceed only* ***two*** *iterations.*

4. Consider the linear system;

$$\begin{aligned} x_1 - 2x_2 + 4x_3 &= 6 \\ 8x_1 - 3x_2 + 2x_3 &= 2 \\ -1x_1 + 10x_2 + 2x_3 &= 4 \end{aligned}$$

   i Solve this system by Gaussian elimination with pivoting. How many row interchanges are needed?

   ii What is the value of determinant?

   iii Obtain the LU decomposition of the system.

   iv Repeat without any row interchanges (only for the first item). Do you get the same results? Why?