Çankaya University
Mcs 331 Numerical Methods
Final Examination
Dec 31, 2014 09.30 – 11.30
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

DURATION: 120 minutes

◇ **Answer all the questions.**
◇ **Write the solutions explicitly and clearly. Use the numerical terminology.**
◇ **You are allowed to use Formulae Sheet.**
◇ **Calculator is allowed.**
◇ **You are not allowed to use any other electronic equipment in the exam.**
◇ **I declare hereby that I fulfilled the requirements for the attendance according to the University regulations and I accept that my examination will not be valid otherwise.**

Question	Grade	Out of
1		15
2		10
3		15
4		20
5		20
6		20
7		20
TOTAL		120

1. Answer the following questions, **choose only 4 of them.**

   i What are the advantages and disadvantages of numerical analysis?

   ii Describe the general working of a bracketing method. What are the assumptions for this family of methods?

   iii Describe truncation and round-off errors. Give example.

   iv Describe the concept of ill-condition. Give an example.

   v What does singularity mean for a matrix? Make a comparison of singular and nonsingular matrices.

   vi What information can be obtained from the condition number of a matrix?

   vii What are the differences between the interpolation and curve fitting?

2. Consider the matrix

$$A = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 1 & 3 \\ -3 & 0 & 5 \end{bmatrix}$$

i Get the inverse of the matrix through **either** Gaussian elimination **or** Gauss-Jordan method.

ii Check your result: $AA^{-1} = I$

3. For the given data points;

x	y
1	1.06
2	1.12
3	1.34
5	1.78

i construct the divided-difference table.

ii interpolate for $x = 4$.

iii extrapolate for $x = 5.5$.

4. A material is tested for cyclic fatigue failure whereby a stress (S), in MPa, is applied to the material and the number of cycles (N) needed to cause failure is measured. The results are in the table below.

X	Y
Cycles (N)	Stress (S)
1	1100
10	1000
100	925
1000	800
10000	625
100000	550
1000000	420

When a log-log plot of stress versus cycles is generated, the data trend **shows a linear relationship;** $Y' = aX' + b$. Use least-squares method to determine a best-fit equation for this data. **Hints:**

i Start by taking logarithms of the data.

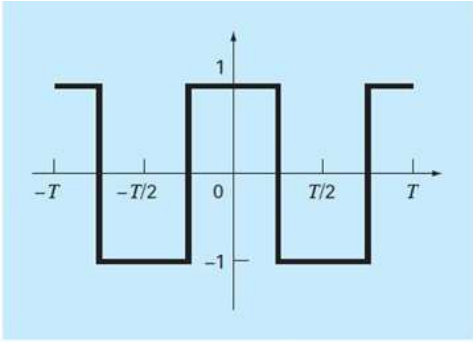
ii Construct the normal equations.

iii Find the values a and b.

iv Determine the best-fit equation for $Y(X)$.

5. Write the expression to economize the the Maclaurin series for e^{2x} with the precision 0.08 by using Chebyshev polynomials. Do not perform the calculation.

6. Use the Fourier series to approximate the square wave function (see Figure).

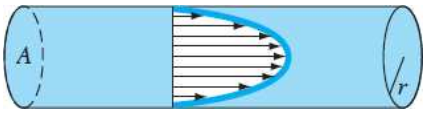


$$f(t) = \begin{cases} -1 & \text{-T/2 < t < -T/4;} \\ 1 & \text{-T/4 < t < T/4;} \\ -1 & \text{T/4 < t < T/2.} \end{cases}$$

i Find the Fourier coefficients. Do not evaluate the integrals. **Hint:** Odd or even function?

ii Write the Fourier series expansion for this function up to 2^{nd} term.

7. If the velocity distribution of a fluid flowing through a pipe is known (see Figure), the flow rate Q (that is, the volume of water passing through the pipe per unit time) can be computed by $Q = \int \upsilon dA$, where υ is the velocity and A is the pipe's cross-sectional area. For a circular pipe, $A = \pi r^2$ and $dA = 2\pi r dr$. Therefore, $Q = \int_0^r \upsilon(2\pi r)dr$, where r is the radial distance measured outward from the center of the pipe.



If the velocity distribution is given by

$$\upsilon = 2(1 - \frac{r}{r_0})^{1/6}$$

where r_0 is the total radius (in this case, 3 cm), compute Q using the *Composite Trapezoidal Rule.*

r_i	q_i
0.0000	0.0000
0.5000	
3.0000	

i First fill the table within the four digit accuracy.

ii Approximate the integral with a step size of $h = 0.5$.

iii Approximate the integral with a step size of $h = 1.0$.

iv Estimate the *error* in your answers;

- Exact value of the integral is 44.7419. Find the errors for parts ii and iii.
- Also use the global error formula to find the errors for parts ii and iii.
- Analyze and compare these error values.